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A Revision to Gödel's Incompleteness Theorem by Neutrosophy

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Abstract: According to Smarandache's neutrosophy, the Gödel's incompleteness theorem contains the truth, the falsehood, and the indeterminacy of a statement under consideration. It is shown in this paper that the proof of Gödel's incompleteness theorem is faulty, because all possible situations are not considered (such as the situation where from some axioms wrong results can be deducted, for example, from the axiom of choice the paradox of the doubling ball theorem can be deducted; and many kinds of indeterminate situations, for example, a proposition can be proved in 9999 cases, and only in 1 case it can be neither proved, nor disproved). With all possible situations being considered with Smarandache's neutrosophy, the Gödel's Incompleteness theorem is revised into the incompleteness axiom: Any proposition in any formal mathematical axiom system will represent, respectively, the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-}0, 1^{+}[$. Considering all possible situations, any possible paradox is no longer a paradox. Finally several famous paradoxes in history, as well as the so-called unified theory, ultimate theory, $\cdots$, etc. are discussed.

Key words: Smarandache's Neutrosophy, Gödel's Incompleteness theorem, Incompleteness axiom, paradox, unified theory.

The most celebrated results of Gödel are as follows.

Gödel's First Incompleteness Theorem: *Any adequate axiomatizable theory is incomplete.*

Gödel's Second Incompleteness Theorem: *In any consistent axiomatizable theory which can encode sequences of numbers, the consistency of the system is not provable in the system.*

In literature, the Gödel's incompleteness theorem is usually stated by *any formal mathematical axiom system is incomplete, because it always has one proposition that can neither be proved, nor disproved.*

Gödel's incompleteness theorem is a significant result in the history of mathematical logic, and has greatly influenced to mathematics, physics and philosophy among others. But, any

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theory cannot be the ultimate truth. Accompanying with the science development, new theories will replace the old ones. That is also for the Gödel's incompleteness theorem. This paper will revise the Gödel's Incompleteness theorem into the incompleteness axiom with the Smarandache's neutrosophy.

§1. An Introduction to Smarandache's Neutrosophy

Neutrosophy is proposed by F.Smarandache in 1995. *Neutrosophy* is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

This theory considers every notion or idea $\langle A\rangle$ together with its opposite or negation $\langle Anti-A\rangle$ and the spectrum of *neutralities* $\langle Neut-A\rangle$, i.e., notions or ideas located between the two extremes, supporting neither $\langle A\rangle$ nor $\langle Anti-A\rangle$). The $\langle Neut-A\rangle$ and $\langle Anti-A\rangle$ ideas together are referred to as $\langle Non-A\rangle$.

Neutrosophy is the base of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics used in engineering applications, especially for software and information fusion, medicine, military, cybernetics and physics, etc..

Neutrosophic Logic is a general framework for unification of existent logics, such as the fuzzy logic, especially intuitionistic fuzzy logic, paraconsistent logic, intuitionistic logic,$\cdots$, etc.. The main idea of Neutrosophic Logic (NL) is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-}0,1^{+}[$ without necessarily connection between them.

More information on Neutrosophy may be found in references [1-3].

§2. Some Errors in the Proof of Gödel's Incompleteness Theorem

It has been pointed out some errors in the proofs of Gödel's first and second incompleteness theorems in the reference [4]. This paper will again show that the proof of Gödel's incompleteness theorems contain some errors, but from other point of view. It will be shown that in the proof of Gödel's incompleteness theorem, all possible situations are not considered.

First, in the proof, the following situation is not considered: *wrong results can be deduced from some axioms*. For example, from the axiom of choice a paradox, the doubling ball theorem, can be deduced, which says that a ball of volume 1 can be decomposed into pieces and reassembled into two balls both of volume 1. It follows that in certain cases, the proof of Gödel's incompleteness theorem may be faulty.

Second, in the proof of Gödel's incompleteness theorem, only four situations are considered, that is, one proposition can be proved to be true, cannot be proved to be true, can be proved to be false, cannot be proved to be false and their combinations such as one proposition can neither be proved to be true nor be proved to be false. But those are not all possible situations. In fact, there may be many kinds of indeterminate situations, including it can be proved to be true in some cases and cannot be proved to be true in other cases; it can be proved to be false in some cases and cannot be proved to be false in other cases; it can be proved to be true in some cases and can be proved to be false in other cases; it cannot be proved to be true in some cases and cannot be proved to be false in other cases; it can be proved to be true in some cases and can neither be proved to be true, nor be proved to be false in other cases; and so on.

Because so many situations are not considered, we may say that the proof of Gödel's incompleteness theorem is faulty, at least, is not one with all sided considerations.

In order to better understand each case, we consider an extreme situation where one proposition as shown in Gödel's incompleteness theorem can neither be proved, nor disproved. It may be assumed that this proposition can be proved in 9999 cases, only in 1 case it can neither be proved, nor disproved. We will see whether or not this situation has been considered in the proof of Gödel's incompleteness theorem.

Some people may argue that, this situation is equivalent to that of a proposition can neither be proved, nor disproved. But the difference lies in the distinction between the part and the whole. If one case may represent the whole situation, many important theories cannot be applied. For example the general theory of relativity involves singular points; the law of universal gravitation does not allow the case where the distance r is equal to zero. Accordingly, whether or not one may say that the general theory of relativity and the law of universal gravitation cannot be applied as a whole? Similarly, the situation also cannot be considered as the one that can be proved. But, this problem may be easily solved with the neutrosophic method.

Moreover, if we apply the Gödel's incompleteness theorem to itself, we may obtain the following possibility: *in one of all formal mathematical axiom systems, the Gödel's incompleteness theorem can neither be proved, nor disproved.*

If all possible situations can be considered, the Gödel's incompleteness theorem can be improved in principle. But, with our boundless universe being ever changing and being extremely complex, it is impossible considering all possible situations. As far as considering all possible situations is concerned, the Smarandache's neutrosophy is a quite useful way, and possibly the best. Therefore this paper proposes to revise the Gödel's incompleteness theorem into the incomplete axiom with Smarandache's neutrosophy.

§3. **The Incompleteness Axiom**

Considering all possible situations with Smarandache's neutrosophy, one may revise the Gödel's Incompleteness theorem into the incompleteness axiom following.

Any proposition in any formal mathematical axiom system will represent the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-}0, 1^{+}[$, *respectively.*

§4. **Several Famous Paradoxes in History**

The proof of Gödel's incompleteness theorem has a close relation with some paradoxes. However, after considering all possible situations, any paradox may no longer be a paradox.

Now we discuss several famous paradoxes in history.

Example 1. *The Barber paradox, one of Russell's paradoxes.*

Consider all men in a small town as members of a set. Now imagine that a barber puts up a sign in his shop that reads *I shave all those men, and only those men, who do not shave themselves.* Obviously, we may divide the set of men in this town into two subsets, those who shave themselves, and those who are shaved by the barber. To which subset does the barber himself belong? The barber cannot belong to the first subset, because if he shaves himself, he will not be shaved by the barber, or by himself; he cannot not belong to the second subset as well, because if he is really shaved by the barber, or by himself, he will not be shaved by the barber.

Now we will see from where comes the contradiction.

The contradiction comes from the fact that the barber's rule does not take all possible situations into consideration.

First, we should divide the set of men in this town into three subsets, those who shave themselves, those who are shaved by the barber, and those who neither shave themselves, nor are shaved by the barber. This contradiction can be avoided by the neutrosophy as follows. If the barber belongs to the third subset, no contradiction will appear. For this purpose, the barber should declare himself that he will be the third kind of person, and from now on, he will not be shaved by anyone; otherwise, if the barber's mother is not a barber, he can be shaved by his mother.

Second, the barber cannot shave all men in this town. For example, the barber cannot shave those who refuse to be shaved by the barber. Therefore, if the barber is the one who cannot shave himself and "who refuse to be shaved by the barber" , no contradiction will occur.

There also exist indeterminate situations to avoid the contradiction. The barber may say: *If I meet men from another universe, I will shave myself, otherwise I will not shave myself.*

Example 2. *Liar's paradox, another Russell's paradox.*

Epimenides was a Cretan who said that *all Cretans are liars*. Is this statement true or false? If this statement is true, he (a Cretan) is a liar, therefore, this statement is false; if this statement is false, that means that he is not a liar, this statement will be true. Therefore, we always come across a contradiction.

Now we will see from where comes the contradiction.

First, here the term "liar" should be defined. Considering all possible situations, a "liar" can be one of the following categories: those whose statements are all lies; those whose statements are partly lies, and partly truths; those whose statements are partly lies, partly truths and sometimes it is not possible to judge whether they are truths or lies. For the sake of convenience, at this movement we do not consider the situation where it is not possible to judge whether the statements are true or false.

Next, the first kind of liar is impossible, i.e., a Cretan could not be a liar whose statements are all lies. This conclusion can not be reached by deduction, instead, it is obtained through experience and general knowledge. With the situation where a liar's statements are partly truths, and partly lies, Epimenides' statement *all Cretans are liars*, will not cause any contradiction. According to the definitions of liar of the second category and the fact that Epimenides' statements could not be all lies, this particular statement of Epimenides' can be true and with his other statements being possibly lies, Epimenides may still be a liar.

This contradiction can be avoided by the neutrosophy as follows.

For this statement of *all Cretans are liars*, besides true or false, we should consider the situation where it is not possible to judge whether the statement is true or false. According to this situation, this *Russell's paradox* can be avoided.

Example 3. *Dialogue paradox.*

Considering the following dialogue between two persons A and B.

A: *what B says is true.*

B: *what A says is false.*

If the statement of A is true, it follows that the statement of B is true, that is, the statement what A says is false is true, which implies that the statement of A must be false. We come to a contradiction.

On the other hand, if the statement of A is false, it follows that the statement of B must be false, that is, the statement what A says is false is false, which implies that the statement of A must be true. We also come to a contradiction.

So the statement of A could neither be true nor false.

Now we will see that how to solve this contradiction.

It should be noted that, this dialogue poses a serious problem. If A speaks first, before B says anything, how can A know whether or not what B says is true? Otherwise, if B speaks first, B would not know whether what A says is true or false. If A and B speak at the same time, they would not know whether the other's statement is true or false.

For solving this problem, we must define the meaning of *lie*. In general situations a *lie* may be defined as follows:

with the knowledge of the facts of cases, a statement does not show with the facts.

But in order to consider all possible situations, especially those in this dialogue, another definition of lie must be given. For the situation when one does not know the facts of the case, and one makes a statement irresponsibly, can this statement be defined as a lie? There exist two possibilities: *it is a lie, and it is not a lie.* For either possibility, the contradiction can be avoided.

Consider the first possibility, i.e., it is a lie.

If A speaks first, before B makes his statement, it follows that A does not know the facts of the case, and makes the statement irresponsibly, it is a lie. Therefore the statement of A is false. B certainly also knows this point, therefore B's statement: what A says is false is a truth.

Whereas, if B speaks first before A makes his statement, it follows that B does not know the facts of the case, and makes the statement irresponsibly, it is a lie. Therefore the statement of B is false. A certainly also knows this point, therefore A's statement: *what B says is true* is false.

If A and B speak at the same time, it follows that A and B do not know the facts of the case, and make their statements irresponsibly, these statements are all lies. Therefore, the statements of A and B are all false.

Similarly, consider the second possibility, i.e., it is not a lie, the contradiction can be also avoided.

If we do not consider all the above situations, what can we do? With a lie detector! The results of the lie detector can be used to judge whose statement is true, whose statement is false.

§5. On the So-Called Unified Theory, Ultimate Theory and So on

Since Einstein proposed the theory of relativity, the so-called unified theory, ultimate theory and so on have made their appearance.

Not long ago, some scholars pointed out that if the physics really has the unified theory, ultimate theory or theory of everything, the mathematical structure of this theory also is composed by the finite axioms and their deductions. According to the Gödel's incompleteness theorem, there inevitably exists a proposition that cannot be derived by these finite axioms and their deductions. If there is a mathematical proposition that cannot be proved, there must be some physical phenomena that cannot be forecasted. So far all the physical theories are both inconsistent, and incomplete. Thus, the ultimate theory derived by the finite mathematical principles is impossible to be created.

The above discussion is based on the Gödel's incompleteness theorem. With Smarandache's neutrosophy and the incompleteness axiom, the above discussion should be revised.

For example, the proposition *this theory is the ultimate theory* should represent respective the truth (T), the falsehood (F) and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^-0, 1^+[$.

Now we discuss the proposition *Newton's law of gravity is the ultimate theory of gravitation* (Proposition A).

According to the Gödel's incompleteness theorem, the ultimate theory is impossible, therefore, the above proposition is 0% true, 0% indeterminate, and 100% false. It may be written as $(0, 0, 1)$.

While according to the incomplete axiom, we may say that the Proposition A is 16.7% true, 33.3% indeterminate, and 50% false. It may be written as $(0.167, 0.333, 0.500)$. The reason for this sentence is on the following.

Consider the containing relation between the ultimate theory of gravitation and Newton's law of gravity. According to the incompleteness axiom, the proposition the ultimate theory of gravitation contains Newton's law of gravity (Proposition B) should represent respective the truth (T), the falsehood (F) and the indeterminacy (I). For the sake of convenience, we may assume that $T = I = F = 33.3\%$.

If the Proposition B is equivalent to the Proposition A , the Proposition A also is 33.3% true, 33.3% indeterminate, and 33.3% false. But they are not equivalent. Therefore we have to see how the ultimate theory of gravitation contains Newton's law of gravity. As is known, to establish the field equation of the general theory of relativity, one has to do a series of mathematical reasoning according to the principle of general covariance and so on, with Newton's law of gravity as the final basis. Suppose that the ultimate theory of gravitation is similar to the general theory of relativity, it depends upon some principle and Newton's law of gravity. Again this principle and Newton's law of gravity are equally important, they all have the same share of truthfulness, namely 16.7% (one half of 33.3%), but the 16.7% shared by this principle may be added to 33.3% for falsehood. Therefore, the Proposition A is 16.7% true, 33.3% indeterminate, and 50% false. It may be written as $(0.167, 0.333, 0.500)$.

This conclusion indicates that Newton's law of universal gravitation will continue to occupy a proper position in the future gravitational theory.

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