

# Reconstruction of ghost scalar fields

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In literature, a large number of approaches have been done to reconstruct the potential and dynamics of the scalar fields by establishing a connection between holographic/Ricci/new agegraphic/ghost energy density and a scalar field model of dark energy. In most of these attempts, the analytical form of the potentials in terms of the scalar field have not been reconstructed due to the complexity of the equations involved. In the present work, we establish a correspondence between ghost dark energy and quintessence, tachyon and dilaton scalar field models in anisotropic Bianchi type-I universe to reconstruct the dynamics of these scalar fields.

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## I. INTRODUCTION

Based on the recent observational evidences: SNe-Ia[1], WMAP[2], SDS[3], X-Ray[4] and Planck-2013[5], it is accepted that the universe has a phase transition from decelerating to accelerating and expands with accelerating velocity due to the presence of an unknown mysterious component namely the dark energy. Investigating the origin and the nature of this enigmatic component has been one of the great challenges in modern cosmology.

Ghost dark energy[6–12] is one of the interesting models of dark energy and was recently proposed. Among plenty of models, the so called Veneziano ghost dark energy[13] is proposed to discuss the U(1)A problem in low-energy effective theory of QCD[14–16], but it is completely decoupled from the physical sector[17–19]. The Veneziano ghost dark energy seems to be unphysical in the quantum field theory in Minkowski spacetime, but exhibits an important non-trivial physical influence in the expanding universe and this remarkable effect gives rise to a vacuum energy density  $\rho_D \sim H\Lambda_{QCD}^3 \sim (10^{-3}eV)^4$  (with  $H \sim 10^{-33}eV$  and  $\Lambda_{QCD} \sim 100eV$  we have the right magnitude for the force accelerating the Universe today)[20]. This numerical coincidence means that the ghost dark energy model gets rid of fine-tuning problem.

On the other hand, scalar fields can be regarded as an effective description of the dark Universe and naturally arise in particle physics including the String/M theory and super-symmetric field theories, hence scalar fields are expected to reveal the dynamical mechanism and the nature of the dark Universe[21]. Fundamental theories such as string/M theory provide many possible scalar field candidates, but they do not predict its potential  $V(\phi)$  uniquely[21].

In the present work, we are interested in that if we consider the ghost dark energy model as the underlying theory of dark energy, how the low-energy effective scalar field model can be used to describe it. On this purpose, we reconstruct the potential and the dynamics of scalar field models including quintessence, tachyon and dilaton

according to the results we obtained for the Ghost dark energy. We can establish a correspondence between the ghost dark energy and scalar field models, and describe ghost dark energy in this case effectively by making use of these scalar fields.

## II. GHOST DARK ENERGY SCENARIO

The metric representation of the Bianchi type-I spacetime is given as

$$ds^2 = -dt^2 + A^2(t)dx^2 + B^2(t)dy^2 + C^2(t)dz^2, \quad (1)$$

where  $A$ ,  $B$  and  $C$  are the cosmic scale factors along the  $x$ ,  $y$  and  $z$  axes, respectively, and they measure the expansion of the universe[22]. For the case  $A = B = C$ , the Bianchi type-I model reduces to the flat Friedmann-Robertson-Walker spacetime.

We consider the following energy-momentum tensor

$$T_{\nu}^{\mu} = T_{(m)\nu}^{\mu} + T_{(e)\nu}^{\mu} + T_{(r)\nu}^{\mu}. \quad (2)$$

Here, the superscripts  $m$ ,  $e$  and  $r$  denote dark matter, ghost dark energy and radiation, respectively. The corresponding energy-momentum tensors are defined as

$$T_{(m)\nu}^{\mu} = (-\rho_m, p_m, p_m, p_m), \quad (3)$$

$$T_{(e)\nu}^{\mu} = (-\rho_e, p_e^x, p_e^y, p_e^z), \quad (4)$$

$$T_{(r)\nu}^{\mu} = (-\rho_r, p_r^x, p_r^y, p_r^z). \quad (5)$$

We can introduce skewness parameters to parameterize these anisotropies. The skewness parameters for anisotropic ghost dark energy and anisotropic radiation can be defined as[23]

$$\xi_1 = \frac{p_e^x - p_e^y}{3\rho_e}, \quad (6)$$

$$\xi_2 = \frac{p_e^z - p_e^x}{3\rho_e}, \quad (7)$$

$$\zeta_1 = \frac{p_r^z - p_r^x}{3\rho_r}, \quad (8)$$

$$\zeta_2 = \frac{p_r^z - p_r^x}{3\rho_r}. \quad (9)$$

On the other hand, the corresponding equations of states for dark matter, ghost dark energy and dark radiation, respectively, are

$$p_m = \rho_m \omega_m, \quad p_e^i = \rho_e \omega_e^i, \quad p_r^i = \rho_r \omega_r^i, \quad (10)$$

where  $i = (x, y, z)$ . Now, using these definitions and skewness parameters  $\xi_1, \xi_2, \zeta_1, \zeta_2$  we can write[23]

$$T_{(e)\nu}^\mu = [-\rho_e, \rho_e \omega_e, \rho_e(\omega_e + 3\xi_1), \rho_e(\omega_e + 3\xi_2)], \quad (11)$$

$$T_{(r)\nu}^\mu = [-\rho_r, \rho_r \omega_r, \rho_r(\omega_r + 3\zeta_1), \rho_r(\omega_r + 3\zeta_2)]. \quad (12)$$

By making use of equations (11) and (12) in the definition (2) one can find

$$\begin{aligned} T_\nu^\mu = & [-\rho_m, p_m, p_m, p_m] \\ & + [-\rho_e, \rho_e \omega_e, \rho_e(\omega_e + 3\xi_1), \rho_e(\omega_e + 3\xi_2)] \\ & + [-\rho_r, \rho_r \omega_r, \rho_r(\omega_r + 3\zeta_1), \rho_r(\omega_r + 3\zeta_2)]. \end{aligned} \quad (13)$$

Next, the corresponding field equations are[22]

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{A}\dot{C}}{AC} = \rho_m + \rho_e + \rho_r, \quad (14)$$

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} = -\rho_m \omega_m - \rho_e \omega_e - \rho_r \omega_r, \quad (15)$$

$$\begin{aligned} \frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} = & -\rho_m \omega_m - \rho_e(\omega_e + 3\xi_1) \\ & -\rho_r(\omega_r + 3\zeta_1), \end{aligned} \quad (16)$$

$$\begin{aligned} \frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = & -\rho_m \omega_m - \rho_e(\omega_e + 3\xi_2) \\ & -\rho_r(\omega_r + 3\zeta_2). \end{aligned} \quad (17)$$

Also, the continuity equations,  $T_{;\nu}^{\mu\nu}$ , take the form[22]:

$$\dot{\rho}_m + 3H(1 + \omega_m)\rho_m = Q, \quad (18)$$

$$\dot{\rho}_e + 3H(1 + \omega_e + \Gamma_1)\rho_e = -Q', \quad (19)$$

$$\dot{\rho}_r + 3H(1 + \omega_r + \Gamma_2)\rho_r = Q' - Q, \quad (20)$$

where

$$3\Gamma_1 = \xi_1(3 - 2R + S) + \zeta_1(3 - 2S + R), \quad (21)$$

$$3\Gamma_2 = \xi_2(3 - 2R + S) + \zeta_1(3 - 2S + R). \quad (22)$$

Here, we give the mean expansion rate[22] as an average Hubble rate  $H$  as

$$3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}. \quad (23)$$

The difference of expansion rates as the Hubble normalized shear parameters  $R$  and  $S$  are given as[24]

$$HR = \frac{\dot{A}}{A} - \frac{\dot{B}}{B}, \quad (24)$$

$$HS = \frac{\dot{A}}{A} - \frac{\dot{C}}{C}. \quad (25)$$

Also, we introduced  $Q$  and  $Q'$  due to the mutual interaction[25–29]. Negative values of  $Q$  corresponds to energy transfer from dark matter sector to the other two sectors, positive values of  $Q'$  means there is an energy transfer from ghost dark energy sector to the other two constituents, and  $Q > Q'$  case describes energy loss for radiation sector[25]. It is reported recently that *Abell Cluster A586* observational evidences show a transition from dark energy sector to dark matter sector and vice versa[30, 31]. Moreover, this event may effectively appear as a self-conserved dark energy, with a non-trivial equation of state mimicking quintessence or phantom, as in the  $\Lambda$ XCDM scenario[32–34]. Nevertheless, the significance of this interaction is not clearly identified[35]. To be general, in this work, we choose the following expressions for the interaction terms:

$$Q = 3b^2 H(\rho_m + \rho_e + \rho_r) = 3b^2 H \rho_e(1 + r_1 + r_2), \quad (26)$$

$$Q' = 3b'^2 H(\rho_m + \rho_e + \rho_r) = 3b'^2 H \rho_e(1 + r_1 + r_2), \quad (27)$$

here  $b$  and  $b'$  are coupling parameters for the interaction, while  $r_1 = \frac{\rho_m}{\rho_e}$  and  $r_2 = \frac{\rho_r}{\rho_e}$  are the ratios for energy densities. It is obvious that signs of  $b^2$  and  $b'^2$  indicate the directions of energy transitions. The case with  $b = b' = 0$  represents the non-interacting model. In some cases,  $b^2$  and  $b'^2$  are taken in the range  $[0, 1]$ [36]. Galactic clusters and CMB observations show that the coupling parameters  $b^2, b'^2 < 0.025$ , i.e. small but positive constants of order of the unity[37, 38]. The negative coupling parameter cases are avoided due to the violation of gravitational thermodynamics laws.

The generalized Friedmann equation turns out to be in the form

$$H^2 = \frac{\rho_m + \rho_e + \rho_r}{3\beta}, \quad (28)$$

where

$$\beta = 1 - \frac{R^2 + S^2 - RS}{9}. \quad (29)$$

Next, by defining the following dimensionless density parameters

$$\Omega_m = \frac{\rho_m}{3\beta H^2}, \quad (30)$$

$$\Omega_e = \frac{\rho_e}{3\beta H^2}, \quad (31)$$

$$\Omega_r = \frac{\rho_r}{3\beta H^2}, \quad (32)$$

we can rewrite the generalized Friedmann equation as

$$1 = \Omega_m + \Omega_e + \Omega_r. \quad (33)$$

Moreover, the Friedman equation can also be rewritten in a very elegant form

$$\sum_{i=m,e,r} \Omega_i \equiv 1, \quad (34)$$

where

$$\Omega_i \equiv (\Omega_m, \Omega_e, \Omega_r). \quad (35)$$

The ghost energy density is proportional to the Hubble parameter[39]

$$\rho_e = \lambda H. \quad (36)$$

Here  $\lambda$  is a constant of order  $\Lambda_{QCD}^3$  and  $\Lambda_{QCD} \sim 100 MeV$  is QCD mass scale. Taking a time derivative in both sides of relation (36) and using Friedmann equation (28), we obtain

$$\dot{\rho}_e = -\frac{3\lambda H^2}{2\beta} \left\{ 1 + \frac{\dot{\beta}}{3H} + \omega_m \Omega_m + (\omega_e + \Gamma_1) \Omega_e + (\omega_r + \Gamma_2) \Omega_r \right\}. \quad (37)$$

Thence, inserting this relation into the continuity equation (19) yields

$$\omega_e = \frac{2\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] - \frac{\dot{\beta}}{3H(\Omega_e - 2)} - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{\Omega_e - 2}. \quad (38)$$

When we take  $R = S = \xi_1 = \xi_2 = \zeta_1 = \zeta_2 = 0$ , Bianchi type-I spacetime filled with anisotropic dark fluid reduces to the flat Friedmann-Robertson-Walker universe which is filled with isotropic dark fluid. Using these values in equation (38), we have

$$\omega_e = \frac{1}{\Omega_e - 2} \left[ 1 - \omega_m \Omega_m - \omega_r \Omega_r + \frac{2b'^2}{\Omega_e} \right]. \quad (39)$$

On the other hand, for the ghost dark energy interacting with dust fluid (pressureless dark matter plus pressureless dark radiation), we find the following equation

$$\omega_e = \frac{1}{\Omega_e - 2} \left[ 1 + \frac{2b'^2}{\Omega_e} \right] \quad (40)$$

which is the same as obtained by Sheykhi et al.[40]. In the late time where  $\Omega_e \rightarrow 1$ , the equation of state parameter,  $\omega_e$ , of interacting ghost dark energy behaves like phantom energy,  $\omega_e = -(1 + 2b'^2) < -1$  independent of the value of coupling parameter  $b'^2$ [40]. For the non-interacting case,  $b' = 0$ , one can obtain

$$\omega_e = \frac{1}{\Omega_e - 2}. \quad (41)$$

At the early time where  $\Omega_e \ll 1$ , equation (41) gives  $\omega_e = -\frac{1}{2}$ , while at the late time where  $\Omega_e \rightarrow 1$  the ghost dark energy mimics a cosmological constant ( $\omega_e = -1$ )[40].

### III. CORRESPONDENCE WITH SCALAR FIELDS

In this part of the work, a connection[21, 40–44] between interacting ghost dark energy and various scalar fields will be established by equating the equations of state for these models with the equation of state parameter of interacting ghost dark energy obtained in (38).

#### A. Reconstructing ghost quintessence

In order to implement a correspondence between ghost dark energy and quintessence scalar field, we consider the quintessence scalar field model of dark energy is the effective underlying theory. The action for quintessence is defined as[45]

$$S_q = -\frac{1}{2} \int d^4x \sqrt{-g} [g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + 2V(\phi)], \quad (42)$$

where  $V(\phi)$  is the potential of quintessence. The quintessence field is defined by an ordinary time-dependent and homogeneous scalar which is minimally coupled to gravity, but with a particular potential that leads to the accelerating universe[46]. Taking a variation of the action (42) with respect to the inverse metric tensor  $g^{\mu\nu}$  yields the energy-momentum tensor of the quintessence field:

$$T_{\mu\nu}^q = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} g^{\lambda\delta} \partial_\lambda \phi \partial_\delta \phi - g_{\mu\nu} V(\phi). \quad (43)$$

Therefore, for the quintessence scalar field, energy and pressure densities are written as[47, 48]

$$\rho_q = \frac{\dot{\phi}^2}{2} + V(\phi), \quad p_q = \frac{\dot{\phi}^2}{2} - V(\phi), \quad (44)$$

hence the equation of state parameter of the quintessence is obtained as

$$\omega_q = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \quad (45)$$

It is seen that the universe accelerates for  $\dot{\phi}^2 < V(\phi)$  if  $\omega_q < -\frac{1}{3}$  [46, 47]. In order to implement the correspondence between the ghost dark energy and quintessence scalar field, we identify  $\rho_q = \rho_e$  and  $\omega_q = \omega_e$ . From this point of view, we obtain that

$$\begin{aligned} \dot{\phi}^2 &= (1 + \omega_e)\rho_e \\ &= 3\beta H^2 \Omega_e \left[ 1 + \frac{2\beta}{\Omega_e - 2} \left( 1 + \Gamma_1 - \frac{1}{2\beta} + \frac{b'^2}{\Omega_e} \right) \right. \\ &\quad \left. + \frac{\frac{\dot{\beta}}{3H} + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{2 - \Omega_e} \right], \quad (46) \end{aligned}$$

$$\begin{aligned} V(\phi) &= \frac{1}{2}(1 - \omega_e)\rho_e \\ &= \frac{3}{2}\beta H^2 \Omega_e \left[ 1 - \frac{2\beta}{\Omega_e - 2} \left( 1 + \Gamma_1 - \frac{1}{2\beta} + \frac{b'^2}{\Omega_e} \right) \right. \\ &\quad \left. - \frac{\frac{\dot{\beta}}{3H} + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{2 - \Omega_e} \right]. \quad (47) \end{aligned}$$

By making use of equation (46) we get

$$\begin{aligned} \frac{\dot{\phi}}{H} &= \sqrt{3\beta\Omega_e} \left[ 1 + \frac{2\beta}{\Omega_e - 2} \left( 1 + \Gamma_1 - \frac{1}{2\beta} + \frac{b'^2}{\Omega_e} \right) \right. \\ &\quad \left. + \frac{\frac{\dot{\beta}}{3H} + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{2 - \Omega_e} \right]^{\frac{1}{2}}. \quad (48) \end{aligned}$$

Now, we define a new variable,  $x = \ln \sqrt[3]{ABC}$ , to rewrite the result (48) in another form. Using this new variable, we can write

$$\frac{d}{dx} = H \frac{d}{dt}, \quad (49)$$

and we get

$$\phi' = \frac{\dot{\phi}}{H}, \quad (50)$$

where a prime denotes derivative with respect to the new variable  $x$ . Hence, integrating equation (48) with respect to the new variable  $x$  yields

$$\begin{aligned} \phi(u) - \phi(u_0) &= \int_{u_0}^u \frac{du}{u} \sqrt{3\beta\Omega_e} \left[ 1 + \frac{\dot{\beta}}{3H(2 - \Omega_e)} \right. \\ &\quad \left. + \frac{\omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{2 - \Omega_e} \right. \\ &\quad \left. + \frac{2\beta}{\Omega_e - 2} \left( 1 + \Gamma_1 - \frac{1}{2\beta} + \frac{b'^2}{\Omega_e} \right) \right]^{\frac{1}{2}} \quad (51) \end{aligned}$$

here  $u = \sqrt[3]{ABC}$  and  $u_0$  represents the present value. For simplicity, we can consider the limiting case  $R = S = \xi_1 = \xi_2 = \zeta_1 = \zeta_2 = 0$ . Also, we can assume  $\omega_m = \omega_r = 0$  (the ghost quintessence interacting with dust fluid). Thus, we have

$$\phi(u) - \phi(u_0) = \sqrt{3} \int_{u_0}^u \frac{du}{u} \sqrt{\frac{\Omega_e}{2 - \Omega_e} \left( 1 - \Omega_e - \frac{2b'^2}{\Omega_e} \right)} \quad (52)$$

and

$$V(\phi) = \frac{3H^2\Omega_e}{2(2 - \Omega_e)} \left( 3 - \Omega_e + \frac{2b'^2}{\Omega_e} \right). \quad (53)$$

The results we give in equations (52) and (53) are the same as obtained in Ref[21].

## B. Reconstructing ghost tachyon

Now, we are in a position to establish the correspondence between the ghost dark energy and tachyon scalar field. The tachyon field has been proposed as one of the possible candidates for the dark energy. The tachyon field has very interesting equation-of-state parameter which smoothly interpolates between  $-1$  and  $0$ [46, 49]. Therefore, the tachyonic field can be taken into account as a suitable candidate for the inflation at high energy as well as a source of dark energy[50, 51]. The effective Lagrangian density of the tachyonic field is defined as

$$\mathcal{L}_T = V(\phi) \sqrt{1 - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi}, \quad (54)$$

where  $V(\phi)$  is the potential of tachyon and  $g^{\mu\nu}$  represents the inverse metric tensor. For the tachyonic field, the energy and pressure densities are given, respectively, as[47]

$$\rho_T = V(\phi)(1 - \dot{\phi}^2)^{-1/2}, \quad (55)$$

$$p_T = -V(\phi)(1 - \dot{\phi}^2)^{1/2}. \quad (56)$$

Thence, we can easily find the equation of state parameter of tachyon field as

$$\omega_T = \dot{\phi}^2 - 1. \quad (57)$$

It is important to mention here that  $-1 < \dot{\phi} < 1$  is the required condition to describe a real tachyonic energy density[40]. Nevertheless,  $-1 < \omega_T < 0$  is the corresponding constraint for the equation-of-state parameter of tachyon. The tachyon field can describe a universe with accelerated expansion, but it cannot behave like phantom energy[46]. Comparing  $\rho_e$  and  $\rho_T$ , we get the following expression of the tachyon potential:

$$V(\phi) = \rho_e(1 - \dot{\phi}^2)^{1/2}, \quad (58)$$

and using equations (38) and (57), we can write

$$\begin{aligned}\dot{\phi}^2 &= 1 + \omega_e \\ &= 1 + \frac{2\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] - \frac{\dot{\beta}}{3H(\Omega_e - 2)} \\ &\quad - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{\Omega_e - 2}.\end{aligned}\quad (59)$$

Thence, the potential of ghost tachyon field is written as

$$\begin{aligned}V(\phi) &= 3\beta H^2 \Omega_e \left\{ \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{\Omega_e - 2} \right. \\ &\quad \left. - \frac{2\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] + \frac{\dot{\beta}}{3H(\Omega_e - 2)} \right\}^{\frac{1}{2}}\end{aligned}\quad (60)$$

The relation of kinetic term  $\dot{\phi}^2$ , equation (59), and the expression of the tachyonic potential  $V(\phi)$ , equation (58), show that these quantities may exist if it is provided that  $-1 \leq \omega_e \leq 0$ . This condition implies that the phantom energy sector can not be crossed in a universe with an accelerated expansion.

Defining a new variable  $x = \ln u$  with  $u = \sqrt[3]{ABC}$  yields  $\dot{\phi} = H \frac{d\phi}{d \ln u}$ . Hence, we can write

$$\begin{aligned}H \frac{d\phi}{d \ln u} &= \left\{ 1 + \frac{2\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] - \frac{\dot{\beta}}{3H(\Omega_e - 2)} \right. \\ &\quad \left. - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{\Omega_e - 2} \right\}^{\frac{1}{2}}.\end{aligned}\quad (61)$$

And, integrating yields

$$\begin{aligned}\phi(u) &= \phi(u_0) + \int_{u_0}^u \frac{1}{uH} \left\{ 1 + \frac{2\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] \right. \\ &\quad \left. - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{\Omega_e - 2} - \frac{\dot{\beta}}{3H(\Omega_e - 2)} \right\}^{\frac{1}{2}} du,\end{aligned}\quad (62)$$

where  $u_0$  represents the present value. Now, we assume  $R = S = \xi_1 = \xi_2 = \zeta_1 = \zeta_2 = 0$  (isotropic Bianchi type I universe filled with isotropic dark fluid) and  $\omega_m = \omega_r = 0$  (the ghost tachyon interacting with dust fluid). In this way, we get

$$\phi(u) = \phi(u_0) + \int_{u_0}^u \frac{du}{uH} \sqrt{\frac{\Omega_e^2}{2 - \Omega_e} \left( 1 - \Omega_e - \frac{2b'^2}{\Omega_e} \right)},\quad (63)$$

and

$$V(\phi) = 3H^2 \Omega_e \sqrt{\frac{1 + \frac{2b'^2}{\Omega_e}}{2 - \Omega_e}}.\quad (64)$$

These results are the same as obtained in Ref[47].

### C. Reconstructing ghost dilaton

The dilaton-gravity action is given as

$$S_D = \int d^4x \sqrt{-g} [R - 2\partial_\mu \phi \partial_\nu \phi - V(\phi)].\quad (65)$$

Dilaton field's action is more general than the action of Brans-Dicke theory in that we have a dilaton potential. A dilaton is a hypothetical particle of a scalar field  $\phi$  and described by using the lower-energy limit of string theory[52]. The coefficient of the kinematic term of the dilaton field can be negative in the Einstein frame which implies that the dilaton behaves as a phantom-type scalar field[47]. The energy and pressure densities of the dilatonic dark energy model are defined, respectively, as[53, 54]

$$\rho_D = 3he^{\delta\phi} \chi^2 - \chi,\quad (66)$$

$$p_D = he^{\delta\phi} \chi^2 - \chi,\quad (67)$$

where  $h$  and  $\delta$  are positive constants and  $\chi = \frac{1}{2}\dot{\phi}^2$ . The negative coefficient of the kinematic term makes a phantom-like behavior for dilaton field[46]. Therefore, The equation-of-state parameter for the dilaton scalar field is defined as

$$\omega_D = \frac{he^{\delta\phi} \chi - 1}{3he^{\delta\phi} \chi - 1}.\quad (68)$$

Now, we can implement the correspondence between the dilatonic scalar field and ghost dark energy. Equating the relation (68) with equation (38) gives

$$\begin{aligned}h\chi e^{\delta\phi} &= \frac{\omega_e - 1}{3\omega_e - 1} \\ &= \left( \frac{2\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] - \frac{\dot{\beta}}{3H(\Omega_e - 2)} \right. \\ &\quad \left. - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{\Omega_e - 2} - 1 \right) \\ &\quad \times \left( \frac{6\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] - \frac{3\dot{\beta}}{3H(\Omega_e - 2)} \right. \\ &\quad \left. - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{(\Omega_e - 2)/3} - 1 \right)^{-1}.\end{aligned}\quad (69)$$

Next, using  $\chi = \frac{1}{2}\dot{\phi}^2$ , the relation (69) can be rewritten in the following form

$$\begin{aligned}e^{\frac{\delta\phi}{2}} \dot{\phi} &= \sqrt{\frac{2}{h}} \left( \frac{2\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] - \frac{\dot{\beta}}{3H(\Omega_e - 2)} \right. \\ &\quad \left. - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{\Omega_e - 2} - 1 \right)^{\frac{1}{2}}\end{aligned}$$

$$\times \left( \frac{6\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] - \frac{3\dot{\beta}}{3H(\Omega_e - 2)} - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{(\Omega_e - 2)/3} - 1 \right)^{-\frac{1}{2}}. \quad (70)$$

Integrating equation (70) with respect to  $u = \sqrt[3]{ABC}$  gives

$$e^{\frac{\delta\phi(u)}{2}} = e^{\frac{\delta\phi(u_0)}{2}} + \frac{\delta}{\sqrt{2h}} \int_{u_0}^u \frac{du}{uH} \times \left( \frac{2\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] - \frac{\dot{\beta}}{3H(\Omega_e - 2)} - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{\Omega_e - 2} - 1 \right)^{\frac{1}{2}} \times \left( \frac{6\beta}{\Omega_e - 2} \left[ 1 + \Gamma_1 + \frac{b'^2}{\Omega_e} \right] - \frac{3\dot{\beta}}{3H(\Omega_e - 2)} - \frac{1 + \omega_m \Omega_m + \Gamma_1 \Omega_e + (\omega_r + \Gamma_2) \Omega_r}{(\Omega_e - 2)/3} - 1 \right)^{-\frac{1}{2}}. \quad (71)$$

Considering the limiting case  $R = S = \xi_1 = \xi_2 = \zeta_1 = \zeta_2 = 0$  (isotropic Bianchi type I universe filled with isotropic dark fluid) and  $\omega_m = \omega_r = 0$  (the ghost dilaton interacting with dust fluid) we can obtain the same result as obtained in Ref[55] which is given as

$$e^{\frac{\delta\phi(u)}{2}} = e^{\frac{\delta\phi(u_0)}{2}} + \frac{\delta}{\sqrt{2h}} \int_{u_0}^u \frac{du}{uH} \sqrt{\frac{\frac{2b'^2}{\Omega_e} - \Omega_e + 3}{\frac{6b'^2}{\Omega_e} - \Omega_e + 5}}. \quad (72)$$

## IV. CONCLUSIONS

The well-known *Ghost Dark Energy* model has recently been proposed to explain the dark energy dominated universe. It is known that the scalar field models of dark energy can be considered as an effective theory to investigate the nature of dark universe. Hence, the reconstruction of the scalar fields based on some dark energy models give important results. This point motivated us to reconstruct the quintessence, tachyon and dilaton models of dark energy based on the ghost dark energy. It is important to mention here that the resulting models with the reconstructed potentials are unique single-scalar models that can reproduce the ghost dark energy evolution of the universe.

In this paper, we established a connection between the scalar field model of dark energy including quintessence, tachyon, dilaton energy density and anisotropic ghost dark energy which is in interaction with dark matter and anisotropic dark radiation in the Bianchi-type I universe. It is important to mention here that these correspondences are very important to understand how various candidates of dark energy are mutually related to each other. Such scalar fields have very exciting feature of understanding the phantom crossing while the reconstructed scalar potential has interesting physical implications in cosmology.

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