Hydrodynamic Analogue for Curved Space-Time and General Relativity

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ABSTRACT

An attempt is made to simulate the curved space-time of General Relativity by comparison with the characteristics of a potential vortex at steady state sometimes called a free vortex. In this paper, we present results of our study of the potential vortex as an analogy with gravity and General Relativity. Analyses of potential vortex characteristics demonstrate that an analogy exists between potential vortex flow, General Relativity and the gravitational fields around massive objects. An analogue model of gravity is developed which simulates the potential vortex model of space-time observed by Gravity Probe-B[1, 2] experiments. Statements by Gravity Probe-B researchers that the region around Earth resembles a "space-time vortex" lend credibility to the premise of this paper that potential vortex flow and gravity are related. Albert Einstein introduced the first analogue model for General Relativity that describes space-time as a membrane upon which objects move along geodesics. Analogue models provide a unique and viable means of analyzing complex space-time in situations that would normally be beyond our means to explore using conventional methods. Further, by exploring the dynamics of the analogue model, we are able to recover the equivalent form of the Einstein field equation starting from the usual equations of hydrodynamics and vector dynamics and make predictions about space-time and gravity using these observations. An implication for the existence of an analogy between space-time and the potential vortex is the possibility that space-time can be considered a superfluid forming the surface of our universe.

I. INTRODUCTION

This paper proposes that an analogy exists between the potential vortex and General Relativity (GR), which theorizes the curvature of space-time and the force of gravity, are directly related to the distribution of energy-momentum as specified by the Einstein field equation. The proposed analogy between the potential vortex and General Relativity may provide a laboratory experiment that will explain how curved space-time engenders gravitational attraction. This paper also proposes that in general a potential vortex is formed when approaching all non-rotating and rotating massive objects that have a finite central region with the characteristics of a rotational solid vortex core and potential vortex outer flow. To illustrate the analogy between General Relativity and the potential vortex an apparatus was designed and fabricated to generate a standing potential vortex for laboratory experiments.

Analogue models of gravity have been around for many years, and in fact have been in existence since the infancy of General Relativity itself. In recent years, however, they seem to be enjoying a tremendous upsurge in popularity. The reason for this is clear: these models are proving to be extremely useful in providing new and deeper insights into the mysteries of curved space-time, and other aspects of General Relativity. In particular, the work by Unruh[3] paved the way for such models to be used as a means to understand the physics of wave propagation on effective curved space-time. While the work by Unruh is not related to the analogy being proposed, our claim is that by using the potential vortex analogue model it is feasible to study the effects of gravity near a massive object and therefore gain valuable insight into the nature of the gravity field surrounding such an object. With this purpose in mind, we have succeeded in creating a potential vortex that forms our analogue of a gravitational field. As demanded by GR, we wish to build an analog model of gravity in which it is the presence of matter, which dictates what the metric for the space-time should be, in accordance with the Einstein field equations. We find that a potential vortex is the source of the energy–momentum that changes the geometry of space-time from being flat to curved, and tells us what the appropriate metric should be.

Yet another issue that we hope to address in our model is being able to reproduce the dynamical aspects of GR, especially obtaining the Einstein field equations naturally from the equations of hydrodynamics and vector mechanics. It is a known fact that none of the analog models currently in use are capable of reproducing the Einstein field equations, so these models only deal with the kinematical aspects of GR, and so cannot be made to extend to the dynamics of GR. This is a serious drawback, since any successful candidate for a quantum theory of gravity should be able to lead to the Einstein equations.

II. POTENTIAL VORTEX SOLUTION

A potential vortex or tornado is often approximated by the flow around a drain hole at the bottom of a container. As an approximation to this phenomenon an experiment to simulate a three dimensional potential vortex was performed[4]. This experiment using a rapidly rotating central cylinder located on the axis of a cylindrical basin filled with water to generate a standing potential vortex is illustrated in Figure 1a. By comparison a draining potential vortex, illustrated in Figure 4b, shows just how complex defining and measuring a vortex can be if the flow is not stationary. For these comparisons we define the vortex as stationary to remove unnecessary complexity of the steady state solution.

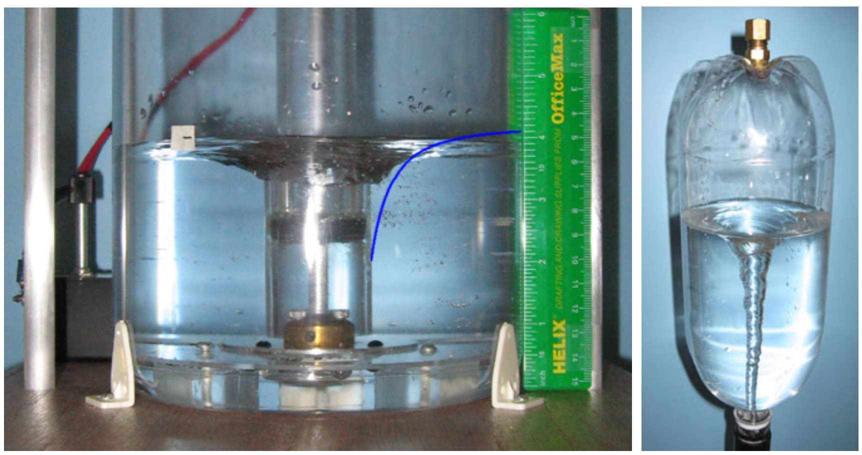

FIG. 1. Potential vortex. (a) Standing free vortex, (b) Draining free vortex.

For our standing potential vortex a rotating cylindrical core forms the inner fluid boundary around which the potential vortex is created. The rotation of the inner solid boundary or vortex core maintains an approximately non-viscous or inviscid boundary condition on the outer boundary or far field of the flow. The rotating inner boundary's non-slip or viscous boundary drags circumferential layers of fluid that generate the potential vortex. The rotating inner core of the experiment represents the rotating inner boundary of the potential vortex. The circumferential velocity for the potential vortex at steady state is:

$$u_\theta = \frac{V_0 \ r_0}{r} \ or \ \frac{u_\theta}{V_0} = \frac{r_0}{r}. \qquad (1)$$

Where r is measured from the center of the system, V_0 is the rotation velocity at the surface of the inner vortex core and r_0 represents the radius of the inner rotating cylinder. Velocity in the radial direction, u_r is zero and r_1 represents the outer radius of the stationary cylindrical basin. During initial startup, viscous interactions between subsequent layers of fluid generate a time varying velocity profile and free surface deflection. The transient phase of flow cannot be considered irrotational. However, at steady state the velocity profile and free surface deflection of the resulting fluid flow can be approximated by an irrotational solution based on the viscous Navier Stokes equations in the circumferential, radial, and vertical (θ, r, z) directions when the proper boundary conditions are imposed. The Navier Stokes solution for the potential vortex generates a velocity distribution that is incompressible where $\nabla \cdot \boldsymbol{V} = \boldsymbol{0}$ and irrotational where $\nabla \times \boldsymbol{V} = \boldsymbol{0}$ throughout the flow field at steady state. The final solution for the steady state potential vortex is the Bernoulli equation, that holds everywhere in the flow including the free surface where p = p_{atm} and forms a series of constant-pressure surfaces having the form of a second-order hyperboloid, that is, z varies inversely with r^2. Where the Bernoulli equation is now expressed as[5].

$$p = \frac{\rho K^2}{2 r^2} + \rho g z = C. \tag{2}$$

Where $K = V_0 r_0$ and the constant, C must be determined to compute the shape of the fluid surface at p = p_{atm} which represents the free surface of the potential vortex. After solving for fluid height, z using the Bernoulli equation, then applying the pressure boundary condition $p = p_{atm}$ on the free surface to determine the constant C, a free surface of constant pressure p_{atm} has the following shape as a function of r at steady state for a potential vortex or free vortex as it is sometimes called.

$$z_{p=p1} = C_1 - \frac{K^2}{2 g r^2}. \text{ Where } C_1 = z_0 + \frac{K^2}{2g}. \tag{3}$$

Typical free surface shapes for potential vortex flow are illustrated in Figure 2. Where in these illustrations the plots correspond to the results of the experiment for the 10 Hz and 15 Hz rotation rate of the inner cylinder. The plots displayed in Figure 2 use Eqn. 3 where the free surface displacement, Z and radial distance, r are normalized by the inner surface radius, r_0.

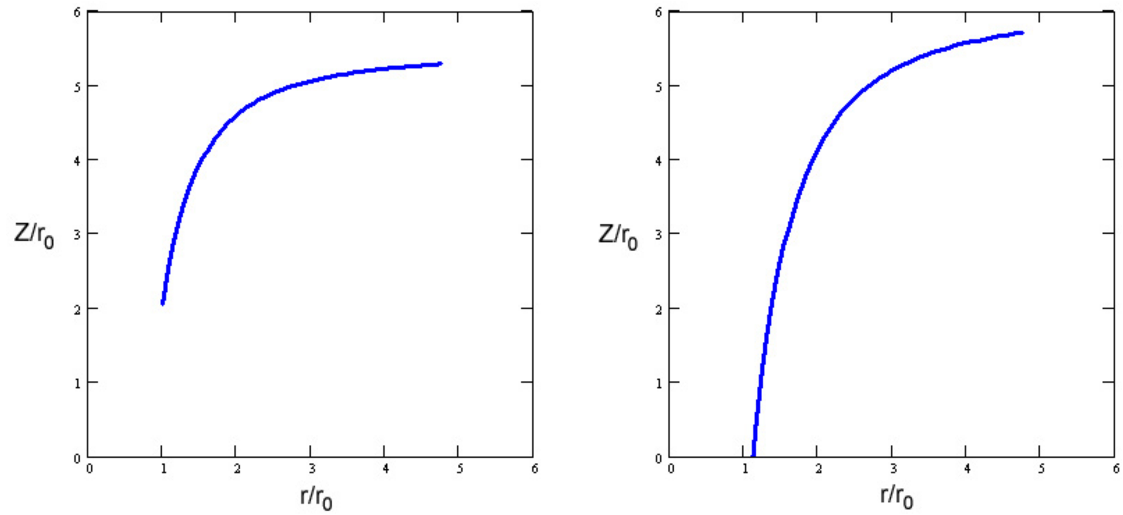

FIG. 2. Theoretical potential vortex surface shapes, (a) 10 Hz and (b) 15 Hz rotation rates.

D. Frame dragging and potential vortex surface velocity compared

Similar to potential vortex flow an angular velocity component is imparted to a particle as it approaches a rotating massive object like a rotating block hole. Gravitational frame dragging increases the angular velocity of an object under the influence of strong rotating gravitational fields. Frame dragging causes a gyroscopic precession also known as the Lense-Thirring effect, which is what Gravity Probe-B measured while in orbit around the Earth. Figure 3b plots the frame dragging influence of an object with the mass of the Sun rotating at 40 revolutions per day on inward falling objects within 5 radii of the central mass. The frame dragging velocity profile plotted in Figure 3b looks similar to the velocity profile plotted in Figure 3a for potential vortex flow. This is further evidence of the analogy that is hypothesized to exist between gravitational attraction and the potential vortex. The following equation for frame dragging angular velocity[6] and precession velocity[7] is used to generate the velocity profile displayed in Figure 3b.

$$\omega_\theta = \frac{2\,m\,a\,r}{(r^2+a^2)^2}, \tag{4}$$

$$V_\theta = r\,\omega_\theta, \tag{5}$$

Where, the remaining variables used in ω_θ are defined as follows:

$$m = \frac{GM}{c^2}, a = \frac{J}{cM}, J = I_z\omega \text{ and } I_z = \frac{2}{5}\,M\,{r_0}^2. \tag{6}$$

The comparison between the potential vortex velocity distribution displayed in Figure 3a and the frame dragging effect plotted in Figure 3b indicate that an analogy does indeed exist between the potential vortex and the rotation of massive objects like black holes.

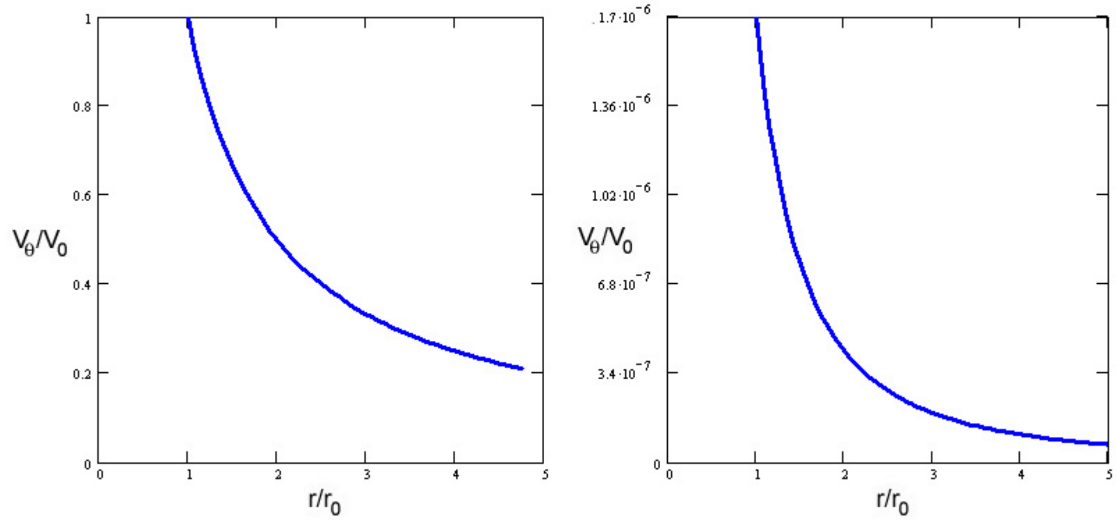

FIG. 3. (a) Potential vortex velocity ratio, Eqn. 1. (b) Frame dragging velocity ratio, Eqn. 5.

III. COMPARISON WITH GRAVITY PROBE B

A. Gravity Probe B space-time observations

Gravity Probe-B (GP-B) is a NASA space-based physics mission to experimentally investigate Albert Einstein's theory of General Relativity. GP-B accurately measured geodetic deformation or the inward warping of space-time around a gravitational body. In addition, GP-B accurately measured frame dragging or the amount a spinning massive object pulls space-time with it as it rotates around its axis. The GP-B experiment measured *geodetic deformation* and *frame dragging* deformation by precisely measuring the precession or displacement angles of the spin axes of four ultra precise gyroscopes over the course of a year and comparing these experimental results with predictions from Einstein's theory of General Relativity. Geodetic deformation and frame dragging are observable analogous effects for a potential vortex, as discussion will illustrate.

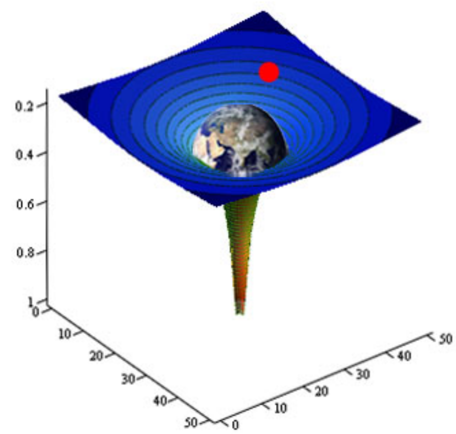

FIG. 4. GP-B measured geodetic and frame dragging deformation of Earth space-time.

1. Geodetic deformation

Geodetic deformation is the degree to which massive objects bend space-time. Where, according to Einstein's general theory of relativity, gravitation is a manifestation of the curvature of space-time causing light and particles of matter to travel along geodesics. A geodesic is the shortest line between two points that lies in a given surface. For curved space-time two separate geodesics that start off parallel will eventually cross or intersect when approaching a massive object. The intersection of initially parallel geodesics causes gravitational tidal effects when traveling within a gravitational field. For example, two particles in free fall in a gravitational field will initially move parallel to each other as they approach an object. However, because the particles are moving on radial paths to the center of the object they will seem to move toward each other if the distance traveled is great enough. The relative displacement of two or more objects traveling along the geodesics of a very massive object is called geodesic deviation or deformation. This description of geodetic deformation is clearly illustrated in Figure 1a and Figure 1b for the potential vortex.

2. Frame dragging

Frame dragging is the degree to which rotating massive objects drag space-time around their spin axis altering the displacement angle of space-time. Where, the torsion of space-time around the spin axis of a massive object causes a swirling or torsional effect not previously measured. The space-time vortex observed by GP-B is a natural consequence of a notional rotational solid vortex causing the torsion of space-time and the deformation that alters the relative displacement angles of space-time around massive objects. This description of frame dragging deformation is clearly illustrated in Figure 1a and Figure 1b for the potential vortex.

B. Free and forced vortices as analogies for space-time

From a fluid dynamics point of view, which is the point of view expressed by the official Gravity Probe B experiment, a potential vortex is created by the rotation of an inner solid boundary while maintaining a non-interacting and therefore inviscid outer boundary in the far field of space-time. The inner boundary no-slip velocity boundary condition initially drags layers of fluid or space-time around the inner core to generate a space-time vortex. The rotating inner boundary is represented by a rotating inner core, which is an analogy for a spinning massive object like the Earth, star or black hole. Therefore, using this analogy a rotating massive object creates a potential vortex in space-time by the interaction of rotating layers of space-time starting from the surface of an inner rotating boundary. The circumferential velocity profile for an incompressible and irrotational potential vortex at steady state is, $u_\theta = \frac{V_0\ r_0}{r}$, where r is measured from the center of the system, V_0 is the rotation velocity of the surface of the inner core and r_0 represents the radius of the inner rotating core. The radial component of fluid velocity, u_r is zero. The observation by GP-B for the space-time vortex around the Earth exactly matches the description of a potential vortex. Therefore, a potential vortex seems to be a reasonable analogy for the space-time around massive objects based on the observed boundary conditions and the shape of space-time observed by GP-B around massive objects. However, a forced or solid vortex is created by the rotation of an outer boundary, which initially transports vorticity by the viscous interaction of each subsequent fluid layer starting from a rotating outer boundary. In addition, a forced vortex undergoing a solid body rotation is rotational and does not fit the description of a superfluid or the quantum mechanical analogy of space-time. The circumferential velocity profile for a forced vortex at steady state is, u_θ = r ω, where r is measured from the center of the system and ω is the rate of rotation (rad/sec). The radial component of the fluid velocity, u_r is zero for the entire flow field. This does not sound like what Gravity Probe-B observed.

Therefore, a forced vortex is not a good analogy for the space-time around a massive object like the Earth, Sun or black hole. However, a solid vortex may be an analogy for the singularity at the center of massive objects like black holes. Therefore, a potential vortex, which is common in nature, fits as a possible analogy with General Relativity, gravitation and the space-time around massive objects. Therefore, a potential vortex generates by analogy what can be called **geodetic deformation** as illustrated in Figure 3, which is the free surface deformation of a rotating fluid with a central rotating core. In addition, a rotating potential vortex generates by analogy what can be called a **frame dragging** effect due to the twisting action of the fluid as it rotates around its central core as illustrated in Figure 1, Figure 2 and Figure 3a. In conclusion, geodetic deformation and frame dragging as defined by General Relativity have analogous effects in potential vortex flow.

IV. DEVELOPING THE SPACE-TIME POTENTIAL VORTEX ANALOGY

A. Derivation of the potential vortex metric

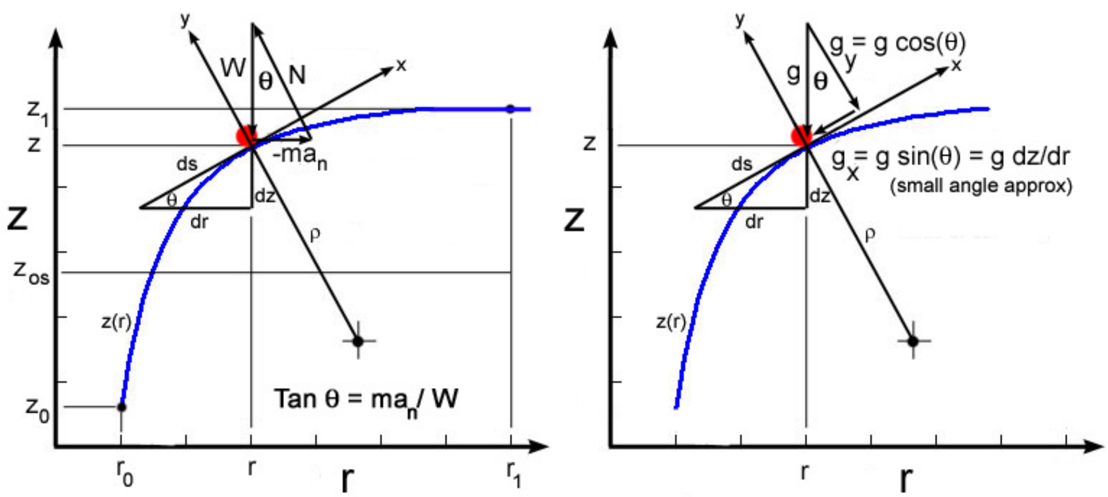

FIG. 5. (a) Potential vortex geometry. (b) Potential vortex local slope, acceleration and curvature.

The local surface shape of a potential vortex that supports an orbiting object, allows an object to orbit at a velocity that is a function of core rotation rate, radius to the object on the free surface and free surface curvature. The velocity of an object on the free surface is due to the dynamic equilibrium between object inertia, ma_n object weight, W and the local surface deformation or curvature of the free surface upon which the object rests.

Specifically, the local slope of the potential vortex free surface under an orbiting object allows the object to be in dynamic equilibrium under the action of its own weight, W normal force, N exerted by the surface and the inertia vector, $-ma_n$ directed opposite to a_n. Where, the centripetal acceleration, a_n is directed away from the center of the circular path of the object when the object is in orbit around the rotating core of the potential vortex. The local deformation or curvature of the potential vortex free surface causes the object to orbit in the direction of rotation of the inner surface. Therefore, the velocity of the object orbiting at a point on the free surface is purely a function of the physical characteristics of the inner surface having some angular rate of rotation and radius. This analysis implies local curvature of a potential vortex allows the object to orbit on the surface and objects placed on the surface move at the same orbital velocity as the potential vortex in those locations. These observations are used to develop the equations that generate the curvature and energy-momentum equations analogous to there counterparts in General Relativity. The potential vortex surface velocity distribution and potential vortex surface shape are used to support the proposition that a potential vortex is analogous to the space-time predicted by General Relativity and the resulting curvature and energy-momentum components of the flow are analogous to their counterparts in General Relativity.

The following discussion uses the cylindrical coordinate system and surface geometry for potential vortex flow and illustrates the similarity between the potential vortex, General Relativity and space-time curvature around massive objects. Using the continuity, momentum and energy equations of fluid dynamics and the associated boundary conditions for potential vortex flow the free surface of a potential vortex is defined by the following equation and is also illustrated in Figure 5.

$$z = z_0 + \frac{{V_0}^2}{2g}\left(1 - \frac{{r_0}^2}{r^2}\right), \tag{7}$$

The slope, (dz/dr) of the free surface at z(r) is after taking the derivative of Eq. (7).

$$\frac{dz}{dr} = \frac{K^2}{gr^3}, \tag{8}$$

Acceleration (g_x) along the free surface, z(r) is due to the external **g** field of the Earth.

$$g_x = \boldsymbol{g}\,\frac{dz}{dr} = \boldsymbol{g}\left(\frac{K^2}{gr^3}\right). \text{ Where } K = V_0 r_0, \tag{9}$$

Equate acceleration along the curve, g_x to the gravitational acceleration[8], $\boldsymbol{a} = \frac{GM}{r^2}$ to find the acceleration of a particle along the free surface, z(r) in terms of standard gravitational constants, G and M. Then, setting $g_x = \frac{GM}{r^2}$ equal to $g_x = \frac{K^2}{r^3}$ implies $K^2 = GMr$ and the acceleration along the potential vortex surface, z(r) is.

$$g_x = \frac{K^2}{r^3} = \frac{GMr}{r^3} = \frac{GM}{r^2}. \tag{10}$$

To develop the space-time metric of a potential vortex use Eq. (8) or the slope of the free surface where the orbiting particle is located where the equation is repeated below.

$$\frac{dz}{dr} = \frac{K^2}{gr^3}, \tag{11}$$

And the standard line element for cylindrical coordinates is.

$$ds^2 = dr^2 + r^2\, d\phi^2 + \mathrm{d}z^2\,, \tag{12}$$

We can express the line element in a succinct way by writing the coordinates with indices and summing as the following equation illustrates.

$$ds^2 = g_{ab}(x)\, dx^a dx^b, \tag{13}$$

Where, $g_{ab}(x)$ are components of the 2 X 2 matrix called the space-time metric. The line element (Eq. 13) in cylindrical coordinates can be written and rearranged as.

$$ds^2 = \left(\frac{dz}{dr}\right)^2 dr^2 + dr^2 + r^2\, d\phi^2, \tag{14}$$

$$ds^2 = [1+\left(\frac{dz}{dr}\right)^2]\, dr^2 + r^2\, d\phi^2, \tag{15}$$

Then, after substituting Eq. 11 into Eq. 15 the line element, ds^2 becomes the following.

$$ds^2 = \left[1 + \left(\frac{K^2}{g\, r^3}\right)^2\right] dr^2 + r^2 d\phi^2, \tag{16}$$

In summation, the line element for the potential vortex surface is represented by the following equation in cylindrical coordinates.

$$ds^2 = \left[1 + \left(\frac{K^2}{g\, r^3}\right)^2\right] dr^2 + r^2 d\phi^2. \text{ Where } K = V_0\, r_0, \tag{17}$$

Finally, the metric matrix in cylindrical coordinates for the potential vortex becomes.

$$g_{ab} = \begin{bmatrix} 1 + \left(\frac{K^2}{g\, r^3}\right)^2 & 0 \\ 0 & r^2 \end{bmatrix}. \tag{18}$$

B. Derivation of the potential vortex field equation

Finally, the potential vortex equivalent to Einstein's field equation is derived using the fluid dynamic equations for the curvature of a potential vortex that are analogous to gravitational attraction. To develop the field equation for the potential vortex, central mass, M is expressed in terms of the central rotating cylinder velocity, V_0 central rotating cylinder radius, r_0 object orbital radius, r object orbital velocity, u_θ and gravitational constant, G and is equivalent to $F = Ma$ or force equals mass times acceleration. The equivalent potential vortex central mass is derived by equating gravitational circular orbital velocity to potential vortex orbital velocity.

$$u_{gravity} = \sqrt{\frac{GM}{r}} \text{ Equated to } u_{vortex} = \frac{V_0 r_0}{r}, \tag{19}$$

After some algebra the equivalent potential vortex central mass, M is found to be.

$$M = \frac{V_0 r_0}{G} u_\theta, \tag{20}$$

Then, after solving for GM using Eqn. 20.

$$GM = V_0 r_0 u_\theta, \tag{21}$$

Then after multiplying both sides of Eqn. 21 by ${r_0}^3$, $c^2$and after pulling out the quantity $\frac{c^2}{G}$, the equivalent to Einstein's field equation for potential vortex flow is.

$$R_{\mu\nu} = \frac{G}{c^4} T_{\mu\nu}, \tag{22}$$

Where the equivalent curvature for potential vortex motion is.

$$R_{\mu\nu} = \frac{M}{{r_0}^3}\left(\frac{G}{c^2}\right), \tag{23}$$

And the equivalent potential vortex energy-momentum is. Where $\mu = \nu = 1{,}2{,}3$.

$$T_{\mu\nu} = \frac{V_0 u_\theta}{{r_0}^2}\left(\frac{c^2}{G}\right). \tag{24}$$

Finally, the fluid potential vortex curvature tensor[9, 10] is defined.

$$R_{\mu\nu} = \begin{bmatrix} \frac{M}{{r_0}^3}\left(\frac{G}{c^2}\right) & 0 & 0 \\ 0 & \frac{M}{{r_0}^3}\left(\frac{G}{c^2}\right) & 0 \\ 0 & 0 & -2\frac{M}{{r_0}^3}\left(\frac{G}{c^2}\right) \end{bmatrix}, \tag{25}$$

And, the fluid potential vortex energy-momentum tensor[9, 10] is defined.

$$T_{\mu\nu} = \begin{bmatrix} \frac{V_0 u_\theta}{r_0^2}\left(\frac{c^2}{G}\right) & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{bmatrix}. \tag{26}$$

Where, $p = \rho g_{H2O}\, z$ is the isotropic rest frame pressure, ρg_{H2O} is the specific weight of water and z is the height measured from the surface of the potential vortex. By assuming an isotropic fluid the density and pressure are assumed equal in all directions. The coordinates r, θ, and z are in the cylindrical coordinate system that describes the potential vortex. An analysis of these equations using Mathcad confirms the units for curvature, $R_{\mu\nu}$ and the units for energy-momentum, $T_{\mu\nu}$ are equivalent to similar components in the Einstein field equations. Please see the results in Section C where the units for curvature and energy-momentum are confirmed.

C. Unit analysis validation of the potential vortex field equation

The following results from a MathCAD spreadsheet analysis of the potential vortex field equation for vortex orbital motion confirm the units for curvature, $R\ (m^{-2})$ and the units for energy-momentum, $T\ (\frac{erg}{cm^3})$ are identical to their counterparts in General Relativity and gravitation. The following MathCAD results used input data from Table I.

TABLE I. Summary of unit analysis input data.

Universal Gravitational Constant	$6.673 * 10^{-11}\ newton\ m^2/kg^2$
Radius to surface of rotating central core	$r_0 = 6.37 * 10^6\ m$
Radius to location of orbiting object	$R_{orbit} = 5r_0$
Mass of central object (Sun)	$M = 1.995 * 10^{30}$kg

After solving for V_0 from Eqn. 20 and by defining $r = R_{orbit}$, the surface velocity of the example rotating central core required for generating a space-time potential vortex is.

$$V_0 = \sqrt{\frac{R_{orbit} G M}{r_0^2}},\ V_0 = 1.022 * 10^4 km/sec. \tag{27}$$

As a check, the mass of the example gravitational object from Eqn. 19 is.

$$M = \frac{V_0^2 r_0^2}{R_{orbit} G},\ M = 1.995 * 10^{30}\ kg. \tag{28}$$

The strength of the example rotating central core of the space-time potential vortex is.

$$K = V_0 r_0, K = 6.512 * 10^{13} m^2/sec. \quad (29)$$

From Newtonian gravitational theory the circular orbit at distance R_{orbit} is.

$$u_\theta = \sqrt{\frac{GM}{R_{orbit}}}, u_\theta = 2.044 * 10^3 km/sec. \quad (30)$$

Using Eqn. 23 the free vortex curvature equivalent[9, 10] to Einstein's field curvature[6] is.

$$R = \frac{M}{{r_0}^3}\left(\frac{G}{c^2}\right), R = 5.731 * 10^6\ Tm^{-2}. \quad (31)$$

Where, R is free vortex curvature and the units of curvature, Tm are equal to 10^{12} meters. Then, energy-momentum is determined using equation 31 as follows.

$$T = \frac{V_0 u_\theta}{{r_0}^2}\left(\frac{c^2}{G}\right), T = 6.936 * 10^{27} erg/cm^3. \quad (32)$$

Finally, using Eqn. 22 the resulting equivalent free vortex field equation is.

$$R = \frac{G}{c^4} T. \quad (33)$$

To check the results it was found, $\frac{G}{c^4} T = 5.731 * 10^6\ Tm^{-2}$, identical value computed previously for R using the equivalent potential vortex field equation. Therefore, this example Mathcad analysis confirms units of curvature, R and units for energy-momentum, T are correct and agree with similar results from General Relativity for Einstein's field equation, $R_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu}$. Where, it is interesting to note that $\frac{G}{c^4}$ is the inverse of the Planck Force, a fundamental quantity of gravitational and electromagnetic energy.

D. Extracting potential vortex space-time from General Relativity

This paper proposes the potential vortex is analogous to the space-time metrics of stationary and rotating massive objects like planets, stars and black holes. The Schwarzschild space-time metric represented by Eq. 34 describes the space-time distribution around slowly rotating and stationary massive objects like the planet Earth and the Sun. However, the Kerr metric represented by Eq. 35 describes the space-time distribution around rapidly rotating massive objects like a Kerr black hole [6, 9, 10].

$$ds^2 = -c^2\left[1-\frac{2GM}{rc^2}\right]dt^2+\left[1-\frac{2GM}{rc^2}\right]^{-1}dr^2+r^2d\theta^2+r^2\sin\theta^2\,d\phi^2, \quad (34)$$

$$ds^2 = -c^2\left[1-\frac{2GM}{rc^2}\right]dt^2- \frac{4aGM}{rc^2}\,drd\phi+\left[1-\frac{2GM}{rc^2}+\frac{a^2}{r^2}\right]^{-1}dr^2+\left[1+\frac{a^2}{r^2}+\frac{2a^2GM}{r^3\,c^2}\right]r^2d\phi^2, \quad (35)$$

Before extracting the equation that describes a free surface in the equatorial plane of the Schwarzschild space-time metric the following illustrations provide insight into the analogy between a potential vortex and the space-time around stationary and rapidly rotating massive objects. Where the following establishes the boundary conditions for the analysis.

$$\theta=\frac{\pi}{2},\ d\theta=0,\ d\phi=0,\ dt>0. \quad (36)$$

Where dt is chosen to plot the upper half of the **Einstein-Rosen Bridge**[9]. First, Figure 6 displays the space-time distribution for $r>2GM$ around massive objects using the Schwarzschild space-time metric (Eqn. 34). That is, Figure 6 displays the space-time distribution when the radius is greater than the Schwarzschild radius on the equatorial plane of stationary massive objects like the Earth, Sun or black hole. The form of the curve closely resembles the theoretical potential vortex shapes illustrated in Figures 1-3. Second, Figure 7 displays the normalized space-time distribution for $r>2GM$ on the equatorial plane for rapidly rotating massive objects that are described by the Kerr space-time metric (Eqn. 35). Similar for stationary objects, the space-time of rotating objects closely resembles the experimental potential vortex shapes illustrated in Figures 1-3. Because the space-time interval is normalized by the maximum space-time interval in the range of interest the specific parameters of the metric displayed in Figures 6-7 are not important. However, for this analysis the parameters use the frame-dragging data from

Figure 3 where the mass of the object is one Sun and the object rotates at 40 revolutions per day for the Kerr metric analysis. However, the results plotted in Figures 6-7 are interesting because the results closely match the shape of a potential vortex supporting the premise of this paper that there is an analogy between General Relativity and the potential vortex.

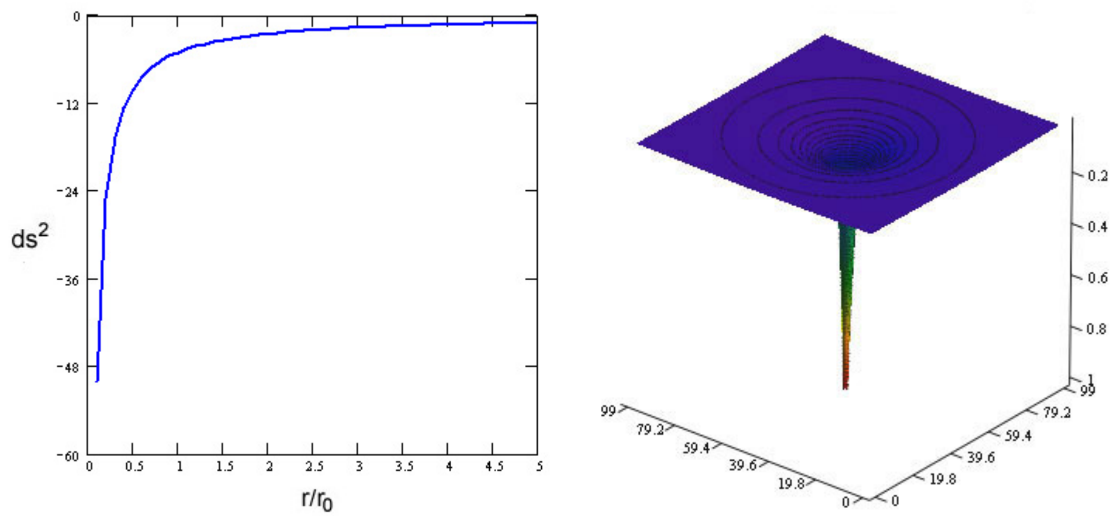

FIG. 6. Normalized Schwarzschild space-time, (a) plane perpendicular to spin axis and (b) 3D view.

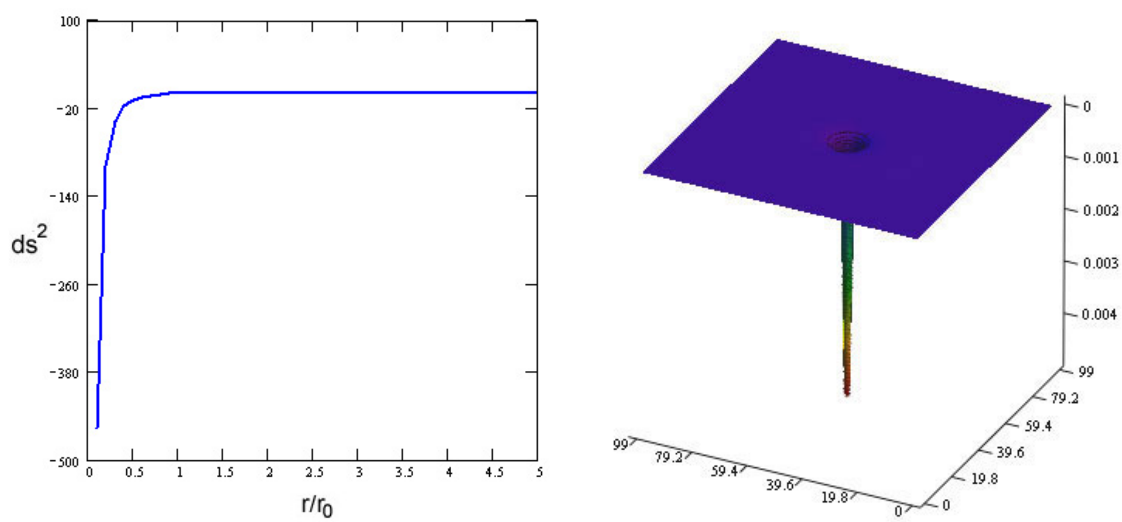

FIG. 7. Normalized Kerr space-time, (a) plane perpendicular to spin axis and (b) 3D view.

Now that we have illustrated the basic similarity between the potential vortex and the space-time interval distribution, ds^2 around stationary and rotating massive objects this section will derive the equation describing the free surface of Schwarzschild space-time or the shape of space-time around stationary massive objects. The complete solution of the Schwarzschild metric [6, 9, 10] in 4D spherical coordinates (one time and three space) should have the same signature as the Minkowski metric when written as before for the Schwarzschild metric, Eqn. 34:

$$ds^2 = -c^2\left[1-\frac{2GM}{rc^2}\right]dt^2 + \left[1-\frac{2GM}{rc^2}\right]^{-1}dr^2 + r^2 d\theta^2 + r^2\sin\theta^2\,d\phi^2, \quad (37)$$

To match the 2 dimensions of the potential vortex line element choose the following conditions to establish the equatorial plane of the analysis.

$$\theta = \frac{\pi}{2}, \quad d\theta = 0, \quad dt = 0, \quad (38)$$

This reduces the Schwarzschild metric to the following equation.

$$ds^2 = \left[1-\frac{2GM}{rc^2}\right]^{-1}dr^2 + r^2 d\phi^2, \quad (39)$$

Insert this metric into the 3D cylindrical space of the vortex using this metric.

$$ds^2 = dz^2 + dr^2 + r^2\,d\phi^2, \quad (40)$$

On the surface of z = z(r), Eq. (40) can be written as.

$$ds^2 = \left(\frac{dz}{dr}\right)^2 dr^2 + dr^2 + r^2\,d\phi^2, \quad (41)$$

$$ds^2 = [1+\left(\frac{dz}{dr}\right)^2]\,dr^2 + r^2\,d\phi^2, \quad (42)$$

Comparing Eq. (39) and Eq. (42) on the surface, z = z(r).

$$[1+\left(\frac{dz}{dr}\right)^2]\,dr^2 + r^2\,d\phi^2 = \left[1-\frac{2GM}{rc^2}\right]^{-1}dr^2 + r^2 d\phi^2, \quad (43)$$

Then, after some algebra derive the slope of the Schwarzschild vortex surface, dz/dr.

$$\frac{dz}{dr} = \sqrt{\frac{2GM}{rc^2-2GM}}, \quad (44)$$

After integration of Eq. 44 the surface z(r) has the following form for massive non-rotating objects based on the Schwarzschild metric of General Relativity.

$$z = 2\sqrt{\frac{2\,G\,M}{c^2}\left(r-\frac{2\,G\,M}{c^2}\right)}. \quad (45)$$

Where Eqn. 45 represents the equation that describes the free surface of Schwarzschild space-time or the shape of space-time around stationary massive objects that are similar to the shape of a potential vortex previously displayed in Figures 1-2.

E. Extracted Schwarzschild space-time and potential vortex compared

The space-time surface extracted from Schwarzschild space-time using Eq. 45 illustrates the similarity between General Relativity and the potential vortex. The similarity between the Schwarzschild space-time shape in Figure 8a and the potential vortex surface in Figure 8b demonstrate the relationship. For the distribution of Schwarzschild space-time, r represents the radial distance from the center of a massive object while r_0 represents the radius of the massive object. For the potential vortex, r represents the radius from the rotating core while r_0 represents the radius of the rotating core. Finally, z is the deflection of space-time in Figure 8a and potential vortex surface deflection in Figure 8b.

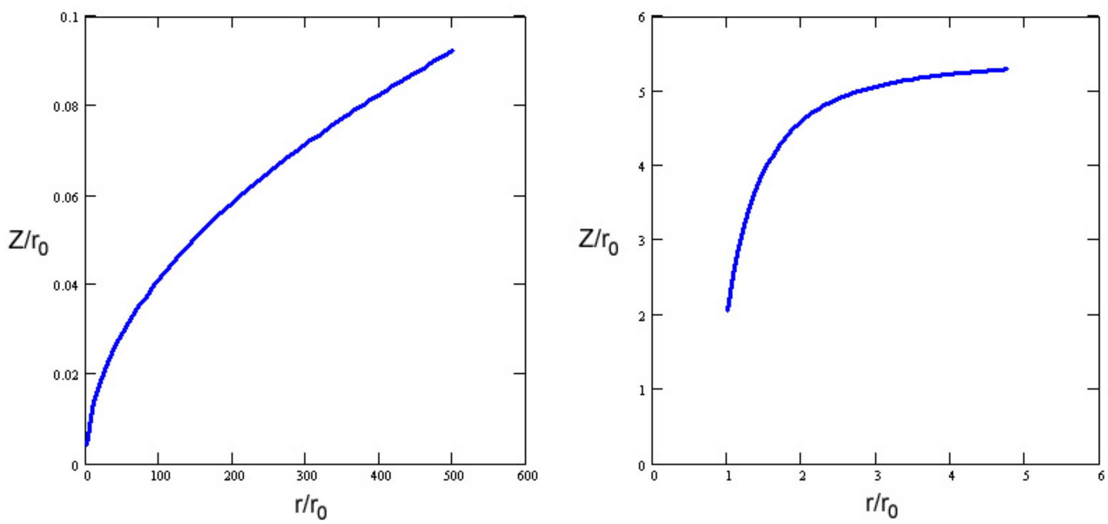

FIG. 8. (a) Sun's Schwarzschild space-time surface (Eq. 45). (b) Potential vortex surface 10 Hz.

The surface deflections extracted from Schwarzschild space-time in Figure 8a and the potential vortex surface in Figure 8b illustrates the similarity between General Relativity and potential (free) vortex flow. This similarity is also illustrated in Figures 6-7 where Schwarzschild and Kerr space-time approximate the shape of a potential vortex for stationary and rotating objects using the concepts of General Relativity. Therefore, the potential vortex seems to be a reasonable analogy for the space-time around non-rotating and rotating massive objects based on the observed shape of space-time near and around those objects as these illustrations demonstrates.

V. DISCUSSION

A. PREDICTIONS OF THE FREE VORTEX ANALOGY

The analogy between the fluid free vortex and GR implies that massive objects generate a space-time potential vortex due to the existence of a circulating inner rotational region known as a solid vortex. The potential vortex, which is irrotational at steady state, is generated by a rotational inner solid vortex that forms the region at the heart of every potential vortex. The solution of the fluid flow continuity and momentum equations proves that within every irrotational potential vortex exists a rotational solid vortex. Only during startup do viscous interactions play a part in generating the steady state free-surface shape and velocity profile of a potential vortex. Therefore, a transient solution of the potential vortex is not required for an analogy with General Relativity. However, the steady state solution of a potential vortex is irrotational and the fluid interactions are inviscid similar to the space-time around massive objects. In effect it is postulated that all matter-energy has an intrinsic spin or vortex strength, K that is characteristic of all massive objects that curve space-time due to the presence of mass-energy. The constant, K for a potential vortex equals $K = V_0 r_0$. Where, K in the potential flow equation can be put in terms of gravitational constant, G and mass, M of an object that sets up a potential vortex in space-time with the following orbital velocity profile.

$$u_\theta = \frac{K}{r}. \tag{45}$$

This paper proposes that a potential vortex is formed when approaching all non-rotating and rotating massive objects that have a finite central region with the characteristics of a rotational solid vortex. Similar to a fluid potential vortex starting from a non-rotating state, space-time will swirl through the central core to form a potential vortex. Further, it is postulated that if the mass of an object exceeds the Schwarzschild limit a swirling rotational solid vortex core that uses torsion to spin space-time replaces the usual singularity at the heart of every black hole. Therefore, potential vortex theory may eliminate the need for the usual singularity at the heart of a black hole that could form the basis of a theory that connects quantum mechanics or the physics of the very small and General Relativity or the physics of the very large. Also implied in these assumptions is the premise that a black hole is a bridge to another universe with a black hole on our side and a white hole on the other side that connects two membrane universes. Where, one membrane is our universe and the other membrane is a parallel universe located a Planck distance apart in space-time connected by an Einstein-Rosen Bridge.

B. Warp drive propulsion

An implication for the existence of a superfluid potential vortex substratum is that interesting fluid mechanical characteristics of space-time can be revealed. Specifically, an interesting by product of a superfluid substratum is the Magnus effect. The Magnus effect is the force exerted on a rapidly spinning cylinder or sphere moving through air or another fluid in a direction at an angle to the axis of spin. The sideways force is responsible for the swerving of balls when hit or thrown with spin. For example, if an object composed of energy-momentum rotates in the gravitational field of another massive object a Magnus effect based on the superfluid of space-time will impart a sideways force on the object and an associated acceleration in the substratum. In exactly the same way the surrounding fluid is deformed by a spinning object, space-time will be compressed on one side of the object and expanded on the other side of the object generating an imbalance in space-time. The deformed space-time surrounding the spinning object could be called a warp bubble that uses the imbalance within space-time to propel an object perpendicular to the field lines of the surrounding superfluid. Speeds approaching the speed of light are not practical but exotic materials are not required for a device based on this technology.

Many aspects of this hypothesis are similar to Miguel Alcubierre's paper[11] where General Relativity was employed to illustrate how a warp bubble using opposing regions of expanding and contracting space-time can theoretically propel a starship to velocities approaching and even exceeding the speed of light. According to the results expressed in Miguel Alcubierre's paper, gravity and acceleration are not distinguishable and are caused by the curvature of space-time. A warp bubble is a specific warp metric solution of General Relativity and is a combination of positive and negative energy fields that pushes and pulls our starship forward to bring our destination to us just like a conveyer belt. The exotic ingredient required to make a warp bubble according to this theory is negative energy, which has the unusual property of being able to make ordinary matter fall up in a gravitational field. According to Alcubierre's paper the space-time in front of a warp bubble is compressed pulling our destination to us. At the same time the space-time behind a warp bubble is expanding pushing us to our destination. The compression and expansion process happens in an instant and at many times the speed of light making faster than light travel possible. The combination of positive and negative energy produces an expansion of space behind the bubble and a contraction of space in front of the bubble. In other words, creating space behind the bubble pushes us to our destination and destroying space in front of the bubble pulls us to our destination. This mechanism allows travel many times faster than the speed of light relative to the Earth without exceeding the speed of light in our local frame of reference, the warp bubble. The main requirement for implementing Alcubierre's warp drive is negative energy, which is a property of the vacuum in space where subatomic particles smaller than an atom dart into and out of existence almost instantaneously. However, the forces produced by the Magnus effect on space-time proposed here are more modest and do not require exotic negative energy as specified by Alcubierre's warp drive. At most a modest acceleration can be realized using the effect described here within the gravitational influence of a very massive object.

C. Removing the black hole singularity

Further, it is postulated that if the mass of an object exceeds the Schwarzschild limit a swirling rotational solid vortex core that uses torsion to spin space-time replaces the usual singularity at the heart of every black hole. Therefore, potential vortex theory may eliminate the need for the usual singularity at the heart of a black hole that could form the basis of a theory that connects quantum mechanics or the physics of the very small and General Relativity or the physics of the very large. Also implied in these assumptions is the premise that a black hole is a bridge to another universe with a black hole on our side and a white hole on the other side that connects two membrane universes. Where, one membrane is our universe and the other membrane is another universe located a Planck distance apart in space-time.

D. Validation of the General Relativity-potential vortex analogy

The following 7 points illustrate that an analogy exists between the potential vortex and the generation of space-time curvature around massive objects as predicted by Einstein's theory of General Relativity. These points are based on observations from the potential vortex experiment and GP-B, the free-surface shape extracted from Schwarzschild's space-time, a unit analysis of the curvature and energy-momentum components of the potential vortex and the analogous components from Einstein's Field Equation and finally, observations from black hole dynamics compared to potential vortex dynamics.

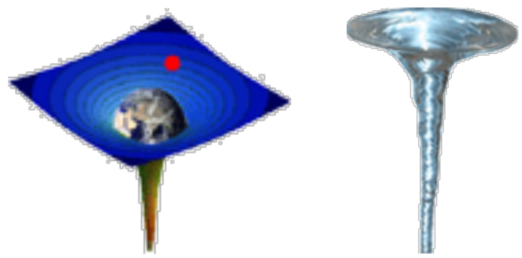

FIG. 9. (a) Earth space-time measured by GP-B. (b) Potential vortex analogy.

1. The rotating inner boundary of a potential vortex is represented by a rotating inner boundary, which is an analogy for spinning massive objects like the Earth, star or black hole. Where, the potential vortex finite inner velocity boundary condition and zero velocity outer boundary condition eliminate the solid vortex as a potential analogy for General Relativity and gravitation and make the potential vortex a logical alternative.

2. A potential vortex seems to be a reasonable analogy for the space-time around non-rotating and rotating massive objects based on the observed shape of space-time around those objects. The surface deflection extracted from Schwarzschild space-time illustrates the similarity between General Relativity and potential vortex flow. This similarity is illustrated in Figures 7-8 and the 3-dimensional illustrations in Figure 9 where the space-time around massive objects approximates the characteristics of a potential vortex.

3. Similar to objects rotating around black holes, objects that rotate around a potential vortex can only move in the same direction as the rotating inner boundary. Where, the local deformation or curvature of the vortex free surface causes the object to orbit at

some characteristic orbital velocity depending on the radius of orbit. Therefore, the velocity of the object orbiting at a point on the free surface is purely a function of the physical characteristics of the inner boundary having some angular rate of rotation and radius.

4. Similar to the singularity that exists at the center of every black hole at the heart of every potential vortex exists a small but finite solid vortex that may provide clues about the nature of the black hole singularity. Where, it is postulated that a swirling finite solid vortex at the center of a black hole uses torsion to spin space-time replacing the usual singularity at the heart of a black hole.

5. As verified by these experiments a potential vortex is incompressible and irrotational at steady state as is the superfluid postulated to exist using quantum gravity. This paper proposes that an analogy exists between an irrotational potential vortex and the description of space-time known as superfluid vacuum theory (SVT) where space is filled with a frictionless fluid that is isotropic and inviscid at zero temperature.

6. An analysis of the field equation for potential vortex orbital motion confirm the units for curvature, R (m^{-2}) and the units for energy-momentum, T $(\frac{erg}{cm^3})$ are identical to their counterparts in the Einstein field equation from General Relativity. In addition, the form of the potential vortex field equation is similar to the form of the Einstein field equation derived from General Relativity.

7. The orbital velocity of a potential vortex is derivable from the curvature or slope of the potential vortex surface as explained in Section-IV and illustrated in Figure 5.

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