A New Principle of Conservation of Energy

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Abstract

In classical mechanics, this paper presents a new principle of conservation of energy which is invariant under transformations between reference frames and which can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

Definitions of Work, K and U

If we consider a system of N particles then the total work W done by the forces acting on the system of particles, the total kinetic energy K of the system of particles and the total potential energy U of the system of particles, are as follows:

$$W = \sum_{i=1}^{N} + \left(\int_1^2 \mathbf{F}_i \cdot d\bar{\mathbf{r}}_i + \Delta\, \frac{1}{2}\, \mathbf{F}_i \cdot \bar{\mathbf{r}}_i \right)$$

$$\Delta K = \sum_{i=1}^{N} \Delta \left(\frac{1}{2}\, m_i\, \bar{\mathbf{v}}_i \cdot \bar{\mathbf{v}}_i + \frac{1}{2}\, m_i\, \bar{\mathbf{a}}_i \cdot \bar{\mathbf{r}}_i \right)$$

$$\Delta U = \sum_{i=1}^{N} - \left(\int_1^2 \mathbf{F}_i \cdot d\bar{\mathbf{r}}_i + \Delta\, \frac{1}{2}\, \mathbf{F}_i \cdot \bar{\mathbf{r}}_i \right)$$

where $\bar{\mathbf{r}}_i = \mathbf{r}_i - \mathbf{r}_{cm}$, $\bar{\mathbf{v}}_i = \mathbf{v}_i - \mathbf{v}_{cm}$, $\bar{\mathbf{a}}_i = \mathbf{a}_i - \mathbf{a}_{cm}$, $\mathbf{r}_i$, $\mathbf{v}_i$ and $\mathbf{a}_i$ are the position, the velocity and the acceleration of the i-th particle, $\mathbf{r}_{cm}$, $\mathbf{v}_{cm}$ and $\mathbf{a}_{cm}$ are the position, the velocity and the acceleration of the center of mass of the system of particles, m_i is the mass of the i-th particle, and $\mathbf{F}_i$ is the net force acting on the i-th particle.

Theorems of K and U

In a system of N particles, the total work W done by the forces acting on the system of particles is equal to the change in the total kinetic energy K of the system of particles.

$$W = +\Delta K$$

In a system of N particles, the total work W done by the conservative forces acting on the system of particles is equal and opposite in sign to the change in the total potential energy U of the system of particles.

$$W = -\Delta U$$

Conservation of Energy

In a system of N particles, if the non-conservative forces acting on the system of particles do not perform work then the total (mechanical) energy of the system of particles remains constant.

$$K + U = constant$$

General Observations

The new principle of conservation of energy is invariant under transformations between reference frames.

The new principle of conservation of energy can be applied in any reference frame (rotating or non-rotating) (inertial or non-inertial) without the necessity of introducing fictitious forces.

The new principle of conservation of energy would be valid even if Newton's third law of motion were false in an inertial reference frame.

The new principle of conservation of energy would be valid even if Newton's three laws of motion were false in a non-inertial reference frame.

Annex

Work and Potential Energy

If we consider an isolated system of N particles and if Newton's third law of motion is valid then the total work W done by the forces acting on the system of particles and the total potential energy U of the system of particles, are as follows:

$$W = \sum_{i=1}^{N} + \left(\int_1^2 \mathbf{F}_i \cdot d\mathbf{r}_i + \Delta \, \frac{1}{2} \, \mathbf{F}_i \cdot \mathbf{r}_i \right)$$

$$\Delta U = \sum_{i=1}^{N} - \left(\int_1^2 \mathbf{F}_i \cdot d\mathbf{r}_i + \Delta \, \frac{1}{2} \, \mathbf{F}_i \cdot \mathbf{r}_i \right)$$

where $\mathbf{r}_i$ is the position of the i-th particle, and $\mathbf{F}_i$ is the net force acting on the i-th particle.

Kinetic Energy

If we consider a system of N particles then the total kinetic energy K of the system of particles can also be expressed as follows:

$$K = \sum_{i=1}^{N} \frac{1}{2} \, m_i \, \mathbf{v}_i \cdot \mathbf{v}_i - \frac{1}{2} \, m_{cm} \, \mathbf{v}_{cm} \cdot \mathbf{v}_{cm} + \sum_{i=1}^{N} \frac{1}{2} \, m_i \, \mathbf{a}_i \cdot \mathbf{r}_i - \frac{1}{2} \, m_{cm} \, \mathbf{a}_{cm} \cdot \mathbf{r}_{cm}$$

that is:

$$K = \sum_{i=1}^{N} \sum_{j>i}^{N} \left(\frac{1}{2} \, \frac{m_i \, m_j}{m_{cm}} \, (\mathbf{v}_i - \mathbf{v}_j) \cdot (\mathbf{v}_i - \mathbf{v}_j) + \frac{1}{2} \, \frac{m_i \, m_j}{m_{cm}} \, (\mathbf{a}_i - \mathbf{a}_j) \cdot (\mathbf{r}_i - \mathbf{r}_j) \right)$$

where $\mathbf{r}_i$, $\mathbf{v}_i$, $\mathbf{a}_i$, $\mathbf{r}_j$, $\mathbf{v}_j$, $\mathbf{a}_j$, $\mathbf{r}_{cm}$, $\mathbf{v}_{cm}$, $\mathbf{a}_{cm}$ are the positions, the velocities and the accelerations of the i-th particle, of the j-th particle and of the center of mass of the system of particles, and m_i, m_j, m_{cm} are the masses of the i-th particle, of the j-th particle and of the center of mass of the system of particles.