THE SHORT PROOFS

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Dedicated to my Parents and my Brother

ABSTRACT. The short proofs of the Fermat's Last Theorem for even $n \geq 4$.

I. INTRODUCTION

It is known that for each primitive Pythagorean triple (x, y, z) there exist two relatively prime natural numbers u, v such that $u - v$ is positive and odd.

Moreover

$$x^2 + y^2 = z^2 \wedge (x+y)^2 + [\pm(x-y)]^2 = 2z^2, \tag{1}$$

where z is odd because for all $a, b \in \mathbb{N}$: $\frac{1}{2}\left[(2a+1)^2 + (2b+1)^2\right]$ is odd.

II. THE FERMAT'S LAST THEOREM FOR EVEN n

Theorem 1 (FLT)**.** *For all* $n \in \{4, 6, 8, ...\}$ *the equation*

$$X^n + Y^n = Z^n$$

has no primitive solutions in $\mathbb{N}_1$.

Proof of the Main Theorem. Suppose that for some $n \in \{4, 6, 8, ...\}$ the equation

$$X^n + Y^n = Z^n$$

has primitive solutions $[X, Y, Z]$ in $\mathbb{N}_1$.

A. The Proof of the Main Theorem for $4 \mid n$.

We assume that for some relatively prime natural numbers u, v such that $u - v$ is positive and odd:

$$\left[u^2 - v^2 = \left(X^{\frac{n}{4}}\right)^2 \wedge 2uv = Y^{\frac{n}{2}} \wedge u^2 + v^2 = Z^{\frac{n}{2}}\right].$$

Thus on the strength of (1):

$$\left[2u^2 = \left(X^{\frac{n}{4}}\right)^2 + \left(Z^{\frac{n}{4}}\right)^2 \wedge v - X^{\frac{n}{4}} = \pm X^{\frac{n}{4}} \wedge X^{\frac{n}{4}} + v = Z^{\frac{n}{4}}\right] \Rightarrow$$

$$\left(3X^{\frac{n}{4}} = Z^{\frac{n}{4}} \vee X^{\frac{n}{4}} = Z^{\frac{n}{4}}\right) \Rightarrow \gcd(X, Z) > 1,$$

which is inconsistent with $\gcd(X, Z) = 1$. ✠

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B. The Proof of the Main Theorem for $4 \nmid n$.

We assume that for some $m \in \{3, 5, 7, ...\}$ and for some mutually coprime odd natural numbers a, b, c, d and for some relatively prime natural numbers u, v such that $u - v$ is positive and odd:

$$\Big[2m = n \wedge (abcd)^{2m} = (u+v)^2 (u-v)^2 = (X^m)^2 = (Z^m + Y^m)(Z^m - Y^m) \wedge$$

$$(ab)^m = u + v \wedge (cd)^m = u - v \wedge u^2 + v^2 = Z^m \wedge 2uv = Y^m \wedge$$

$$b^{2m} = Z + Y \wedge d^{2m} = Z - Y \wedge (bd)^{2m} = Z^2 - Y^2 \wedge \ 4 \mid Y\Big],$$

which is inconsistent with $4 \nmid Y$ [2]. This is the proof. □

III. The Proofs Of The Fermat's Last Theorem For $n = 4$

Theorem 2. *The equation*

$$Z^4 - Y^4 = x^2$$

has no primitive solutions in $\mathbb{N}_1$.

Proof. Suppose that the equation

$$Z^4 - Y^4 = x^2$$

has the primitive solutions $[Z, Y, x]$ in $\mathbb{N}_1$.

A. The Proof For Odd x.

We assume that for some mutually coprime natural numbers Z, Y, p, q, where only Y is even:

$$\left[\left(Z^2 + Y^2 = p^2 \wedge Z^2 - Y^2 = q^2 \text{ [1] }\right) \wedge 2Z^2 = p^2 + q^2 \wedge pq = x\right].$$

Thus on the strength of (1):

$$(p = Y + q \wedge \pm q = q - Y) \Rightarrow (p = q \vee p = 3q) \Rightarrow \gcd(p, q) > 1,$$

which is inconsistent with $\gcd(p, q) = 1$. ✠

B. The Proof For Even x.

We assume that for some relatively prime natural numbers u, v such that $u - v$ is positive and odd:

$$\left(u^2 + v^2 = Z^2 \wedge u^2 - v^2 = Y^2 \wedge 2uv = x \wedge 2u^2 = Z^2 + Y^2\right).$$

Thus on the strength of (1):

$$(Z = Y + v \wedge \pm Y = v - Y) \Rightarrow (Z = 3Y \vee Z = Y) \Rightarrow \gcd(Z, Y) > 1,$$

which is inconsistent with $\gcd(Z, Y) = 1$. This is the proof. □

Corollary 1. *The equation* $Z^4 - Y^4 = x^2$ *has no primitive solutions* $[Z, Y, X]$ *in* $\mathbb{N}_1$, *where* $X = \sqrt{x}$. *This is the corollary.*

References

[1] Dolan, S. : "Fermat's method of descente infinie", Mathematical Gazette 95, July 2011, 269-271.

[2] Guła, L.W. : http://www.ijetae.com/files/Volume2Issue12/IJETAE_1212_14.pdf

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