Neutrosophic Rings II

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Abstract: This paper is the continuation of the work started in [12]. The present paper is devoted to the study of ideals of neutrosophic rings. Neutrosophic quotient rings are also studied.

Key Words: Neutrosophic ring, neutrosophic ideal, pseudo neutrosophic ideal, neutrosophic quotient ring.

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§1. Introduction

The concept of neutrosophic rings was introduced by Vasantha Kandasamy and Florentin Smarandache in [1] where neutrosophic polynomial rings, neutrosophic matrix rings, neutrosophic direct product rings, neutrosophic integral domains, neutrosophic unique factorization domains, neutrosophic division rings, neutrosophic integral quaternions, neutrosophic rings of real quarternions, neutrosophic group rings and neutrosophic semigroup rings were studied. In [12], Agboola et al further studied neutrosophic rings. The structure of neutrosophic polynomial rings was presented. It was shown that *division algorithm* is generally not true for neutrosophic polynomial rings and it was also shown that a neutrosophic polynomial ring $\langle R \bigcup I \rangle [x]$ cannot be an Integral Domain even if R is an Integral Domain. Also in [12], it was shown that $\langle R \bigcup I \rangle [x]$ cannot be a *unique factorization domain* even if R is a unique factorization domain and it was also shown that every non-zero neutrosophic principal ideal in a neutrosophic polynomial ring is not a neutrosophic prime ideal. The present paper is however devoted to the study of ideals of neutrosophic rings and neutrosophic quotient rings are also studied.

§2. Preliminaries and Results

For details about neutrosophy and neutrosophic rings, the reader should see [1] and [12].

Definition 2.1 *Let $(R, +, \cdot)$ be any ring. The set*

$$\left\langle R \bigcup I \right\rangle = \{a + bI : a, b \in R\}$$

is called a neutrosophic ring generated by R and I under the operations of R, where I is the neutrosophic element and $I^2 = I$.

If $\langle R \bigcup I \rangle = \langle \mathbb{Z}_n \bigcup I \rangle$ with $n < \infty$, then $o(\langle \mathcal{Z}_n \cup I \rangle) = n^2$. Such a $\langle R \bigcup I \rangle$ is said to be a commutative neutrosophic ring with unity if $rs = sr$ for all $r, s \in \langle R \bigcup I \rangle$ and $1 \in \langle R \bigcup I \rangle$.

Definition 2.2 *Let $\langle R \bigcup I \rangle$ be a neutrosophic ring. A proper subset P of $\langle R \bigcup I \rangle$ is said to be a neutrosophic subring of $\langle R \bigcup I \rangle$ if $P = \langle S \cup nI \rangle$, where S is a subring of R and n an integer, P is said to be generated by S and nI under the operations of R.*

Definition 2.3 *Let $\langle R \bigcup I \rangle$ be a neutrosophic ring and let P be a proper subset of $\langle R \bigcup I \rangle$ which is just a ring. Then P is called a subring.*

Definition 2.4 *Let T be a non-empty set together with two binary operations $+$ and $\cdot$. T is said to be a pseudo neutrosophic ring if the following conditions hold:*

(1) *T contains elements of the form $a + bI$, where a and b are real numbers and $b \neq 0$ for at least one value;*
(2) *$(T, +)$ is an abelian group;*
(3) *$(T, \cdot)$ is a semigroup;*
(4) *$\forall x, y, z \in T$, $x(y + z) = xy + xz$ and $(y + z)x = yx + zx$.*

Definition 2.5 *Let $\langle R \bigcup I \rangle$ be any neutrosophic ring. A non-empty subset P of $\langle R \bigcup I \rangle$ is said to be a neutrosophic ideal of $\langle R \bigcup I \rangle$ if the following conditions hold:*

(1) *P is a neutrosophic subring of $\langle R \bigcup I \rangle$;*
(2) *for every $p \in P$ and $r \in \langle R \bigcup I \rangle$, $rp \in P$ and $pr \in P$.*

If only $rp \in P$, we call P a left neutrosophic ideal and if only $pr \in P$, we call P a right neutrosophic ideal. When $\langle R \bigcup I \rangle$ is commutative, there is no distinction between rp and pr and therefore P is called a left and right neutrosophic ideal or simply a neutrosophic ideal of $\langle R \bigcup I \rangle$.

Definition 2.6 *Let $\langle R \bigcup I \rangle$ be a neutrosophic ring and let P be a pseudo neutrosophic subring of $\langle R \bigcup I \rangle$. P is said to be a pseudo neutrosophic ideal of $\langle R \bigcup I \rangle$ if $\forall p \in P$ and $r \in \langle R \bigcup I \rangle$, $rp, pr \in P$.*

Example 2.7 Let $\langle \mathcal{Z} \cup I \rangle$ be a neutrosophic ring of integers and let $P = \langle n\mathcal{Z} \cup I \rangle$ for a positive integer n. Then P is a neutrosophic ideal of $\langle \mathcal{Z} \cup I \rangle$.

Example 2.8 Let $\langle R \bigcup I \rangle = \left\{ \begin{bmatrix} x & y \\ 0 & z \end{bmatrix} : x, y, z \in \langle \mathcal{R} \cup I \rangle \right\}$ be the neutrosophic ring of 2×2 matrices and let $P = \left\{ \begin{bmatrix} x & y \\ 0 & 0 \end{bmatrix} : x, y \in \langle \mathcal{R} \cup I \rangle \right\}$. Then P is a neutrosophic ideal of $\langle \mathcal{Z} \cup I \rangle$.

Theorem 2.9 *Let $\langle \mathcal{Z}_p \cup I \rangle$ be a neutrosophic ring of integers modulo p, where p is a prime number. Then:*

(1) *$\langle \mathcal{Z}_p \cup I \rangle$ has no neutrosophic ideals and*
(2) *$\langle \mathcal{Z}_p \cup I \rangle$ has only one pseudo neutrosophic ideal of order p.*

Proposition 2.10 *Let P, J and Q be neutrosophic ideals (resp. pseudo neutrosophic ideals) of a neutrosophic ring $\langle \mathcal{R} \cup I \rangle$. Then*

> (1) $P + J$ *is a neutrosophic ideal (resp. pseudo neutrosophic ideal) of $\langle \mathcal{R} \cup I \rangle$;*
> (2) PJ *is a neutrosophic ideal (resp. pseudo neutrosophic ideal) of $\langle \mathcal{R} \cup I \rangle$;*
> (3) $P \cap J$ *is a neutrosophic ideal (resp. pseudo neutrosophic ideal) of $\langle \mathcal{R} \cup I \rangle$;*
> (4) $P(JQ) = (PJ)Q$;
> (5) $P(J + Q) = PJ + PQ$;
> (6) $(J + Q)P = JP + QP$.

Proof The proof is the same as in the classical ring. □

Proposition 2.11 *Let $\langle \mathcal{R} \cup I \rangle$ be a neutrosophic ring and let P be a subset of $\langle \mathcal{R} \cup I \rangle$ t. Then P is a neutrosophic ideal (resp. pseudo neutrosophic ideal) iff the following conditions hold:*

> (1) $P \neq \emptyset$;
> (2) $a, b \in P \Rightarrow a - b \in P$;
> (3) $a \in P, r \in \langle R \cup I \rangle \Rightarrow ra, ar \in P$.

Proof the proof is the same as in the classical ring. □

Proposition 2.12 *Let $\langle \mathcal{R} \cup I \rangle$ be any neutrosophic ring. Then $\langle \mathcal{R} \cup I \rangle$ and $< 0 >$ are neutrosophic ideals of $\langle \mathcal{R} \cup I \rangle$.*

Proposition 2.13 *Let $\langle \mathcal{R} \cup I \rangle$ be a neutrosophic ring with unity (no unit in $\langle \mathcal{R} \cup I \rangle$ since I^{-1} does not exist in $\langle \mathcal{R} \cup I \rangle$) and let P be a neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$. If $1 \in P$ then $P = \langle \mathcal{R} \cup I \rangle$.*

Proposition 2.14 *Let $\langle \mathcal{R} \cup I \rangle$ be a neutrosophic ring with unity (no unit in $\langle \mathcal{R} \cup I \rangle$ since I^{-1} does not exist in $\langle \mathcal{R} \cup I \rangle$) and let P be a pseudo neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$. If $1 \in P$ then $P \neq \langle \mathcal{R} \cup I \rangle$.*

Proof Suppose that P is a pseudo neutrosophic ideal of the neutrosophic ring $\langle \mathcal{R} \cup I \rangle$ with unity and suppose that $1 \in P$. Let r be an arbitrary element of $\langle \mathcal{R} \cup I \rangle$. Then by the definition of P, $r.1 = r$ should be an element of P but since P is not a neutrosophic subring of $\langle \mathcal{R} \cup I \rangle$, there exist some elements $b = x + yI$ with $x, y \neq 0$ in $\langle \mathcal{R} \cup I \rangle$ which cannot be found in P. Hence $P \neq \langle \mathcal{R} \cup I \rangle$. □

Proposition 2.15 *Let $\langle \mathcal{R} \cup I \rangle$ be a neutrosophic ring and let $a = x + yI$ be a fixed element of $\langle \mathcal{R} \cup I \rangle$. Suppose that $P = \{ra : r \in \langle R \cup I \rangle\}$ is a subset of $\langle \mathcal{R} \cup I \rangle$.*

> (1) *If $x, y \neq 0$, then P is a left neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$;*
> (2) *If $x = 0$, then P is a left pseudo neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$.*

Proof (1) is clear. For (2), if $x = 0$ then each element of P is of the form sI for some $s \in R$. Hence $P = \{0, sI\}$ which is a left pseudo neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$. □

Theorem 2.16 *Every ideal of a neutrosophic ring $\langle \mathcal{R} \cup I \rangle$ is either neutrosophic or pseudo neutrosophic.*

Proof Suppose that P is any ideal of $\langle \mathcal{R} \cup I \rangle$. If $P \neq (0)$ or $P \neq \langle \mathcal{R} \cup I \rangle$, then there exists a subring S of R such that for a positive integer n, $P = < S \cup nI >$. Let $p \in P$ and $r \in \langle \mathcal{R} \cup I \rangle$. By definition of P, $rp, pr \in P$ and the elements rp and pr are clearly of the form $a + bI$ with at least $b \neq 0$. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad$ $\square$

Definition 2.17 *Let $\langle \mathcal{R} \cup I \rangle$ be a neutrosophic ring.*

(1) *If P is a neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$ generated by an element $r = a + bI \in \langle \mathcal{R} \cup I \rangle$ with $a, b \neq 0$, then P is called a neutrosophic principal ideal of $\langle \mathcal{R} \cup I \rangle$, denoted by (r).*

(2) *If P is a pseudo neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$ generated by an element $r = aI \in \langle R \cup I \rangle$ with $a \neq 0$, then P is called a pseudo neutrosophic principal ideal of $\langle \mathcal{R} \cup I \rangle$, denoted by (r).*

Proposition 2.18 *Let $\langle \mathcal{R} \cup I \rangle$ be a neutrosophic ring and let $r = a + bI \in \langle \mathcal{R} \cup I \rangle$ with $a, b \neq 0$.*

(1) *(r) is the smallest neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$ containing r;*

(2) *Every pseudo neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$ is contained in some neutrosophic ideal of $\langle \mathcal{R} \cup I \rangle$.*

Proposition 2.19 *Every pseudo neutrosophic ideal of $\langle \mathcal{Z} \cup I \rangle$ is principal.*

Definition 2.20 *Let $\langle R \cup I \rangle$ be a neutrosophic ring and let P be a neutrosophic ideal (resp. pseudo neutrosophic ideal) of $\langle R \cup I \rangle$.*

(1) *P is said to be maximal if for any neutrosophic ideal (resp. pseudo neutrosophic ideal) J of $\langle R \cup I \rangle$ such that $P \subseteq J$ we have either $J = M$ or $J = \langle R \cup I \rangle$.*

(2) *P is said to be a prime ideal if for any two neutrosophic ideals (resp. pseudo neutrosophic ideals) J and Q of $\langle R \cup I \rangle$ such that $JQ \subseteq P$ we have either $J \subseteq P$ or $Q \subseteq P$.*

Proposition 2.21 *Let $\langle R \cup I \rangle$ be a commutative neutrosophic ring with unity and let P be a neutrosophic ideal (resp. pseudo neutrosophic ideal) of $\langle R \cup I \rangle$. Then P is prime iff $xy \in P$ with x and y in $\langle R \cup I \rangle$ implies that either $x \in P$ or $y \in P$.*

Example 2.22 In $\langle \mathcal{Z} \cup I \rangle$ the neutrosophic ring of integers:

(1) (nI) where n is a positive integer is a pseudo netrosophic principal ideal.

(2) (I) is the only maximal pseudo neutrosophic ideal.

(3) (0) is the only prime neutrosophic ideal (resp. prime pseudo neutrosophic ideal).

Definition 2.23 *Let $\langle R \cup I \rangle$ be a commutative neutrosophic ring and let $x = a + bI$ be an element of $\langle R \cup I \rangle$ with $a, b \in R$.*

(1) *If $a, b \neq 0$ and there exists a positive integer n such that $x^n = 0$ then x is called a strong neutrosophic nilpotent element of $\langle R \cup I \rangle$.*

(2) *If $a = 0, b \neq 0$ and there exists a positive integer n such that $x^n = 0$ then x is called a weak neutrosophic nilpotent element of $\langle R \cup I \rangle$.*

(3) *If $b = 0$ and there exists a positive integer n such that $x^n = 0$ then x is called an ordinary nilpotent element of $\langle R \cup I \rangle$.*

Example 2.24 In the neutrosophic ring $\langle \mathcal{Z}_4 \cup I \rangle$ of integers modulo 4, 0 and 2 are ordinary nilpotent elements, $2I$ is a weak neutrosophic nilpotent element and $2 + 2I$ is a strong neutrosophic nilpotent element.

Proposition 2.25 *Let $\langle R \cup I \rangle$ be a commutative neutrosophic ring.*

(1) *The set of all strong neutrosophic nilpotent elements of $\langle R \cup I \rangle$ is not a neutrosophic ideal.*

(2) *The set of all weak neutrosophic nilpotent elements of $\langle R \cup I \rangle$ is not a neutrosophic ideal.*

(3) *The set of all nilpotent (ordinary, strong and weak neutrosophic) elements of the commutative neutrosophic ring $\langle R \cup I \rangle$ is a neutrosophic ideal of $\langle R \cup I \rangle$.*

Definition 2.26 *Let $\langle R \cup I \rangle$ be a neutrosophic ring and let P be a neutrosophic ideal of $\langle R \cup I \rangle$. Let $\langle R \cup I \rangle / P$ be a set defined by*

$$\langle R \cup I \rangle / P = \{r + P : r \in \langle R \cup I \rangle\}.$$

If addition and multiplication in $\langle R \cup I \rangle / P$ are defined by

$$(r + P) + (s + P) = (r + s) + P,$$
$$(r + P)(s + P) = (rs) + P, r, p \in \langle R \cup I \rangle,$$

it can be shown that $\langle R \cup I \rangle / P$ is a neutrosophic ring called the neutrosophic quotient ring with P as an additive identity.

Definition 2.27 Let $\langle R \cup I \rangle$ be a neutrosophic ring and let P be a subset of $\langle R \cup I \rangle$.

(1) *If P is a neutrosophic ideal of $\langle R \cup I \rangle$ and $\langle R \cup I \rangle / P$ is just a ring, then $\langle R \cup I \rangle / P$ is called a false neutrosophic quotient ring;*

(2) *If P is a pseudo neutrosophic ideal of $\langle R \cup I \rangle$ and $\langle R \cup I \rangle / P$ is a neutrosophic ring, then $\langle R \cup I \rangle / P$ is called a pseudo neutrosophic quotient ring;*

(3) *If P is a pseudo neutrosophic ideal of $\langle R \cup I \rangle$ and $\langle R \cup I \rangle / P$ is just a ring, then $\langle R \cup I \rangle / P$ is called a false pseudo neutrosophic quotient ring.*

Example 2.28 Let $< \mathcal{Z}_6 \cup I > = \{0, 1, 2, 3, 4, 5, I, 21, 3I, 4I, 5I, 1 + I, 1 + 2I, 1 + 3I, 1 + 4I, 1 + 5I, 2 + I, 2 + 2I, 2 + 3I, 2 + 4I, 2 + 5I, 3 + I, 3 + 2I, 3 + 3I, 3 + 4I, 3 + 5I, 4 + I, 4 + 2I, 4 + 3I, 4 + 4I, 4 + 5I, 5 + I, 5 + 2I, 5 + 3I, 5 + 4I, 5 + 5I\}$ be a neutrosophic ring of integers modulo 6.

(1) If $P = \{0, 2, I, 2I, 3I, 4I, 5I, 2 + I, 2 + 2I, 2 + 3I, 2 + 4I, 2 + 5I\}$, then P is a neutrosophic ideal of $< \mathcal{Z}_6 \cup I >$ but $< \mathcal{Z}_6 \cup I > / P = \{P, 1 + P, 3 + P, 4 + P, 5 + P\}$ is just a ring and thus $< \mathcal{Z}_6 \cup I > / P$ is a false neutrosophic quotient ring.

(2) If $P = \{0, 2I, 4I\}$, then P is a pseudo neutrosophic ideal of $< \mathcal{Z}_6 \cup I >$ and the quotient ring

$< \mathcal{Z}_6 \cup I > /P = \{P, 1+P, 2+P, 3+P, 4+P, 5+P, I+P, (1+I)+P, (2+I)+P, (3+I)+P, (4+I)+P, (5+I)+P\}$ is a pseudo neutrosophic quotient ring.

(3) If $P = \{0, I, 2I, 3I, 4I, 5I\}$, then P is a pseudo neutrosophic ideal and the quotient ring. $< \mathcal{Z}_6 \cup I > /P = \{P, 1+P, 2+P, 3+P, 4+P, 5+P\}$ is a false pseudo neutrosophic quotient ring.

Definition 2.29 *Let $\langle R \cup I \rangle$ and $\langle S \cup I \rangle$ be any two neutrosophic rings. The mapping $\phi : \langle R \cup I \rangle \rightarrow \langle S \cup I \rangle$ is called a neutrosophic ring homomorphism if the following conditions hold:*

(1) *ϕ is a ring homomorphism;*
(2) *$\phi(I) = I$.*

If in addition ϕ is both $1-1$ and onto, then ϕ is called a neutrosophic isomorphism and we write $\langle R \cup I \rangle \cong \langle S \cup I \rangle$.

The set $\{x \in \langle R \cup I \rangle : \phi(x) = 0\}$ is called the kernel of ϕ and is denoted by $Ker\phi$.

Theorem 2.30 *Let $\phi : \langle R \cup I \rangle \rightarrow \langle S \cup I \rangle$ be a neutrosophic ring homomorphism and let $K = Ker\phi$ be the kernel of ϕ. Then:*

(1) *K is always a subring of $\langle R \cup I \rangle$;*
(2) *K cannot be a neutrosophic subring of $\langle R \cup I \rangle$;*
(3) *K cannot be an ideal of $\langle R \cup I \rangle$.*

Example 2.31 Let $\langle \mathcal{Z} \cup I \rangle$ be a neutrosophic ring of integers and let $P = 5\mathcal{Z} \cup I$. It is clear that P is a neutrosophic ideal of $\langle \mathcal{Z} \cup I \rangle$ and the neutrosophic quotient ring $\langle \mathcal{Z} \cup I \rangle /P$ is obtained as

$$
\begin{aligned}
\langle \mathcal{Z} \cup I \rangle /P \quad = \quad & \{P, 1+P, 2+P, 3+P, 4+P, I+P, 2I+P, 3I+P, 4I+P, \\
& (1+I)+P, (1+2I)+P, (1+3I)+P, (1+4I)+P, (2+I)+P, \\
& (2+2I)+P, (2+3I)+P, (2+4I)+P, (3+I)+P, (3+2I)+P, \\
& (3+3I)+P, (3+4I)+P, (4+I)+P, (4+2I)+P, (4+3I)+P, (4+4I)+P\}.
\end{aligned}
$$

The following can easily be deduced from the example:

(1) $\langle \mathcal{Z} \cup I \rangle /P$ is neither a field nor an integral domain.
(2) $\langle \mathcal{Z} \cup I \rangle /P$ and the neutrosophic ring $< \mathcal{Z}_5 \cup I >$ of integers modulo 5 are structurally the same but then
(3) The mapping $\phi : \langle \mathcal{Z} \cup I \rangle \rightarrow \langle \mathcal{Z} \cup I \rangle /P$ defined by $\phi(x) = x + P$ for all $x \in \langle \mathcal{Z} \cup I \rangle$ is not a neutrosophic ring homomorphism and consequently $\langle \mathcal{Z} \cup I \rangle \not\cong \langle \mathcal{Z} \cup I \rangle /P \not\cong < \mathcal{Z}_5 \cup I >$.

These deductions are recorded in the next proposition.

Proposition 2.32 *Let $\langle \mathcal{Z} \cup I \rangle$ be a neutrosophic ring of integers and let $P = \langle n\mathcal{Z} \cup I \rangle$ where n is a positive integer. Then:*

(1) *$\langle \mathcal{Z} \cup I \rangle /P$ is a neutrosophic ring;*
(2) *$\langle \mathcal{Z} \cup I \rangle /P$ is neither a field nor an integral domain even if n is a prime number;*
(3) *$\langle \mathcal{Z} \cup I \rangle /P \not\cong \langle \mathcal{Z}_n \cup I \rangle$.*

Theorem 2.33 *If P is a pseudo neutrosophic ideal of the neutrosophic ring $\langle \mathcal{Z}_n \cup I \rangle$ of integers modulo n, then*

$$\langle \mathcal{Z}_n \cup I \rangle / P \cong \mathcal{Z}_n.$$

Proof Let $P = \{0, I, 2I, 3I, \cdots, (n-3)I, (n-2)I, (n-1)I\}$. It is clear that P is a pseudo neutrosophic ideal of $\langle \mathcal{Z}_n \cup I \rangle$ and $\langle \mathcal{Z}_n \cup I \rangle / P$ is a false neutrosophic quotient ring given by

$$\langle \mathcal{Z}_n \cup I \rangle / P = \{P, 1+P, 2+P, 3+P, \cdots, (n-3)+P, (n-2)+P, (n-1)+P\} \cong \mathcal{Z}_n. \quad \square$$

Proposition 2.34 *Let $\phi : \langle R \cup I \rangle \to \langle S \cup I \rangle$ be a neutrosophic ring homomorphism.*

(1) *The set $\phi(\langle R \cup I \rangle) = \{\phi(r) : r \in \langle R \cup I \rangle\}$ is a neutrosophic subring of $\langle S \cup I \rangle$;*

(2) *$\phi(-r) = -\phi(r) \quad \forall r \in \langle R \cup I \rangle$;*

(3) *If 0 is the zero of $\langle R \cup I \rangle$, then $\phi(0)$ is the zero of $\phi(\langle R \cup I \rangle)$;*

(4) *If P is a neutrosophic ideal (resp. pseudo neutrosophic ideal) of $\langle R \cup I \rangle$, then $\phi(P)$ is a neutrosophic ideal (resp. pseudo neutrosophic ideal) of $\langle S \cup I \rangle$;*

(5) *If J is a neutrosophic ideal (resp. pseudo neutrosophic ideal) of $\langle S \cup I \rangle$, then $\phi^{-1}(J)$ is a neutrosophic ideal (resp. pseudo neutrosophic ideal) of $\langle R \cup I \rangle$;*

(6) *If $\langle R \cup I \rangle$ has unity 1 and $\phi(1) \neq 0$ in $\langle S \cup I \rangle$, then $\phi(1)$ is the unity $\phi(\langle R \cup I \rangle)$;*

(7) *If $\langle R \cup I \rangle$ is commutative, then $\phi(\langle R \cup I \rangle)$ is commutative.*

Proof The proof is the same as in the classical ring. $\qquad \square$

References

[1] Vasantha Kandasamy W.B. and Florentin Smarandache, *Neutrosophic Rings*, Hexis, Phoenix, Arizona, 2006.

[2] Vasantha Kandasamy W.B. and Florentin Smarandache, *Some Neutrosophic Algebraic Structures and Neutrosophic N-Algebraic Structures*, Hexis, Phoenix, Arizona, 2006.

[3] Vasantha Kandasamy W.B. and Florentin Smarandache, *Basic Neutrosophic Algebraic Structures and their Applications to Fuzzy and Neutrosophic Models*, Hexis, Church Rock, 2004.

[4] Vasantha Kandasamy W.B., Gaussian polynomial rings, *Octogon*, vol.5, 58-59, 1997.

[5] Vasantha Kandasamy W.B., Inner associative rings, *J. of Math. Res. and Expo.*, vol.18, 217-218, 1998.

[6] Vasantha Kandasamy W.B., CN rings, *Octogon*, vol.9, 343-344, 2001.

[7] Vasantha Kandasamy W.B., On locally semi unitary rings, *Octogon*, vol.9, 260-262, 2001.

[8] Vougiouklis Thomas, On rings with zero divisors strong V-groups, *Comment Math. Univ. Carolin J.*, vol.31, 431-433, 1990.

[9] Wilson John S., A note on additive subgroups of finite rings, *J. Algebra*, vol.234, 362-366, 2000.

[10] Florentin Smarandache, *A Unifying Field in Logics: Neutrosophic Logic, Neutrosophy, Neutrosophic Set, Neutrosophic Probability*, (3rd edition), American Research Press, Rehoboth, 2003.

[11] Agboola A.A.A. and Akinola L.S., On the Bicoset of a Bivector Space, *Int. J. of Math. Comb.*, vol.4, 1-8, 2009.

[12] Agboola A.A.A., Akinola A.D., and Oyebola O.Y., Neutrosophic Rings I, *Int. J. of Math. Comb.*, vol.4, 1-14, 2011.

[13] Fraleigh, J.B., *A First Course in Abstract Algebra* (5th Edition), Addison-Wesley, 1994.

[14] Lang S., *Algebra*, Addison Wesley, 1984.

[15] Atiyah M.F. and MacDonald I.G., *Introduction to Commutative Algebra*, Addison-Wesley, 1969.

[16] Herstein I.N., *Topics in Algebra*, John Wiley and Sons, 1964.

[17] Herstein I.N., *Topics in Ring Theory*, University of Chicago Press, 1969.