Scientia Magna
Vol. 1 (2005), No. 2, 49-51

On finite Smarandache near-rings

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Abstract In this paper we study the Finite Smarandache-2-algebraic structure of Finite-near-ring, namely, Finite-Smarandache-near-ring, written as Finite-S-near-ring. We define Finite Smarandache near-ring with examples. We introduce some equivalent conditions for Finite S-near-ring and obtain some of its properties.

Keywords Finite-S-near-ring; Finite-Smarandache-near-ring.

§1. Introduction

In this paper, we studied Finite-Smarandache 2-algebraic structure of Finite-near-rings, namely, Finite-Smarandache-near-ring, written as Finite-S-near-ring. A Finite-Smarandache 2-algebraic structure on a Finite-set N means a weak algebraic structure A_0 on N such that there exist a proper subset M of N, which is embedded with a stronger algebraic structure A_1, stronger algebraic structure means satisfying more axioms, by proper subset means a subset different from the empty set, from the unit element if any, from the whole set [5]. By a Finite-near-ring N, we mean a zero-symmetric Finite- right-near-ring. For basic concept of near-ring we refer to Gunter Pilz [2].

Definition 1. A Finite-near-ring N is said to be Finite-Smarandache-near-ring. If a proper subset M of N is a Finite-near-field under the same induced operations in N.

Example 1 [2]. Let $N = \{0, n_1, n_2, n_3\}$ be the Finite-near-ring defined by:

Let $M = \{0, n_1\} \subset N$ be a Finite-near-field. Defined by

Now $(N, +, .)$ is a Finite-S-near-ring .

Example 2 [4]. Let $N = \{0, 6, 12, 18, 24, 30, 36, 42, 48, 54\}$ (mod 60) be the Finite-near-ring since every ring is a near-ring. Now N is a Finite-near-ring, Whose proper subset $M = \{0, 12, 24, 36, 48\}$ (mod 60) is a Finite-field. Since every field is a near-field, then M is a Finite-near-field. Therefore N is a Finite-S-near-ring.

Theorem 1. Let N be a Finite-near-ring. N is a Finite-S-near-ring if and only if there exist a proper subset M of N , either $M \cong M_c(z_2)$ or Z_p, integers modulo p, a prime number.

Proof. **Part-I**: We assume that N is a Finite-S-near-ring. By definition, there exist a proper subset M of N is a Finite-near-field. By Gunter Pilz Theorem (8.1)[2], either $M \cong$

$M_c(z_2)$ or zero-symmetric. Since $Z_p^{;S}$ is zero-symmetric and Finite-fields implies Z_p, S are zero-symmetric and Finite-near-fields because every field is a near-field. Therefore in particular M is Z_p.

Part-II: We assume that a proper subset M of N, either $M \cong M_c(z_2)$ or Z_p. Since $M_c(z_2)$ and Z_p are Finite-near-fields. Then M is a Finite-near-field. By definition, N is a Finite-S-near-ring.

Theorem. Let N be a Finite-near-ring. N is a Finite-S-near-ring if and only if there exist a proper subset M of N such that every element in M satisfying the polynomial $x^{pm} - x$.

Proof. **Part-I**: We assume that N is a Finite-S-near-ring. By definition, there exist a proper subset M of N is a Finite-near-field. By Gunter Pilz, Theorem (8.13)[2]. If M is a Finite-near-field, then there exist $p \in P, \exists m \in M$ such that $\mid M \mid = p^m$. According to I.N.Herstein[3]. If the Finite-near-field M has p^m element, then every $a \in M$ satisfies $a^{pm} = a$, since every field is a near-field. Now M is a Finite-near-field having p^m element, every element a in M satisfies $a^{pm} = a$. Therefore every element in M satisfying the polynomial $x^{pm} - x$.

Part-II: We assume that there exist a proper subset M of N such that every element in M satisfying the polynomial $x^{pm} - x$, which implies M has p^m element. According to I.N.Herstein[3], For every prime number p and every positive integer m, there is a unique field having p^m element. Hence M is a Finite-field implies M is a Finite-near-field. By definition, N is a Finite-S-near-ring.

Theorem 3. Let N be a Finite-near-ring. N is a Finite-S-near-ring if and only if M has no proper left ideals and $M_0 \neq M$. Where M is a proper sub near-ring of N, in which idempotent commute and for each $x \in M$, there exist $y \in M$ such that $yx \neq 0$.

Proof. **Part-I** :We assume that N is a Finite-S-near-ring. By definition A proper subset M of N is a Finite-near-field. In [1] Theorem (4),it is zero-symmetric and hence every left-ideal is a M-subgroup. Let $M_1 \neq 0$ be a M-subgroup and $m_1 \neq 0 \in M_1$. Then $m_1^{-1} m_1 = 1 \in M_1$. therefore $M = M_1$. Hence M has no proper M-subgroup, which implies M has no proper left ideal.

Part-II: We assume that a proper sub-near-ring M of N has no proper left ideals and $M_0 \neq M$, in which idempotent commute and for each $x \in M$ there exist $y \in M$ such that $yx \neq 0$. Let $x \neq 0$ in M. Let $F(x) = \{m \in M \mid mx = 0\}$. Clearly $F(x)$ is a left ideal. Since there exist $y \in M$ such that $yx \neq 0$. Then $y \notin F(x)$. Hence $F(x) = 0$. Let $\phi : (M, +) \longrightarrow (Mx, +)$ given by $\phi(m) = mx$. Then ϕ is an isomorphism. Since M is finite then $Mx = M$. Now by a theroem(2) in [1], M is a Finite-near-field. Therefore, by definition N is a Finite-S-near-ring.

We summarize what has been studied in

Theorem 4. Let N be a Finite-near-ring. Then the following conditions are equivalent.

1. A proper subset M of N, either $M \cong M_c(z_2)$ or Z_p, integers modulo p, a prime number.

2. A proper subset M of N such that every element in M satisfying the polynomial $x^{pm} - x$.

3. M has no proper left ideals and $M_0 \neq M$. Where M is a proper sub near-ring of N, in which idempotent commute and for each $x \in M$, there exist $y \in M$ such that $yx \neq 0$.

Theorem 5. Let N be a Finite-near-ring. If a proper subset M, sub near-ring of N, in which M has left identity and M is 0-primitive on M^M. Then N is a Finite-S-near-ring.

Proof. By Theorem(8.3)[2], the following conditions are equivalent:

(1) M is a Finite-near-field;

(2) M has left identity and M is 0-primitive on M^M.

Now Theorem is immediate.

Theorem 6. Let N be a Finite-near-ring. If a proper subset M, sub near-ring of N, in which M has left identity and M is simple. Then N is a Finite-S-near-ring.

Proof. By Theorem(8.3)[2], the following conditions are equivalent:

(1) M is a Finite-near-field;

(2) M has left identity and M is simple. Now the Theorem is immediate.

Theorem 7. Let N be a Finite-near-ring. If a proper subset M, sub near-ring of N is a Finite-near-domain, then N is a Finite-S-near-ring.

Proof. By Theorem(8.43)[2], a Finite-near- domain is a Finite-near-field. Therefore M is a Finite-near-field. By definition N is a Finite-S-near-ring.

Theorem 8. Let N be a Finite-near-ring. If a proper subset M of N is a Finite-Integer-domain. Then N is a Finite-S-near-ring.

Proof. By I.N.Herstein[3], every Finite-Integer-domain is a field, since every field is a near-field. Now M is a Finite-near-field. By definition N is a Finite-S-near-ring.

Theorem 9. Let N be a Finite-near-ring. If a proper subset M of N is a Finite-division-ring. Then N is a Finite-S-near-ring.

Proof. By Wedderburn's Theorem(7.2.1)[3], a Finite-division-ring is a necessarily commutative field, which gives M is a field, implies M is a Finite-near-field. By definition N is a Finite-S-near-ring.

References

[1] P.Dheena, On near-field, J. Pure. Appl. Math., **17(3)** (1986), 332-326.

[2] G.Pilz, Near-ring, North Holland, Amsterdam, 1997.

[3] I.N.herstein, Topics in algebra, Wiley Eastern Limited, New Delhi, 1993

[4] R.Padilla, Smarandache algebraic structures,presented to the Universidade do Minho, Baraga, Portugal, 18-23, June, 1999.

[5] PlanetMath, Smarandache n-structure.