

## Smarandacheials

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Let  $n > k \geq 1$  be two integers. Then the Smarandacheial is defined as:

$$!n!_k = \prod_{\substack{0 < |n-k \cdot i| \leq n \\ i \in \mathbb{N}}} (n-k \cdot i)$$

For examples:

1) In the case  $k=1$ :

$$!n!_1 \equiv !n! = \prod_{\substack{\text{conv} \\ 0 < |n-i| \leq n \\ i=0, 1, 2, \dots}} (n-i) = n(n-1)(n-2)\dots(2)(1)(-1)(-2)\dots(-n+2)(-n+1)(-n) = (-1)^n (n!)^2.$$

Thus  $!5! = 5(5-1)(5-2)(5-3)(5-4)(5-6)(5-7)(5-8)(5-9)(5-10) = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot (-1) \cdot (-2) \cdot (-3) \cdot (-4) \cdot (-5) = -14400$ .

The sequence is: 4, -36, 576, -14400, 518400, -25401600, 1625702400, -131681894400, 13168189440000, -1593350922240000, 229442532802560000, -38775788043632640000, 7600054456551997440000, -1710012252724199424000000, ... .

2) In the case  $k=2$ :

a) If  $n$  is odd, then

$$!n!_2 = \prod_{\substack{0 < |n-2i| \leq n \\ i=0, 1, 2, \dots}} (n-2i) = n(n-2)(n-4)\dots(3)(1)(-1)(-3)\dots(-n+4)(-n+2)(-n) = (-1)^{(n+1)/2} (n!)^2.$$

a) If  $n$  is even, then

$$!n!_2 = \prod_{\substack{0 < |n-2i| \leq n \\ i=0, 1, 2, \dots}} (n-2i) = n(n-2)(n-4)\dots(4)(2)(-2)(-4)\dots(-n+4)(-n+2)(-n) = (-1)^{n/2} (n!)^2.$$

Thus:  $!3!_2 = 3(3-2)(3-4)(3-6) = 9$  and  $!4!_2 = 4(4-2)(4-6)(4-8) = 64$ .

The sequence is: 9, 64, -225, -2304, 11025, 147456, -893025, -14745600, 108056025, 2123366400, ... .

3) In the case  $k=3$ :

$$!n!_3 = \prod_{\substack{0 < |n-3i| \leq n \\ i=0, 1, 2, \dots}} (n-3i) = n(n-3)(n-6)\dots$$

Thus  $!7!_3 = 7(7-3)(7-6)(7-9)(7-12) = 7(4)(1)(-2)(-5) = 280$ .

The sequence is: -8, 40, 324, 280, -2240, -26244, -22400, 246400, 3779136, 3203200, -44844800, ... .

4) In the case  $k=4$ :

$$!n!_4 = \prod_{\substack{0 < |n-4i| \leq n \\ i=0, 1, 2, \dots}} (n-4i) = n(n-4)(n-8) \dots .$$

$$\text{Thus } !9!_4 = 9(9-4)(9-8)(9-12)(9-16) = 9(5)(1)(-3)(-7) = 945.$$

The sequence is: -15, 144, 105, 1024, 945, -14400, -10395, -147456, -135135, 2822400, 2027025, ... .

5) In the case  $k=5$ :

$$!n!_5 = \prod_{\substack{0 < |n-5i| \leq n \\ i=0, 1, 2, \dots}} (n-5i) = n(n-5)(n-10) \dots .$$

$$\text{Thus } !11!_5 = 11(11-5)(11-10)(11-15)(11-20) = 11(6)(1)(-4)(-9) = 2376.$$

The sequence is: -24, -42, 336, 216, 2500, 2376, 4032, -52416, -33264, -562500, -532224, -891072, 16039296, ... .

**More general:**

Let  $n > k \geq 1$  be two integers and  $m \geq 1$  another integer. Then the generalized Smarandacheial is defined as:

$$!n!_k^m = \prod_{\substack{0 < |n-k \cdot i| \leq m \\ i \in \mathbb{N}}} (n-k \cdot i)$$

**For examples:**

$$!7!_2 = 7(7-2)(7-4)(7-6)(7-8)(7-10) = 7(5)(3)(1)(-1)(-3) = 315.$$

$$\begin{aligned} !7!_9 &= 7(7-2)(7-4)(7-6)(7-8)(7-10)(7-12)(7-14)(7-16) = 7(5)(3)(1)(-1)(-3)(-5)(-7)(-9) \\ &= -99225. \end{aligned}$$

**References:**

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