Commonsense local realism refutes Bell's theorem.

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Abstract: With Bell (1964) and his EPR-based analysis contradicted by experiments, at least one step in his supposedly commonsense theorem must be false. Using commonsense local realism — the fusion of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively) — we make EPR correlations intelligible by completing the quantum mechanical account in a classical way. Thus refuting the false inequality at the heart of Bell's analysis and the false equality at its core, we reinforce the classical mantra — that correlated tests on correlated things produce correlated results — without mystery. We conclude that Bell's theorem and all related experiments negate naive realism, not commonsense local realism: Einstein's reasonable thing works.

1 Notes to the Reader

> 'In the interest of clearness, it appeared to me inevitable that I should repeat myself frequently, without paying the slightest attention to the elegance of presentation,' Einstein (1916). May this essay bring you many happy hours of fun and critical thinking.

a. Pre-reading: EPR and Bell (1964), available on-line, are taken as read; EPR to the start of page 778, Bell to his equation (15). Other texts are also available via hyperlinks in References B.
b. Terms/notation: See Appendix A. $(\mathbf{u}, \mathbf{v})$ = angle between vectors $\mathbf{u}$, $\mathbf{v}$; $\mathbf{u}\cdot\mathbf{v}$ = inner product.
c. Results: Requiring no loopholes, all results here accord with reputable experimental findings.
d. Errors: Please report errors, typos, etc; critical correspondence is especially welcome.
e. Key words: CLR, dynamic equivalence class, function Q, *gedanken*-restoration, local realism.

2 Introduction

Embracing commonsense local realism (CLR), the fusion of local-causality (no causal influence propagates superluminally) and physical-realism (some physical properties change interactively), we endorse EPR's (1935:777) condition of completeness: Every relevant element of the physical reality must have a counterpart in our physical theory. But we reject the naive realism in Bell (1964; 2004): and the 'nonlocality' so often linked to Bell's theorem and his impossibility proof.

> "Bell's theorem asserts [?] that if certain predictions of quantum theory are correct then our world is non-local. 'Non-local' here means that there exist interactions between events that are too far apart in space and too close together in time for the events to be connected even by signals moving at the speed of light," Goldstein *et al.* (2011).
>
> NB: We accept that those 'certain predictions' are correct! "Indeed it was the explicit representation of quantum nonlocality [in de Broglie-Bohm theory] which started a new wave of investigation in this area [of local causality]. Let us hope that these analyses also may one day be illuminated, perhaps harshly, by some simple constructive model. However that may be, long may Louis de Broglie continue to inspire those who suspect that what is proved by impossibility proofs is lack of imagination," (Bell 2004:167).

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In fact, CLR — a simple constructive model that counters nonlocality and non-realism in full accord with relativity and Einstein's ideas — came to mind and was voiced as I read David Mermin's (1988:14) 'impossibility proof'. Indeed, I offered CLR to David by phone the next day.

> On one supposition I absolutely held fast; that of local-causality, often called Einstein-locality: "The real *factual* situation of the system S_2 is independent of what is done with the system S_1, which is spatially separated from the former," after Einstein (1949:85).

Allowing that natural physical variables and their local interactions alone account for my classical mantra — correlated results produced by correlated tests on correlated things, without mystery — I had unknowingly followed Bell (2004:174): Unlike *observables,* I let natural physical variables — *existents* in my old-fashioned terms; bettered by Bell's *beables* — be elements of reality, things which exist, their existence independent of *measurement* and *observation.*

Further, I argued: the existents here are revealed by interactions and confirmed by robust physical experiments and tests. For the *gedanken* certainties in (3)-(4) below were clear to me.

So, under CLR policy — making weak allowances and taking maths to be the best logic — let's now together bring (3)-(4) to the dynamics of EPR's physics in the context of EPRB [A.1], the experiment in Bell (1964): and let's allow those dynamics do the talking from now on.

To commence this task – our common purpose – confidently, let's cast it in the context of a wholly mathematical version (2) of Bell's (1964) theorem. Let's then identify the false *equality* at the core of Bell's EPR-based analysis therein [4]: the source of his subsequent more famous – but equally false – inequality (2). Let's then refute one of the many CHSH inequalities [5].

With these results based on a particle-by-particle analysis, let's then refute Bell's theorem using continuous variables [6]. Thereby enabled, let's then show that Bell's indifference is misplaced —

> "It is a matter of indifference . . . whether λ denotes a single variable or a set, or even a set of functions, and whether the variables are discrete or continuous," Bell (1964:195).

— that the defects in his work are deep and conceptual: For we'll see how Bell's adoption of d'Espagnat's (1979; 1979a) naive realism leaves him failing to identify EPR's *elements of physical reality* with *beables* – his own coinage [3]. How he misses a key feature of EPRB particle-pairs – their unique variety (15). How he confuses 'causal independence' with 'statistical independence' [7].

Then, demonstrating CLR's utility in the context of Mermin's (1990a) 3-particle GHZ-variant [8] — thereby showing that CLR is equally effective at disentangling the likes of GHZ (1989), GHSZ (1990), CRB (1991) — we can end with conclusions [9], acknowledgments [10], a technical appendix [A], references [B].

3 Our theory -vs- Bell's theorem

> λ may denote "any number of hypothetical additional complementary variables needed to complete quantum mechanics in the way envisaged by EPR," Bell (2004:242).

Here's a wholly mathematical version of Bell's theorem; Bell (1964:(1)-(3), (12)-(14); 2004:14-21):

$$\textbf{If } A(\lambda,\mathbf{a}) = A^{\pm} = \pm 1;\ B(\lambda',\mathbf{b}) = B^{\pm} = \pm 1 = -A(\lambda,\mathbf{b}); \textstyle\int d\lambda\, \rho(\lambda) = 1: \tag{1}$$

$$\textbf{Then } \langle AB\rangle \equiv \textstyle\int d\lambda\, \rho(\lambda) A(\lambda,\mathbf{a}) B(\lambda',\mathbf{b}) = -\int d\lambda\, \rho(\lambda) A(\lambda,\mathbf{a}) A(\lambda,\mathbf{b}) \neq -\mathbf{a}{\cdot}\mathbf{b}. \tag{2}$$

$\langle AB\rangle$ replaces Bell's $P(\vec{a},\vec{b})$; please see Appendix A for other relevant CLR technicalities.

Introduced in the line below his 1964:(3) and based on the paragraph below his 1964:(15), the $\neq$ in (2) is Bell's famous inequality. Here's Bell's (2004:147) explanation of the background to (2):

> "To explain this dénouement without mathematics I cannot do better than follow d'Espagnat (1979; 1979a)." Our paraphrase of d'Espagnat (1979:166) follows:

'One can infer that in every particle-pair [every pair of twins; $p(\lambda)$, $p'(\lambda')$], one particle has the property A^+ and the other has the property A^-, one has property B^+ and one B^-, ... Such conclusions require a subtle but important extension of the meaning assigned to our notation A^+. Whereas previously A^+ was merely one possible outcome of a measurement made on a particle, it is converted by this argument into an attribute of the particle itself.'

For us, preferring 'outcome of a test' to d'Espagnat's (1979:166) 'outcome of a measurement', and concluding that Bell's theorem is based on a restrictive naive realism, we reject any such tamper with our task. On the contrary — given *the fact* that such pairs are twins, *physically correlated at birth by their tightly choreographed birth in a spin-conserving decay* — here's our position:

One can infer that in every particle-pair — every pair of twins, per Bell's abandoned 'genetic' hypothesis (Bernstein 1991:84) — one particle has the property $\lambda \sim +\mathbf{a}$ where $\sim +\mathbf{a}$ is not the outcome A^+ of a test but an equivalence revealed by that outcome:

For we allow that A^+ reveals a previously-hidden preexisting equivalence relation $\sim$ on Λ — the space of λ — see [A.4]. Putting it personally and strongly — to emphasize the difference between an outcome A^+ and a pristine property $\lambda \sim +\mathbf{a}$ — but still in the context of CLR:

A secret and painful operation on my far-off twin sister Alice will certainly reveal — via the outcome A^+ — that she has the property $\lambda \sim +\mathbf{a}$. But it will equally reveal — for we are twins — that I have the property $\lambda' \sim -\mathbf{a}'$: *with related behaviors when I too am tested.* Yet I today remain unperturbed and unbloodied by such a far-off and still unknown-to-me interaction. For:

There are no *messages* in one system from the other. EPRB correlations do not give rise to signaling between noninteracting systems. Of course, however, there may be correlations (eg, those of EPRB) and if something about the second system is given (eg, that it is the other side of an EPRB setup) and something about the overall state (eg, that it is the EPRB singlet state) then inferences from events in one system (eg, A^+: an *up*-counter *Yes*) to events (eg, B^-: a *down*-counter *Yes*) or to properties (eg, $p'(\lambda' \sim -\mathbf{a}')$ in the other system are possible; modifying Bell (2004:208) in our terms:

EPRB correlations allow inferences from A^+ *to* B^- and to related facts like those in (A.5)-(A.6).

Thus we arrive at the key to our analysis: we include all the CLR elements of such implications in the dynamics. For, in the micro-physics here, we allow that there may be 'no infinitesimals by the aid of which an observation might be made without appreciable perturbation' (Heisenberg 1930:63). But we also allow that preexisting pristine properties (ie, beables, properties; such as being a member of a DEC) may be revealed by such perturbations.

So, for us: If a test on a particle reveals an associated DEC, then its pristine twin is a member of a related class: *For such twins are physically correlated at birth by their birth in a spin-conserving decay.* We therefore endorse EPR's elements of physical reality, defined as follows:

"If, without any way disturbing a system, we can predict with certainty (ie, with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality [a beable] corresponding to this physical quantity," EPR (1935:777).

For — given the symmetries in (A.5)-(A.6) — let Alice test $A(\mathbf{a}, \lambda)$ and find A^+; ie, $\lambda \sim +\mathbf{a}$. Then, without further ado or disturbance anywhere, Alice can predict with certainty that

$$B(\lambda', \mathbf{a}') = B(-\lambda, \mathbf{a}') = B(-\lambda \sim -\mathbf{a}', \mathbf{a}') = B(\lambda' \sim -\mathbf{a}', \mathbf{a}') = -1 = B^- : \tag{3}$$

$\mathbf{a}'$ distinguishing Bob's $SGD(\mathbf{a}')$ from Alice's $SGD(\mathbf{a})$ when $\mathbf{a}' = \mathbf{b}' = \mathbf{a}$; per [A.3].

Now in (3), the first equality has Bell's backing; see (1) or Bell (1964:(13). And the relation $B(\lambda' \sim -\mathbf{a}', \mathbf{a}') = -1$ is CLR's very definition of equivalence in Bob's domain. For under these conditions, for all $\mathbf{a}'$ and any number of such tests, $B(\lambda' \sim -\mathbf{a}', \mathbf{a}')$ equals minus one with certainty: a central experimental fact.

Thus, via the equivalence class to which $p(\lambda \sim +\mathbf{a})$ in this test belongs, the corresponding EPR beable in Bob's test is $p'(\lambda' \sim -\mathbf{a}')$. In other words: $p'(\lambda' \sim -\mathbf{a}')$ — the EPR beable that here corresponds to the test result B^- — allows us to complement EPR with a CLR comment:

> Unsurprisingly: Without in any way disturbing particle $p'(\lambda' \sim -\mathbf{a}')$, we can predict with certainty the result $B^- = -1$ of that particle's interaction with Bob's $SGD(\mathbf{a}')$:
>
> $$\text{ie}, p'(\lambda' \sim -\mathbf{a}') \Rightarrow [\lambda' \to \pm\mathbf{a}'\}(\lambda' \cdot \mathbf{a}') = -\mathbf{a}'{\cdot}\mathbf{a}' = -1 = B^-. \tag{4}$$
>
> Moreover, to predict with certainty *any* particular pristine particle's interaction with Bob's $SGD(\pm\mathbf{b}')$: we'd let Alice test that particle's twin with her $SGD(\mp\mathbf{b})$; etc.

Differing markedly from the analysis just given, Bell's naive realism leads to contradictions. For, under CLR/EPRB — as we'll soon confirm at (22); refuting (2) — Bell's 1964:(15) reads thus:

$$1 + \langle BC\rangle = 1 - \mathbf{b}{\cdot}\mathbf{c} \geq |\mathbf{a}{\cdot}\mathbf{c} - \mathbf{a}{\cdot}\mathbf{b}| = |\,\langle AB\rangle - \langle AC\rangle\,| : \tag{5}$$

a relation restricted by Bell's acceptance of d'Espagnat's (1979; 1979a) naive realism; a relation absurd under EPRB in the domain $-\pi/3 < \phi < \pi/3$ if $(\mathbf{a}, \mathbf{b}) = (\mathbf{b}, \mathbf{c}) = \phi$ and $(\mathbf{a}, \mathbf{c}) = 2\phi$.

Despite these constraints, Bell (1964:199) concludes:

> "In a theory in which parameters [sic] are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz-invariant."

To the contrary, we will show that a CLR counter-conclusion prevails:

> In a theory in which hidden properties, revealed by tests, are found to determine the results of individual interactions: there must be a function that accurately tracks the factual inferential consequences of such tests without changing the statistical predictions. Such a theory will be Lorentz-invariant.

Our case against Bell thus foreshadowed, we next refute Bell's (1964) analysis: from fundamental first principles — the CLR custom — and therefore beyond dispute.

4 Bell's 1964 analysis refuted

To derive (5), his 1964:(15), Bell goes beyond our (1)-(2) and invokes a third unit-vector $\mathbf{c}$ in unnumbered equations that follow his 1964:(14). If we number them Bell's (14a) to Bell's (14c), Bell — suspiciously, in our view — equates (14b) to (14a).

In anticipation, let's reveal our hunch — the restriction required for Bell's 'equality' here to go through: For his *equality* to hold here, Bell's theorem will be limited to sock-like entities!

Since A, B, C are discrete, let's replace Bell's integrals with sums and Bell's 1964:(14a) with discrete variables. For generality, let λ be a random variable in $\mathbb{R}^3$; with a uniform distribution and consequent probability zero that two λs or two particle-pairs are the same. Then, with index i uniquely numbering each pair, let n be such that, to an adequate accuracy hereafter:

$$\text{Bell's (14a)} = \langle AB\rangle - \langle AC\rangle = -\frac{1}{n}\sum_{i=1}^{n}[A(\mathbf{a}, \lambda_i)A(\mathbf{b}, \lambda_i) - A(\mathbf{a}, \lambda_{n+i})A(\mathbf{c}, \lambda_{n+i})] \tag{6}$$

$$= \frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a}, \lambda_i)A(\mathbf{b}, \lambda_i)[A(\mathbf{a}, \lambda_i)A(\mathbf{b}, \lambda_i)A(\mathbf{a}, \lambda_{n+i})A(\mathbf{c}, \lambda_{n+i}) - 1]. \tag{7}$$

(7) is the correct discrete form of Bell's (14a). And Bell's (14c) is a valid conclusion from his (14b). So, if Bell's (14b) = Bell's (14a), the related components of (7) and Bell's (14c) should be equal. Let $\stackrel{?}{=}$ identify our suspicion of Bell's equality under these conditions. Then,

$$\text{from Bell's (14c): } \langle BC\rangle \equiv -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b},\lambda_i)A(\mathbf{c},\lambda_i) = -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i}) \tag{8}$$

$$\stackrel{?}{=} -\frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i});\ \text{from (7).} \tag{9}$$

To support Bell's (14a) = Bell's (14b) — and remove our **?** from (9) — we require the impossible $\lambda_i = \lambda_{n+i}$: Impossible because by definition, physical context, and from Bell's own λ-license; $\lambda_i \neq \lambda_{n+i}$. So here's a new — and the first valid — Bell-inequality:

$$\text{Bell 1964 : (14}\mathbf{b}) \neq \text{Bell 1964 : 14(}\mathbf{a}) \tag{10}$$

$$\because \frac{1}{n}\sum_{i=1}^{n} A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i}) \neq \frac{1}{n}\sum_{i=1}^{n} A(\mathbf{b},\lambda_i)A(\mathbf{c},\lambda_i) \tag{11}$$

in general. Of course, a stable cohort of n classical objects — like Bertlmann's socks (Bell 2004:139-158) — would allow a non-destructive test and a follow-up non-destructive retest of the cohort so that: $n+i$ denoted another run of n tests on the same set as the i-series of tests; and in the same order. Then (9) and (11) would be unfettered equalities. For then

$$A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_{n+i})A(\mathbf{c},\lambda_{n+i}) = A(\mathbf{a},\lambda_i)A(\mathbf{b},\lambda_i)A(\mathbf{a},\lambda_i)A(\mathbf{c},\lambda_i) = A(\mathbf{b},\lambda_i)A(\mathbf{c},\lambda_i). \tag{12}$$

So classical objects like socks satisfy Bell's inequality: and we have here the source of those famous — but soon to be shown erroneous — inequalities in (2) and elsewhere. The source too of the error in the CHSH family of inequalities, as we show next.

5 CHSH inequality refuted

Based on Peres' (1995:164) version of the CHSH (1969) inequality, let A_j, B_j, C_j, D_j independently equal ± 1 randomly. Then, in our terms – see (6) and [A.2] – the following conditional truism does not hold in general under EPRB:

$$\text{ie, } A_j(B_j - D_j) + C_j(B_j + D_j) \equiv \pm 2 \text{ does not ensure that} \tag{13}$$

$$A_iB_i + B_{n+i}C_{n+i} + C_{2n+i}D_{2n+i} - A_{3n+i}D_{3n+i} = \pm 2 \text{ [sic];} \tag{14}$$

$$\text{nor that } \mid \langle A_iB_i\rangle + \langle B_{n+i}C_{n+i}\rangle + \langle C_{2n+i}D_{2n+i}\rangle - \langle A_{3n+i}D_{3n+i}\rangle \mid \leq 2 \text{ [sic];} \tag{15}$$

for (14) is a false relation over particle-pairs indexed by $wn+i$; see [A.2]. And (15) is absurd in the domain $-\pi/2 < \phi < \pi/2$ if $(\mathbf{a},\mathbf{b}) = (\mathbf{b},\mathbf{c}) = (\mathbf{c},\mathbf{d}) = \phi$ and $(\mathbf{a},\mathbf{d}) = 3\phi$.

Of course, under the same conditions that deliver equality (12), truism (13) will hold; as it will under *gedanken*-restoration similar to that at [A.6]. But real experiments are conducted under LHS (14) and LHS (15): refuting such facile analyses as (13).

So — though naive realism and Bertlmann's socks will wash in (13) — all naively-realistic EPR-based Bell inequalities fall to our CLR particle-by-particle analysis: thanks to that family of unique twins $p(\lambda_{wn+i})$, $p'(\lambda'_{wn+i})$. And, thanks to them and their first-principle examples, we now move to refute Bell's theorem.

6 Bell's theorem refuted

Pedagogy moving us to take the opposite tack to Bell in (1) — to show the utility of CLR; it makes no difference to the results — let's here focus on λ'. Allowing λ' to be a random beable uniformly distributed over $\mathbb{R}^3$, λ' will be perturbed by $p'(\lambda')$'s interaction with Bob's $SGD(\mathbf{b}')$: for "each [pristine] particle, considered separately, *is* unpolarized here," Bell (2004:82). Representing that interaction by $[\lambda' \to \pm\mathbf{b}'\}$, we'll find $\lambda' \sim \pm\mathbf{b}'$ equiprevalently (ie, with equal prevalence). So, expanding (1) in our terms and then constructing LHS (2):

$$A(\lambda, \mathbf{a}) = [\lambda \to \pm\mathbf{a}\}(\lambda{\cdot}\mathbf{a}) = \pm\mathbf{a}{\cdot}\mathbf{a} = \pm 1, \tag{16}$$

$$B(\lambda', \mathbf{b}') = [\lambda' \to \pm\mathbf{b}'\}(\lambda'{\cdot}\mathbf{b}') = (\pm\mathbf{b}'){\cdot}\mathbf{b}' = \pm 1, \tag{17}$$

$$\int d\lambda\, \rho(\lambda) = \frac{1}{4\pi}\int_0^{4\pi} d\Omega = 1. \tag{18}$$

$$\therefore \langle AB\rangle = \frac{1}{4\pi}\int_0^{4\pi} d\Omega\, [\lambda \to \pm\mathbf{a}\}(\lambda{\cdot}\mathbf{a})[\lambda' \to \pm\mathbf{b}'\}(\lambda'{\cdot}\mathbf{b}'); \tag{19}$$

where Ω is a unit of solid-angle. The function-set of Q-functions and response-functions under the integral is therefore

$$\{[\lambda \to \pm\mathbf{a}\}\,(\lambda{\cdot}\mathbf{a})\,[\lambda' \to \pm\mathbf{b}'\}\,(\lambda'{\cdot}\mathbf{b}')\} \tag{20}$$

Now, working with functions, any Q may be applied to any element in its domain, in any order, to derive $\langle AB\rangle$. However, since there is just one independent variable in EPRB — from $\lambda + \lambda' = 0$ — one Q is superfluous. So, focussing on λ', the progressively reduced function-sets after (20) are:

$$\{(-\lambda'{\cdot}\mathbf{a})\,[\lambda' \to \pm\mathbf{b}'\}\,(\lambda'{\cdot}\mathbf{b}')\} \Rightarrow \{(-\lambda'{\cdot}\mathbf{a})\,[\lambda' \to \pm\mathbf{b}'\}\,(\pm 1)\} \Rightarrow \{(\mp\mathbf{b}' \cdot \mathbf{a})\,(\pm 1)\} \Rightarrow \{-\mathbf{b}'{\cdot}\mathbf{a}\}: \tag{21}$$

or, equivalently, completing (19):

$$\langle AB\rangle = \frac{1}{4\pi}\int_0^{4\pi} d\Omega\,(-\lambda'{\cdot}\mathbf{a})[\lambda' \to \pm\mathbf{b}'\}\lambda'{\cdot}\mathbf{b}' = (\pm 1)(\mp\mathbf{b}'){\cdot}\mathbf{a} = -\mathbf{b}'{\cdot}\mathbf{a} = -\mathbf{a}{\cdot}\mathbf{b}.\ \text{QED:}\ \blacksquare \tag{22}$$

Bell's theorem – represented in (2) consistent with Bell's formulation – is refuted.

In passing: Since the outputs of (20)-(22) are identical, we see that Q eliminates the need for normalizing integrals in expressions like (22): for Q is a normalizing function when, as here, its arguments are normalized.

(22) is the first in a series of correct 'disentangling' CLR results that include GHZ (1989), GHSZ (1990), CRB (1991). But before showing CLR's utility in that department at [8] — via Mermin's (1990; 1990a) 3-particle GHZ-variant — we next refute one of Bell's false opinions.

7 Bell's 'statistical independence' refuted

> "One general issue raised by the debates over locality is to understand the connection between stochastic independence (probabilities multiply) and genuine physical independence (no mutual influence). It is the latter that is at issue in 'locality,' but it is the former that goes proxy for it in the Bell-like calculations. We need to press harder and deeper in our analysis here," Arthur Fine, in Schlosshauer (2011:45).

In that CLR is devoid of subjective beliefs and non-physical entities, we take 'probable' and its derivatives to be loaded terms here; though we have no problem with technical terms like impossible, probability zero or probability one. However, to minimize confusion, we allow that P denotes the *normalized prevalence* (aka *objective probability*).

Let Z denote EPRB and let $P(AB = +1|Z)$ denote the *normalized prevalence* of $AB = +1$ given Z. Then, equating (22) to the standard prevalence relation for binary (± 1) outcomes:

$$\langle AB \rangle = -\mathbf{a}{\cdot}\mathbf{b} = (+1)P(AB = +1|Z) + (-1)[1 - P(AB = +1|Z)]. \tag{23}$$

$$\therefore P(AB = +1|Z) = (1 - \mathbf{a}{\cdot}\mathbf{b})/2 = \sin^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b});\ P(AB = -1|Z) = \cos^2 \tfrac{1}{2}(\mathbf{a}, \mathbf{b}). \tag{24}$$

$$\therefore P(A^+B^+|Z) \neq P(A^+|Z)\,P(B^+|Z);\ \text{etc.,} \tag{25}$$

when A^+ and B^+ are causally independent; ie, causally independent in the sense that neither exerts any direct causal influence on the other. That is, just like the apple and pear crop, we expect a dynamic (and hence a mathematico-logical) connection because of the common-cause physical correlation between them. Just as here, with our Q, we expect DECs to be related because of the physical correlations between closely-related (here, twinned) particles.

In this way (from first principles), we refute Bell's opinion (2004:243) and his move there from his (9) to his (10): that *causal independence* should equate to *statistical independence*, seen as a consequence of *local causality.*

Thus, derived from first principles, (25) responds to Fine's urgings and delivers this result: Given EPRB physical correlations, *statistical independence* does not equate to *causal independence* under *local causality*: nor with pear and apple crops. Rather, like apple and pear crops, there is a physical correlation and hence a consequential dynamical (and therefore a mathematico-logical) relation between them. Just as, with our Q, we have physical correlations and consequent equivalence relations in our maths/logic.

However, in full accord with reciprocal causal independence and local-causality (ie, no causal influence propagates superluminally), two CLR boundary conditions follow: Causally independent of $SGD(\mathbf{b}'), B^\pm, \lambda'$: $A^\pm$ may be causally dependent on any property of $SGD(\mathbf{a})$ or λ. Causally independent of $SGD(\mathbf{a}), A^\pm, \lambda$: $B^\pm$ may be causally dependent on any property of $SGD(\mathbf{b}')$ or λ'.

With (25) another sound result from first principles, we finally demonstrate Q's utility in analyzing and disentangling multiparticle experiments.

8 Understanding Mermin's 3-particle experiment

> Einstein argues that 'EPR correlations can be made intelligible only by completing the quantum mechanical account in a classical way,' after Bell (2004:86). Let's see.

Consider experiment M: Mermin's (1990; 1990a) 3-particle GHZ-variant. Respectively: Three spin-half particles with spin beables λ, μ, ν emerge from a spin-conserving decay such that

$$\lambda + \mu + \nu = \pi. \tag{26}$$

Any pristine beable may thus be represented in terms of its siblings — eg, as (26) is used below in the reduction (30)-(31) or in the transition (32)-(33) — thereby allowing still-relevant Q-functions to supply relevant facts re relevant beable properties.

The particles separate along three straight lines in the y-z plane to interact with SGDs that are orthogonal to the related line of flight. Let a, b, c denote the azimuthal angles of each SGD's principal-axis relative to the positive x-axis; let the test results be A, B, C. Then, extending (16)-(17) appropriately with $\oplus =$ xor:

$$A(a, \lambda) = A^\pm = [\lambda \to a \oplus a + \pi\}\cos(\lambda - a) = \pm 1, \tag{27}$$

$$B(b, \mu) = B^\pm = [\mu \to b \oplus b + \pi\}\cos(\mu - b) = \pm 1, \tag{28}$$

$$C(c, \nu) = C^\pm = [\nu \to c \oplus c + \pi\}\cos(\nu - c) = \pm 1. \tag{29}$$

The function-set of Q-functions and response-functions is therefore

$$\{[\lambda \to a \oplus a + \pi\}; \cos(\lambda - a); [\mu \to b \oplus b + \pi\}; \cos(\mu - b); [\nu \to c \oplus c + \pi\}; \cos(\nu - c)\}. \tag{30}$$

Now, working with functions, any Q may be applied to any element in its domain in any order to derive $\langle ABC\rangle$. However, since there are just two independent variables – see (26) – one Q is superfluous. So, taking just one example: (30) may be reduced to:

$$\{[\lambda \to a \oplus a + \pi\}; \cos(\lambda - a); [\mu \to b \oplus b + \pi\}; \cos(\mu - b); \cos(\pi - \lambda - \mu - c)\}. \tag{31}$$

So, as a physically significant shortcut, (31) will yield $\langle ABC\rangle$ correctly. It being understood that – as with any function – each and every Q-function properly maps its domain to its codomain; and consequently onto the domain of every relevant response-function.

For now, bypassing the shortcut, we employ functions (27)-(29) ordered per (30):

$$\langle ABC\rangle = [\lambda \to a \oplus a + \pi\} \cos(\lambda - a)[\mu \to b \oplus b + \pi\} \cos(\mu - b)[\nu \to c \oplus c + \pi\} \cos(\nu - c) \tag{32}$$

$$= [\lambda \to a \oplus a + \pi\} \cos(\lambda - a)[\mu \to b \oplus b + \pi\} \cos(\mu - b)[\nu \to c \oplus c + \pi\} \cos(\pi - \lambda - \mu - c) \tag{33}$$

$$= [\mu \to b \oplus b + \pi\} \cos(\mu - b)[\nu \to c \oplus c + \pi\} \cos(\pi - a - \mu - c) \oplus - \cos(-a - \mu - c) \tag{34}$$

$$= [\nu \to c \oplus c + \pi\} \cos(\pi - a - b - c) \oplus - \cos(-a - b - c) \oplus - \cos(-a - b - c) \oplus \cos(-a - b - c - \pi) \tag{35}$$

$$= \cos(\pi - a - b - c) \oplus - \cos(-a - b - c) \oplus - \cos(-a - b - c) \oplus \cos(-a - b - c - \pi) \tag{36}$$

$$= - \cos(a + b + c).\ QED.\ \blacksquare \tag{37}$$

$$\therefore P(ABC = +1 \mid M) = \sin^2 \tfrac{1}{2}(a + b + c); \tag{38}$$

$$P(ABC = -1 \mid M) = \cos^2 \tfrac{1}{2}(a + b + c). \tag{39}$$

(37) is the correct result for experiment M, Mermin's (1990a:733) 'crucial minus' sign properly delivered: from (37), $\langle ABC\rangle = -1$ when $a + b + c = 0$. Thus, consistent with the ordinary rules for functions, we classically deliver intelligible EPR correlations. And (31) does the same.

9 Conclusions

Employing commonsense local realistic (CLR) first-principles and elementary functions, we have refuted Bell's theorem and all the Bell-supporting arguments known to us; in our view, beyond dispute. We have also explained 'entanglement' in CLR terms.

We conclude that Bell's theorem and related experiments negate naive realism, not commonsense local realism: for that famous inequality at the heart of Bell's analysis is false. Moreover, with every relevant element of each studied physical reality included in our physical theory — with no other elements, subjective or otherwise — we show that our classical mantra holds true: correlated tests on correlated things do produce correlated results without mystery.

We also show that, for us at least, mathematics is the best logic. For, though associated with hidden-variables, the now discovered dynamic equivalence classes (DECs) are physically real and wholly amenable to mathematical analysis and experimental confirmation. We further note that the antipodean dichotomies associated with the DECs here are powerful discriminators.

Then, making EPR correlations intelligible by completing the quantum mechanical account in a classical way, our CLR theory also corrects the view — eg, Bell (2004:243) and Bell's move there from his (9) to his (10) — that *causal independence* should equate to *statistical independence*, seen as a consequence of *local causality*. For a chain of equivalence, based on physical correlations — not causal influences — links the causally independent outcomes in (1) and in (16)-(17) and in (27)-(29) to the appropriate local-realistic expectations $\langle . \rangle$.

It follows that our foreshadowings in Section 3 represent valid conclusions, expressed in a different way. And with (24) and (38)-(39) typifying our work on EPRB correlations: we associate the $\frac{1}{2}$ in our trigonometric arguments with the intrinsic spin $s = \frac{1}{2}$ of the spin-half particles. Similar analysis with photons — eg, in Aspect (2002) — yields $s = 1$.

Finally, working from first principles, showing that Bell's work is limited by his naive realism, we also eliminate the source of Bell's discomfort (expressed in Bernstein 1991:84). So, refuting Bell at every step and honoring Einstein similarly, we here rephrase and reverse Bell's lament:

> Perfect quantum correlations demand something like the 'genetic' hypothesis: like the triplets linked by λ, μ, ν in (26). It's so reasonable to assume that the particles carry with them programs, correlated in advance, telling them how to behave. This is so rational that when Einstein saw that, and the others refused to see it, he was the rational man. The others were burying their heads in the sand. So it's great that Einstein's idea of a classical locally-causal reality works. The reasonable thing works.

10 Acknowledgments

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A Appendix

A.1 EPRB under CLR

$$\textit{Alice}\}A^{\pm}=\pm 1=(\mathbf{a}\cdot\lambda)\{\pm\mathbf{a}\leftarrow\lambda]\Leftarrow p(\lambda)\cdot\langle\lambda+\lambda'=0\rangle\cdot p'(\lambda')\Rightarrow[\lambda'\rightarrow\pm\mathbf{b}'\}(\lambda'\cdot\mathbf{b}')=\pm 1=B^{\pm}\{\textit{Bob}. \tag{A.1}$$

(A.1) depicts the totality of EPRB under CLR — *nothing relevant missing, nothing irrelevant found* — every relevant element of the subject physical reality having its counterpart in the theory. In that CLR and Einstein's (1949:85) local-causality require neither superluminal signaling nor action-at-a-distance: they're not there.

In this way we dismiss such views as: "... it might be that we have to learn to accept not so much action at a distance, but [the] inadequacy of no action at a distance," Bell (1990:6); "... that is the dilemma. We are led by analyzing this situation to admit that in somehow distant things are connected, or at least not disconnected," Bell (1990:7).

In (A.1), with its *gedanken* (mind's eye) *block-time* view, we see a spin-conserving decay/birth with twins flying apart $\Leftarrow p(\lambda)\cdot\langle\lambda+\lambda'=0\rangle\cdot p'(\lambda')\Rightarrow$ en-route to their destiny with *gedanken* Stern-Gerlach devices (SGDs): function-machines, each built from a squeeze-function $Q=[.\rightarrow\pm\cdot\}$ and a response-function $R=(.,\cdot)$. We see the relevant printed outputs ($A^{\pm}=\pm 1/B^{\pm}=\pm 1$) recording each Up/Down output; appropriately observed by Alice/Bob.

There's also the ability to turn the Rs into diagnostic-functions: from reporting the output of an SGD to analyzing its behavior; etc. As we'll see at [A.4] below.

A.2 λ and λ'

In (A.1), primes ($'$) show $p'(\lambda')$ and other elements in Bob's domain. λ, λ' are index-suppressed twinned antiparallel beables from the set of twinned particles

$$\{p(\lambda_{wn+i}), p'(\lambda'_{wn+i}) \mid w=0,1,2,..; i=1,2,..,n\}; w = \text{run-number when required, eg (14)}. \tag{A.2}$$

λ and λ' are thus spin-half related CLR beables; separable hidden-variables: $\lambda, \lambda' \in \Lambda \subset \mathbb{R}^3$.

A.3 The use of primes ($'$)

It is generally helpful to have primes ($'$) distinguish elements in Bob's domain from similar elements in Alice's domain. In (A.1), parameter **a** represents the principal-axis alignment of Alice's Stern-Gerlach device $SGD(\mathbf{a})$, **a** freely and independently chosen by Alice. Parameter $\mathbf{b}'$ represents the principal-axis alignment of Bob's Stern-Gerlach device $SGD(\mathbf{b}')$; $\mathbf{b}'$ freely and independently chosen by Bob.

So, when required, $SGD(\mathbf{a}')$ means that Bob's setting (indicated by the prime) is equal to Alice's setting (indicated by the $\mathbf{a}$). In other words: Bob and Alice have identical settings with $\mathbf{a}' = \mathbf{b}' = \mathbf{a}$; agreeing, from *their* common perspective, on Up/Down. They have antiparallel settings with $-\mathbf{a}' = \mathbf{b}' = -\mathbf{a}$; agreeing, *from a particle perspective*, on Up/Down (since λ, λ' are themselves antiparallel).

However, in many ways, a fact over-riding such considerations is this: Bob – alone and independent of anything that Alice might do – can prove $p'(\lambda' \sim -\mathbf{a}')$. To do so, Bob simply tests $p'(\lambda')$ with $SGD(\mathbf{a}')$, revealing $\lambda \sim -\mathbf{a}'$ directly! To thus make $p'(\lambda' \sim -\mathbf{a}')$ his own; as well as ours. Then, via *gedanken*-restoration, he reverts $p'(\lambda' \sim -\mathbf{a}')$ to its pristine state — ie, to $p'(\lambda' \sim -\mathbf{a}')$ — and continues on, unabated and undisrupted, with his own independent experimental program.

Under CLR, CLR *gedanken*-restoration is allowable and acceptable because, under CLR *gedanken*-analysis, we clearly understand the particle-dynamics. See [A.5] for further discussion.

A.4 $SGD(\mathbf{a})$, Q-function $Q(\pm\mathbf{a}) \equiv [\lambda \rightarrow \pm\mathbf{a}\}$, DECs

Each *SGD* is a composite function-machine: squeeze-function Q feeds response-function R. In the context of Alice's device $SGD(\mathbf{a})$: $Q(\pm\mathbf{a}) = [\lambda \rightarrow \pm\mathbf{a}\}$; $R(\mathbf{a}) = (\lambda{\cdot}\mathbf{a})$: with related print-out (± 1).

We now turn to R in its role as a diagnostic-function: If $R = (\lambda{\cdot}\mathbf{a}) = \pm 1$, then $\lambda = \pm\mathbf{a} \oplus \lambda \sim \pm\mathbf{a}$. So, since $\sim$ is the *weaker* equivalence, we adopt it as the diagnostic message; in full accord with our CLR policy of weak allowances, and ignoring the fact that $P(\lambda = \pm\mathbf{a} \mid Z) = 0$. It follows that:

$$\mathbf{a}^+ \equiv \{\lambda \in \Lambda \subset \mathbb{R}^3 | \lambda \sim +\mathbf{a} \in V \subset \mathbb{R}^3\},\ \mathbf{a}^- \equiv \{\lambda \in \Lambda \subset \mathbb{R}^3 | \lambda \sim -\mathbf{a} \in V \subset \mathbb{R}^3\}; \tag{A.3}$$

where $\mathbf{a}^\pm$ denotes a dynamic equivalence class (DEC); termed *dynamic* because subject to such transformations as $Q(\pm\mathbf{b}) : \mathbf{a}^\pm \rightarrow \mathbf{b}^\pm$, or $Q(\pm\mathbf{a}) : \mathbf{b}^\pm \rightarrow \mathbf{a}^\pm$, *with relevant prevalencies.*

(A.3) shows that Λ is partitioned dyadically under the mapping $[\lambda \rightarrow \pm\mathbf{a}\}$. So $\sim$ on the elements of Q's domain denotes: "Has the same output/image under Q." With $[+\mathbf{a} \rightarrow +\mathbf{a}\} = [\lambda \rightarrow +\mathbf{a}\}$ — allowing that $\mathbf{a}$ could be an element of Λ — $[. \rightarrow \mathbf{a}\}$ is well-defined under $\sim$ on Λ. The quotient set is a set of two diametrically-opposed extremes: $\Lambda/\sim = \{\mathbf{a}^+, \mathbf{a}^-\}$, a maximal antipodean discrimination.

A.5 The fundamental experiment of CLR

$$p(\lambda_1) \Rightarrow [\lambda_1 \rightarrow \pm\mathbf{v}_k\}(\lambda_1{\cdot}\mathbf{v}_k) = \pm 1 = x : y = \pm 1 = (-\mathbf{v}_k^{'}{\cdot}\lambda_1^{'})\{\mp\mathbf{v}_k^{'} \leftarrow \lambda_1^{'}] \Leftarrow p'(\lambda_1^{'}) :$$
$$xy = +1 : \text{ for all } k = 1, 2, .., \aleph_0, \tag{A.4}$$

for all unit-vectors $\mathbf{v}_k \in V \subset \mathbb{R}^3$ and any number of tests: an important proof of exactness.

That is: In (A.4) we take a single pristine particle-pair $p(\lambda_1), p'(\lambda_1^{'})$ and we test and re-test them in pristine condition. A feat possible using CLR's *gedanken*-restoration program ... — *before you bin this essay, shouting angrily, "You can't do that! Once you test a particle it's destroyed!* — please read this continuation: ... under CLR *gedanken*-analysis! Please read the findings:

A.6 The fundamental findings of CLR

Under (A.4): (a) Q-functions are proven to be such: it is impossible to map one spin-beable to two different outputs/images. (b) The equivalence relation $\sim$ on Λ holds: spin-related beables are equivalent if Q maps them to the same output/image.

So $p(\lambda) = p(\lambda \sim +\mathbf{a}) = p(\mathbf{a}^+)$ reveals the previously-hidden (but related) DEC of its unperturbed and *still-pristine* correlate: ie, in general;

$$p(\lambda) = p(\lambda \sim \pm\mathbf{a}) = p(\lambda \in \mathbf{a}^\pm) = p(\mathbf{a}^\pm) \text{ implies} \tag{A.5}$$

$$p'(\lambda') = p'(\lambda' = -\lambda) = p'(\lambda' \sim \mp\mathbf{a}') = p'(\lambda' \in \mathbf{a}'^{\mp}) = p'(\mathbf{a}'^{\mp}); \text{ and vice-versa, etc :} \quad (A.6)$$

a range of properties (physical facts) suited to many analytic situations.

More formally: $Q : \Lambda \rightarrow V \subset \mathbb{R}^3$ by assigning every object $\lambda \in \Lambda$ to exactly one element $Q(\lambda) \in V$ where V is the space of 3-vectors. Experimental proof of the exactness here is provided thus: The product of the paired outputs (±1) from $SGD(\pm\mathbf{a})$ on $p(\lambda)$ and $SGD(\mp\mathbf{a}')$ on $p(\lambda')$ — for all $\mathbf{a}$ and any number of tests — equals one.

Allowing λ, λ' to be antiparallel random beables, it follows that the mutually-exclusive collectively-exhaustive equiprevalent outputs are here $\sim \pm\mathbf{a}$ and $\sim \pm\mathbf{b}$; to thus highlight the symmetries in EPRB.

A.7 CLR dynamics

CLR dynamics deliver the results of local SGD/particle interactions *as well as their* factual *implications*; updating facts re pristine correlates with a mathematical **If** ...**:** **Then** ...: Converting the source of our inferences (*physical facts*) to relevant physical properties (*other physical facts*) via the mathematical transmission of such facts; independent of vague words and conjectures.

> 'Surely the big — $SGD(\mathbf{a})$ — and the small — $p(\lambda)$ — should merge smoothly with one another? And surely in fundamental physical theory this merging should be described not just by vague words but by precise mathematics?' after Bell (2004:190).
>
> "The concept of 'measurement' becomes so fuzzy on reflection that it is quite surprising to have it appearing in physical theory *at the most fundamental level.* ...does not any *analysis* of measurement require concepts more *fundamental* than measurement? And should not the fundamental theory be about these more fundamental concepts? One line of development towards greater physical precision would be to have the [quantum] 'jumps' [or mergings] in the equations and not just in the talk — so it would come about as a dynamical process in dynamically defined conditions," Bell (2004:117-118).

In the context of EPRB, we take *transformation* to be a concept 'more fundamental than measurement'. Requiring such transformations/mergings in our equations – and not just in the talk – we allow that local interaction between $SGD(\mathbf{a})$ and $p(\lambda)$ transforms both the particle and the device: transforming hidden beables and revealing DECs; eg, $\lambda \in \mathbf{a}^+$. Importantly, a pristine correlate will have a related DEC: ie, $\lambda' \in \mathbf{a}^-$ in this example; with a certain confirmatory $\lambda' \in \mathbf{a}'^-$ by Bob's direct pre-, 'simultaneous' or post-testing of that correlate.

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