

On the Brich and Swinnerton-Dyer conjecture

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The elliptic curve be: $y^2 = x^3 + ax + b$.

Now pointing $P(0, 1)$ and $Q(1, 2)$ on the curve,

which are become two equations set:

$$\begin{cases} 1^2 = 0^3 + a \times 0 + b \\ 2^2 = 1^3 + a \times 1 + b \end{cases}$$

By the next step, $a = 2$ and $b = 1$.

and the elliptic curve is become:

$$y^2 = x^3 + 2x + 1 .$$

but now, let $x = \cos \theta$ and $y = \sin \theta$.

By the next step, the elliptic curve is:

$$\sin^2 \theta = \cos^3 \theta + 2\cos \theta + 1 .$$

because $\sin^2 \theta + \cos^2 \theta = 1$,

the elliptic curve become:

$$1 - \cos^2 \theta = \cos^3 \theta + 2\cos \theta + 1 ,$$

By the next step, the elliptic curve is:

$$\cos^2 \theta + \cos \theta + 2 = 0 ,$$

By the next step, the elliptic curve is :

$$\left(\cos \theta + \frac{1}{2}\right)^2 - \frac{1}{4} + 2 = 0 ,$$

By the next step, $\cos \theta = \frac{1}{2}(\pm\sqrt{7}i - 1)$.

But now, let $\pm\sqrt{7}i - 1 = 1$,

By the next step, $\cos \theta = \frac{1}{2}$, and $\theta = 2k\pi + \frac{\pi}{3}$, ($k = 0, 1, 2, 3, 4, \dots, \infty$) ,

So, $x = \cos(2k\pi + \frac{\pi}{3}) = \frac{1}{2}$ and $y = \sin(2k\pi + \frac{\pi}{3}) = \frac{\sqrt{3}}{2}$, ($k=0,1,2,3,4,\dots,\infty$) .