The new interpretation of arithmetic operation symbols

E.Koorambas

8A Chatzikosta, 11521 Ampelokipi, Athens, Greece
E-mail:elias.koor@gmail.com

(March 5, 2014)

Abstract. We introduce the permutation group of arithmetic operations symbols by getting the permutations of all the common arithmetic operations symbols, with keeping the brackets out of ordering. We find 6 ways of doing the arithmetic operations. Therefore the output of any mathematical formulas depends on which one element of the arithmetical permutation group we work on. We find invariants by the reordering of the arithmetic operation x+y, xy. Working with the irreducible representation of the permutation arithmetic symbols group we define new arithmetic structures called arithmetic particles symbols.

MSC number(s):00A30, 03A05, 20B30

Key-Words: Arithmetic Operation Symbols, Permutation Group

It is certainly both natural and useful to associate positive numbers with length, areas, and volumes. This is an ideal that goes back at least to the ancient Greeks [1], and we use it whenever we draw a graph, a histogram or almost any kind of chart.

Following Howie.J.M. [2], to take a simple example, if we have a triangle ABC, right angles at A, and with AB and AC of length 1, then by the celebrated theorem of Pythagoras the length BC is a number z

$$z^2 = 1+1 = 2 \quad \therefore \quad z = \sqrt{2} \qquad (1.1)$$

But how do we find this result? In any set of numbers we follow a standard algorithm by doing the calculation which children know at the first classes of school namely as ordering of operations [3] .

1.0 Axiom*:*

Let two or more numbers x,y,………, $\in$ A where A is any set of numbers, and let a mathematical formula K made by those numbers, and by the normal operations +, - ,$\times$ / and the brackets ( )

To find K we follow the ordering:

A1. The brackets are going first of all the others

A2. Second we are doing the squares

A3. Third we are doing the multiplications and the divisions

A4. Finally we are doing the sums

That is the way which gives the result $z = \sqrt{2}$ in equation (1.1). More formally this equation is the inner product in the set of real numbers R where (1.1) defines the positive length and thus the space which is the Euclidian one for this case.

1.1. Definition: Let any set of numbers A : if in this set we are following the operations ordering axiom, then we say that we have 123 arithmetic in this; Thus 123 arithmetic is the way of doing the operation which respects the axiom 1.0. The set A could be any set of numbers $\mathbb{N}_{123}, \mathbb{Q}_{123}, \mathbb{Z}_{123}, \mathbb{R}_{123}, \mathbb{C}_{123}$

Let us first give the following notations, assuming that first of all we have the brackets { }, thus we keep them out of the ordering. For the other operations we have the following definition for the arithmetic operations symbols

$1 \equiv (\quad squares \quad)$

$2 \equiv \left(\times(a,b), \div(a,b) \right)$

$3 \equiv \left(+(a,b), -(a,b) \right)$, for all $a,b \in \mathbb{R}$

This is just the normal ordering of 123 arithmetic. Let us now define the Set $\{1,2,3\}$. Let all injective maps S from $\{1,2,3\}$ to $\{1,2,3\}$. Make this into the group [4] for some $a,b \in \mathbb{R}$.There are six elements in S_3. They are:

$E = (1)(2)(3)$, $A = (13)(12)$, $B = (12)(13)$,

$C = (1)(23)$, $D = (2)(13)$, $F = (12)(3)$

This formulation is very important as every element {1,2,3} is denoted by the operation, hence the permutation group S_3 making 5 new ways of doing the arithmetic operation gives 5 new arithmetic's (13)(12), (12)(13),(1)(23), (2)(13),(12)(3) symbols.

2.1. Proposition. The mathematical outcomes from any specific formula K known so far, just belongs to one element of permutation S_3 arithmetic operation symbols group, (1) (2) (3) element, coming from the concept of S_3 arithmetic and respect the 1.0 Axiom.

Proof: here we mean that we respect the specific interpretation of arithmetic operations symbols given by the (1) (2) (3) element S_3 arithmetic operation symbols group.

An example of the effect on a mathematical formula by the reordering of the arithmetic operations symbols is the following: let the function

$$f(z) = z^2 + 1, z \in \mathbb{C} \qquad (2.1)$$

And let that respects the arithmetic S_3 group, in the case of the (123) element, the well known as ordinary. This function is said to be conformal in all it's range except the zero [5]. Let the (321) element that also belongs to the arithmetic S_3 group, the function $f(z)$, is given by

$$f(z) = z(z+1), z \in \mathbb{C} \qquad (2.2)$$

We see that the same function changes under the reordering of arithmetic operation symbols, hence $f(z)$ is conformal respect in all the range except the point $-1/2$. With this example we see that a property of a function $f(z)$ which is the range of where is conformal or not conformal, changes under the action of permutation of arithmetic operation. The possible partitions of the $S_3^{Arithmetic}$ group are $[3]$, $[21]$, $[111]$:

The partition $[3]$ corresponds only to one Young tableau namely

$\boxed{1}\boxed{2}\boxed{3}$

The symmetrizer corresponding to the tableau is

$$P_{Aritmetic} = \begin{Bmatrix} E + (12) + (13) + \\ (23) + (123) + (132) \end{Bmatrix} \qquad (3.1)$$

3.1 Definition: we define the algebraically structure referring of the partition $[3]$ of the $S_3^{Arithmetic}$ group, and represents the symmetrizer operator (3.1) which is the result of combination of doing the arithmetic operations symbols.

The partition $[111]$ corresponds only to one Young tableau namely

$\boxed{1}$

$\boxed{2}$

$\boxed{3}$

The antisymmetrizer corresponding to the tableau is

$$Q_{Aritmetic} = \begin{Bmatrix} E + (12) + (13) \\ +(23) + (123) + (132) \end{Bmatrix} \qquad (3.2)$$

3.2 Definition: we define the algebraically structure referring to the partition $[111]$ of the $S_3^{Arithmetic}$ group, and represents the antisymmetrizer operator (3.2) which is the result of combination of doing the arithmetic operations.

The partition $[21]$ corresponds two different Young tableau namely

$\boxed{1}\boxed{3}$ $\qquad$ $\boxed{1}\boxed{2}$

$\boxed{2}$ $\qquad\quad$ $\boxed{3}$

In the first one the corresponding operators are

$$P=\{E+(13)\},Q=\{E-(12)\} \quad (3.3)$$

The total Young operator is

$$\mathrm{Y}=PQ=\{E+(13)-(12)-(123)\} \quad (3.4)$$

In the second corresponding operators are

$$P=\{E+(12)\},Q=\{E-(13)\} \quad (3.5)$$

$$\mathrm{Y}=PQ=\{E-(13)+(12)-(123)\} \quad (3.6)$$

3.3 Definition: we define the algebraically structure referring to the partition $[21]$ of the $S_3^{Arithmetic}$ group, and represent by the mix symmetrizer operators (3.6), (3.4) which is the result of combination of doing the arithmetic operations.

The possible Young diagrams are

$$\begin{array}{c}\square\\ \square\\ \square\end{array} \qquad \square\square\square \qquad \begin{array}{l}\square\square\\ \square\end{array}$$

The antisymmetric tensor corresponding

$$\mathrm{Y}\begin{pmatrix}\square\\ \square\\ \square\end{pmatrix}=1-(12)-(13) \quad (4.1)$$
$$-(23)+(123)+(132)$$

Its dimension is

$$N(1^3,n)=\binom{n}{3}=\frac{1}{6}n(n-1)(n-2) \quad (4.2)$$

The symmetric tensor corresponding

$$\mathrm{Y}\left(\square\square\square\right)=1+(12)+(13) \quad (4.3)$$
$$+(23)+(123)+(132)$$

Its dimension is

$$N(n)=\frac{1}{6}n(n+1)(n+2) \quad (4.4)$$

For n = 3, we have the following 10 components

$\boxed{1}\boxed{1}\boxed{1}$ $\boxed{1}\boxed{1}\boxed{2}$ $\boxed{1}\boxed{1}\boxed{3}$ $\boxed{1}\boxed{2}\boxed{2}$ $\boxed{1}\boxed{2}\boxed{3}$

$\boxed{1}\boxed{3}\boxed{3}$ $\boxed{2}\boxed{2}\boxed{2}$ $\boxed{2}\boxed{2}\boxed{3}$ $\boxed{2}\boxed{3}\boxed{3}$ $\boxed{3}\boxed{3}\boxed{3}$

The mix symmetric tensor corresponding to arithmetic anion

$$\mathrm{Y}\begin{pmatrix}\boxed{1}\boxed{2}\\ \boxed{3}\end{pmatrix}=(1-P_{13})(1+P_{12}) \quad (4.5)$$

$$\mathrm{Y}\begin{pmatrix}\boxed{1}\boxed{3}\\ \boxed{2}\end{pmatrix}=(1-P_{12})(1+P_{13}) \quad (4.6)$$

Its dimension is found from the product

$$[11]\times[1]=[111]+[21] \quad (4.7)$$

This implies

$$N([2,1],n)=\frac{1}{3}n(n^2-1) \quad (4.8)$$

For n = 3, we have the following 8 components

$\begin{array}{l}\boxed{1}\boxed{1}\\ \boxed{2}\end{array}$ $\begin{array}{l}\boxed{1}\boxed{2}\\ \boxed{2}\end{array}$ $\begin{array}{l}\boxed{1}\boxed{3}\\ \boxed{2}\end{array}$ $\begin{array}{l}\boxed{1}\boxed{1}\\ \boxed{3}\end{array}$

$\begin{array}{l}\boxed{1}\boxed{2}\\ \boxed{3}\end{array}$ $\begin{array}{l}\boxed{1}\boxed{3}\\ \boxed{3}\end{array}$ $\begin{array}{l}\boxed{2}\boxed{2}\\ \boxed{3}\end{array}$ $\begin{array}{l}\boxed{2}\boxed{3}\\ \boxed{3}\end{array}$

we give some examples of the other interpretation of arithmetic operations symbols in the 321 ordering

Example.1

Let $a,b,c\in R$ and the formula K given by

$$K=\times(+(a,b),c)=c(a+b)=ca+cb$$
$$K\in\mathbb{R}_{123} \quad (5.1)$$

The Reform of $K\in\mathbb{R}_{123}$ to the $\mathbb{R}_{321}$ is given by

$$\mathrm{Re-}\ form(K)=K'\in\mathbb{R}_{321}$$
$$K'=\mathrm{Re-}\ form[\times(+(a,b),c)]$$
$$=\mathrm{Re-}\ form[ca+cb]=c(a+c)b \quad (5.1)$$
$$K'\in\mathbb{R}_{321}$$

Example.2

Let a sequence, or series defined in the space $\mathbb{R}_{123}$, there are series in the space $\mathbb{R}_{321}$? If exists a sequence or series, exists always? The answer is yes for both statements. The reordering map f transfers from sequence or series to sequence or series. Let work with example. Let the series

$$\sum_{n=0}^{n=\infty}n^2+1 \text{ defined in the } \mathbb{R}_{123}. \quad (5.2)$$

What is the corresponding sequence in the $\mathbb{R}_{321}$? Appling the reordering map f to the real number set $\mathbb{R}_{123}$ we get the following sequence for $\mathbb{R}_{321}$ given by

$$\mathrm{Re-}\ form(\sum_{n=0}^{n=\infty}n^2+1)=\sum_{n=0}^{n=\infty}n\left(n+1\right) \quad (5.3)$$

This is the corresponding sequence of $\sum_{n=0}^{n=\infty} n^2+1$ in the set $\mathbb{R}_{321}$, because right said of the (5.3) can exist in the set $\mathbb{R}_{123}$, thus (5.3) is also corresponding sequence in the set $\mathbb{R}_{123}$ therefore the relation is one to one corresponding. Thus we have an equivalent between them [4], and if existed in one set must exist in the other one.

Theorem 5.1: For every sequence or series in the X_{123} space respect the 123 orbit of arithmetic there exists always an equivalent sequence in the X_{123} space that respects the 321 orbit of arithmetic and vice versa.

Proof: an equivalent relation between X_{123}, X_{123} spaces ensures that the above theorem exists.

5.1 Definition: We defined invariant from the reordering point of view the relations which are not affected from the reordering of arithmetic operations symbols, like relations with only one arithmetic operations symbols

$$x+y, x\times y, ... \qquad (5.4)$$

By looking the usually arithmetic operations symbols from above, as elements of an ordering set making this set to the symmetric group S_3 and keeping the brackets out of ordering. By S_3 arithmetical group we find the 6 different ways of doing the arithmetical operations yield to different results of mathematical formulas. As we work with the irreducible representation of S_3 arithmetic symbols group we get the following case: the symmetric arithmetical operations symbols (arithmetic bosons) and the antisymmetric of arithmetic operations symbols (arithmetic fermions) and the two mix arithmetic symbols (arithmetic anions).

References

[1] Morris Kline, Mathematical Thought from Ancient to Modern Times (Oxford: Oxford University Press, 1972).

[2] Howie.J.M.Real Analysis Springer - Verag London (2001).

[3] Hazewinkel, Michiel,ed. Encyclopedia of Mathematics, Springer London (2001).

[4] Weiss. M .Group and Geometry .Aberdeen University Press (2001).

[5] Churchill, Brown, Complex Variables and Applications, McGraw-Hill, Inc (1948).