# Relativistic Physics of Force Fields in the Space-Time-Mass Domain

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**Abstract**

We consider physics of reference frames moving with any speed, starting from the inertial field. We point out the different physical behavior of electrodynamic systems with respect to other systems, due to the different physical meaning of mass. These considerations allow to define a generalized concept of field that materializes in the domain Space-Time-Mass where new equations of trasformations for reference frames are used. We define at last physical properties of differential forces of reaction in fields of force.

## 1. Introduction

Tranformations of the Space-Time-Mass (domain STM), that are valid in the Theory of Reference Frames, have been calculated in general[1][2] and we have demonstrated, with reference to fig.1, that they are given by the following equations

$$\mathbf{P}[x,y,z,t] = \mathbf{P'}[x',y',z,t'] + \int_0^t \mathbf{v}\, dt$$

$$dt = \frac{m_o dt'}{m'} \qquad (1)$$

in which **P'** is the point inside the moving reference frame S'[O',x',y',z',t'] with any vector speed $\mathbf{v}[v_x,v_y,v_z]$ with respect to the reference frame supposed at rest S[O,x,y,z,t]. **P** is the same point considered with respect to S; $m_o$ and m' represent masses of the massive point with respect to S and S' respectively.

The second of (1) proves that the existence of different times t and t' is possible in the two reference frames (and consequently a relativistic effect of time), but that effect is related to a mass variation in the two reference frames, that is due to a particular and local physical phenomenon, and it isn' t related to a purely kinematic change of the space-time. The two reference frames therefore proceed synchronous even if masses experience different times (or better different durations) in the used physico-mathematical model. We have proved[1][3][4] mechanical systems don' t have a real variation of inertial mass (i.e. $m'=m_o$) and consequently for mechanical systems transformations (1) of the Space-Time-Mass are

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$$\mathbf{P}[x,y,z,t] = \mathbf{P'}[x',y',z',t'] + \int_0^t \mathbf{v}\,dt \qquad (2)$$

$$t = t'$$

For electromagnetic systems, including light and all radiations with any frequency, the (2) are still valid because they are systems of pure energy and consequently a real variation of mass cannot exist for them.

Electrodynamic systems (charged elementary particles) instead have an electrodynamic mass that changes with the speed and therefore we have to consider the (1) with a relativistic effect of time whose physical meaning has been often specified.

Pursuing our study we will apply equations (1) or (2) to the Newton law that, even if it doesn' t represent the complete law of motion, however is certainly one of more important equations of physics.

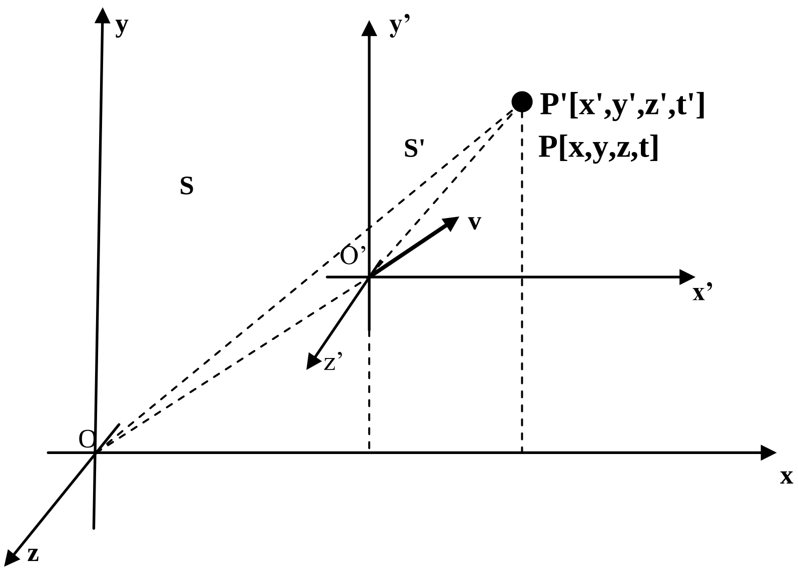


Fig.1 The reference frame S' is moving with linear speed v with respect to the resting reference frame S. The point P' is inside the moving reference frame S', while P is the same point considered with respect to S.

## 2. The inertial field in the STM domain

The inertial field is the field with null force, in which the resultant of all applied forces from outside to the moving inertial reference frame S' is zero. All reference frames with inertial velocity with respect to the reference frame supposed at rest S[O,x,y,z,t] fall under this definition of inertial field. It needs to specify the reference frame supposed at rest isn' t an absolute reference frame but it is at rest with respect to both, physical space where the phenomenon happens and the preferred observer. Inertial reference frames are defined strictly by the Generalized Principle of Inertia[1].

Physics of inertial field is defined by the Principle of Relativity, that claims *"physical laws and mathematical equations that describe them are invariant with respect to all reference frames that are inertial with respect to the reference frame supposed at rest".*

The first consequence of the Principle of Relativity is that all physical quantities are invariant in the inertial field except for those quantities that depend on the relative speed between the moving reference frame S' and the resting reference frame S, as the speed, wavelength, frequency, electrodynamic mass, etc..

**- Mechanical systems and electromagnetic systems (including light)** don' t present variation of mass with respect to reference frames in relative motion and therefore in the inertial field the (2) have to be used, that here, being v constant, we write like this

$$\mathbf{P}[S] = \mathbf{P}'[S'] + \mathbf{v}\, t \qquad t = t' \tag{3}$$

For non linear inertial reference frames transformations equations are in the ref[1]. Supposing inertial motion is linear and it happens along the axis x of S and at initial time t=0 the moving reference frame coincides with the resting reference frame, the constant speed of the origin O' of S' with respect to S is given by v=x/t. We can give a graphic representation of the inertial motion on a kinematic plane with two dimensions and with origin in the point O (fig.2). If we assume in a symbolic manner, for convenience of representation, the speed v of the moving reference frame S'[O',x',y',z',t'] is equal to 1, then it means the space covered by S' with respect to S is equal to time spent for covering it and the speed of S' is represented in the graph by the bisector line of the first quadrant, for which $\alpha$=45° and v=tg$\alpha$=1.

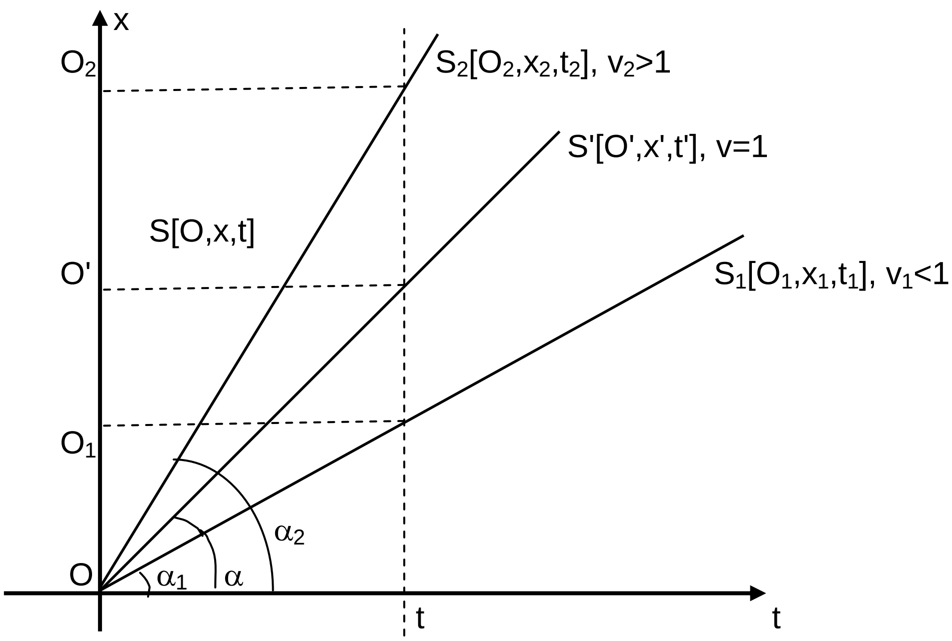


Fig.2 Graphic representation of inertial field and of motion of inertial reference frames S', $S_1$ and $S_2$ with respect to the reference frame supposed at rest S.

The line in graph marked by $v_1<1$ represents the inertial motin of another reference frame $S_1[O_1,x_1,y_1,z_1,t_1=t]$ moving with speed $v_1<v$ and $v_1= tg\alpha_1<1$ with respect to S.
Similarly the line in graph marked by $v_2>1$ represents the inertial motion of a reference frame $S_2[O_2,x_2,y_2,z_2,t_2=t]$ moving with speed $v_2>v$ and $v_2= tg\alpha_2>1$. We supposed also that both moving reference frames, $S_1$ and $S_2$, have the origin coinciding with O at initial time t=0 and both move along the axis x of S, like S'. The three reference frames S', $S_1$ and $S_2$ are inertial with respect to the reference frame S supposed at rest and hence are inertial each other.
In the order of the Theory of Reference Fames the relativistic speed $\mathbf{u_1}$ of $S_1$ with respect to S', at each time t, is given by

$$\mathbf{u_1} = tg\alpha_1 - tg\alpha = \mathbf{v_1} - \mathbf{v} < 0 \qquad (4)$$

and the relativistic speed $\mathbf{u_2}$ of $S_2$ with respect to S' is given by

$$\mathbf{u_2} = tg\alpha_2 - tg\alpha = \mathbf{v_2} - \mathbf{v} > 0 \qquad (5)$$

The negative value of $\mathbf{u_1}$ means only the relativistic speed $\mathbf{u_1}$ is directed in reverse with respect to **v**.
**- For mechanical systems** Newton's law is valid and it has the following expression in the reference frame S

$$\mathbf{F} = m_o\, \mathbf{a_o} = m_o \frac{d\mathbf{u}}{dt} \qquad (6)$$

where suppose that **F**, $\mathbf{a_o}$ and **u** have the same direction as x and $m_o$ is the inertial mass.
For the inertial reference frame S' (the same reasoning is valid also for reference frames $S_1$ and $S_2$) we have $\mathbf{F'}=m'\mathbf{a'}$ in which $\mathbf{F'}=\mathbf{F}$ and $\mathbf{a'}=d\mathbf{u'}/dt'$.
For mechanical systems inertial mass doesn't change with the speed, for which $m'=m_o$.
Because of (3) we have $\mathbf{u'}=\mathbf{u}-\mathbf{v}$ and $\mathbf{a'}=\mathbf{a_o}$, consequently for mechanical systems the Newton law is invariant with respect to all inertial reference frames in concordance with the Principle of Relativity.
**- For electromagnetic and optical (light) systems** the Newton law isn't valid, but with respect to only kinematic aspects the same results of mechanical systems are valid.
In fact if the considered physical system is the motion of light or of an electromagnetic wave that move with the physical speed c with respect to S, as per (3) they move with respect to S', $S_1$ and $S_2$ respectively with relativistic speeds

$$c' = c - v \qquad c_1 = c - v_1 \qquad c_2 = c - v_2. \qquad (7)$$

**- For electrodynamic systems (charged elementary particles)** electrodynamic mass[1][3] must be considered and it changes with the speed according to the well-known relation

$$m = m_o\left(1 - \frac{u^2}{2c^2}\right) \tag{8}$$

where $m_o$ is electrodynamic mass of the resting particle into the reference frame supposed at rest S, m is electrodynamic mass of the moving particle with respect to S.
The Newton law in the resting reference frame S is $\mathbf{F}=m_o\mathbf{a_o}=m\mathbf{a}$, but for the moving inertial reference frame S', relative to electrodynamic systems, we have

$$\mathbf{F'} = m'\mathbf{a'} = \mathbf{F} \tag{9}$$

with

$$m' = m_o\left(1 - \frac{u'^2}{2c^2}\right) \tag{10}$$

where

$$u' = u - v \tag{11}$$

and

$$a' = \frac{a_o}{1 - \frac{u'^2}{2c^2}} \tag{12}$$

The (10) and (12) prove that for electrodynamic systems, unlike mechanical systems, even if in full concordance with the Principle of Relativity, mass and acceleration are non-invariant quantities in the inertial field. Besides for v=0 we have the expression of mass and acceleration at changing of speed with respect to the resting reference frame S.
Graphic representations of electrodynamic mass and acceleration of an electrodynamic particle at changing of the speed are in fig.3 and fig.4.
In fig.3 also the trend of time is represented and it is the same as mass, as per the second of (1)

$$t' = t\left(1 - \frac{u'^2}{2c^2}\right) \tag{13}$$

in which t is time with respect to S.
We observe that for $u'<u'_c=\sqrt{2}\,c$ , where $u'_c$ is the critical speed, particle time t' calculated in the moving laboratory slows down with respect to particle time in S (t'<t) and it implies a virtual contraction of the average life of particle with respect to the reference frame S' of the moving laboratory. Otherwise we can say particle time with respect to the resting reference frame undergoes a dilation with respect to the time in S'. It needs to specify that whether contraction or dilation are virtual because they aren't kinematic but are connected with the variation of electrodynamic mass.
For $u'>u'_c$ time of the moving laboratory becomes negative (t'<0) because also electrodynamic mass becomes negative (antimass) and similarly acceleration becomes negative.

The graph of accelerations shows that at the critical speed acceleration has a singularity, whose interpretation is based on the fact that at the critical speed electrodynamic mass is zero and the constant (non-null) force applied to particle involves an infinetely great acceleration.
Getting over the critical speed, a negative electrodynamic mass (antimass) is generated for which also acceleration has to become negative in order to continue to increase with the speed and to become zero at infinite speed.

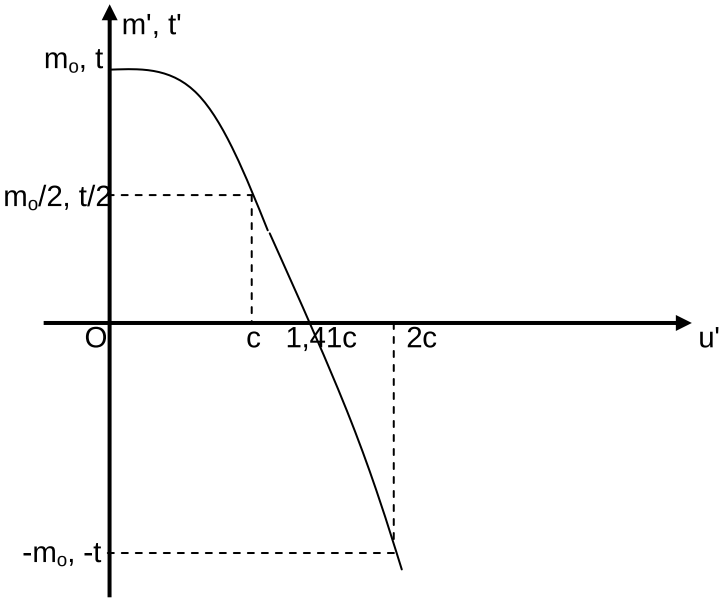


Fig.3 Diagram of electrodynamic mass and time of particle at changing of the speed in the inertial field in which $u'_c=\sqrt{2}\,c=1{,}41c$ is the critical speed.

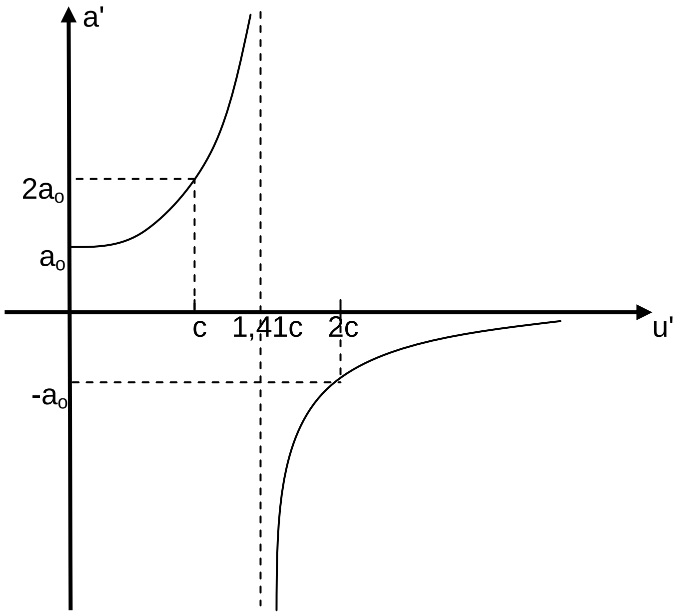


Fig.4 Diagram of acceleration of particle at changing of the speed in the inertial field.

## 3. The field of uniform force in the STM domain

The inertial field is a field characterized by null force, the field of uniform force is a field in which considered reference frames are accelerated in uniform way (constant acceleration) through a constant force with respect to the reference frame supposed at rest.
In that case transformation equations of the Space-Time-Mass (1) become

$$\mathbf{P}[S] = \mathbf{P'}[S'] + \frac{\mathbf{a_v}\, t^2}{2}$$

$$dt = \frac{m_o}{m'}\, dt' \qquad (14)$$

in which $\mathbf{a_v}[a_x,a_y,a_z]$ is the constant acceleration of the moving reference frame S' with respect to the reference frame S supposed at rest.
Supposing that, like in the case of the inertial field, motion happens with variable speed **v** along the axis x, the constant acceleration $\mathbf{a_v}$ is given by $\mathbf{a_v}=\mathbf{v}/t$.
In that case the graphic representation of accelerated motion on a two-dimensional plane (O,x,t) with origin in the point O is parabolic and graphs for three different values of acceleration are drawn in fig.5, where $a_1<a_v$ and $a_2>a_v$.

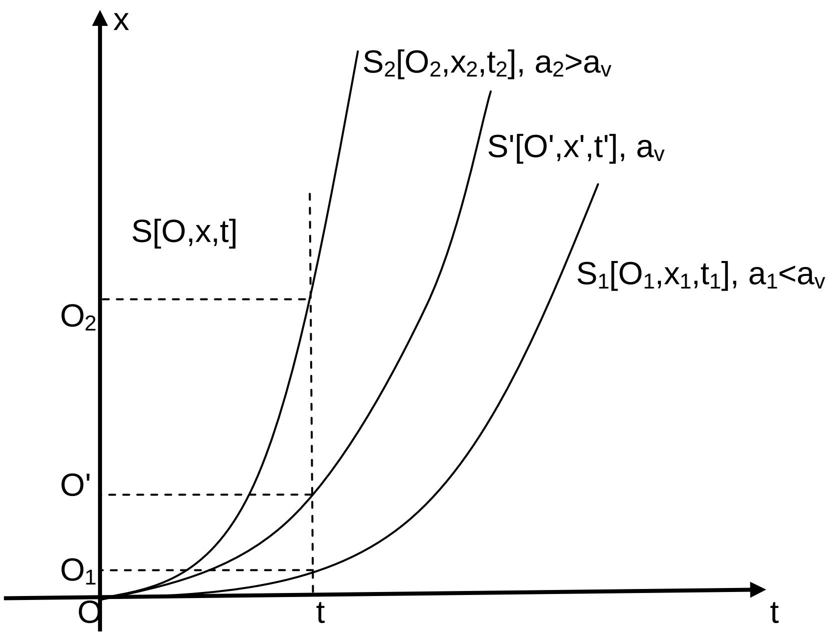


Fig.5 Graphic representation of reference frames S', $S_1$ and $S_2$ moving with constant acceleration with respect to the reference frame supposed at rest S.

Distances travelled by origins O', $O_1$ and $O_2$ of the three reference frames S', $S_1$ and $S_2$ in the time t are

$$x' = \frac{a_v t^2}{2} \qquad x_1 = \frac{a_1 t^2}{2} \qquad x_2 = \frac{a_2 t^2}{2} \qquad (15)$$

Speeds of the three reference frames are

$$v=a_v t \qquad v_1=a_1 t \qquad v_2=a_2 t \qquad (16)$$

The three reference frames start at zero time $t'=t_1=t_2=t=0$ with the same null initial speed, therefore trigonometric tangents of angles that geometric tangents in the origin form with the time axis are null and also angles are null (fig.5).
It's manifest that the three reference frames, being non-inertial and provided with an uniform accelerated motion, are subjected to three constant forces $\mathbf{F'}$, $\mathbf{F_1}$ and $\mathbf{F_2}$ for which, even if they proceed synchronous, relative to the only considered physical process times $t'$, $t_1$ and $t_2$ depend on respective masses $m'$, $m_1$ and $m_2$ and if these change with respect to mass $m_o$ of the resting reference frame S also times change.
**- Mechanical systems** don't have a real variation of mass because of motion, for any speed, and therefore the considered mechanical event proceeds synchronous with the three reference frames and with the resting reference frame: $t'=t_1=t_2=t$.
**- Electromagnetic and optical (light) systems** don't have a real mass and therefore there isn't variation of mass. Consequently still the electromagnetic or optical event proceeds synchronous with the three reference frames and with the resting reference frame.
**- Electrodynamic systems (charged elementary particles)** have a real variation of mass with the speed and therefore they have also a relativistic effect of time.
Let us consider now the Newton law which, with respect to the resting reference frame S, has the expression $\mathbf{F}=m_o\mathbf{a_o}=m_o d\mathbf{u}(t)/dt$ ; with respect to the moving reference frame S' the same Newton law has the expression $\mathbf{F'}=m'\mathbf{a'}=m'du'(t')/dt'$ . Supposing the speed $\mathbf{v}$ of the reference frame S' with respect to the resting reference frame S has the same direction of $\mathbf{u}$, we have

$$\mathbf{u'} = \mathbf{u} - \mathbf{v} \qquad (17)$$

$$m' = m_o\left(1 - \frac{u'^2}{2c^2}\right) \qquad (18)$$

$$dt' = \left(1 - \frac{u'^2}{2c^2}\right)dt \qquad (19)$$

It follows that

$$\mathbf{a'} = \frac{\mathbf{a_o} - \mathbf{a_v}}{1 - \frac{u'^2}{2c^2}} \qquad (20)$$

$$\mathbf{F'} = m'\mathbf{a'} = m_o\mathbf{a_o} - m_o\mathbf{a_v} = \mathbf{F} - m_o\mathbf{a_v} \qquad (21)$$

and assuming that $\mathbf{F_o}=m_o\mathbf{a_v}$ , we have

$$\mathbf{F'} = \mathbf{F} - \mathbf{F_o} \qquad (22)$$

The same reasoning can be repeated also for $S_1$ and $S_2$ and in general for all accelerated reference frames. Preceding relations prove the Newton law isn't invariant for accelerated reference frames and consequently the Principle of Relativity isn't valid for the field of uniform force.

It is easy to test that the same graph of electrodynamic mass, valid in the inertial field (fig.3), is valid also for the field of uniform force, while for acceleration the graph differs for a few values (fig.6).

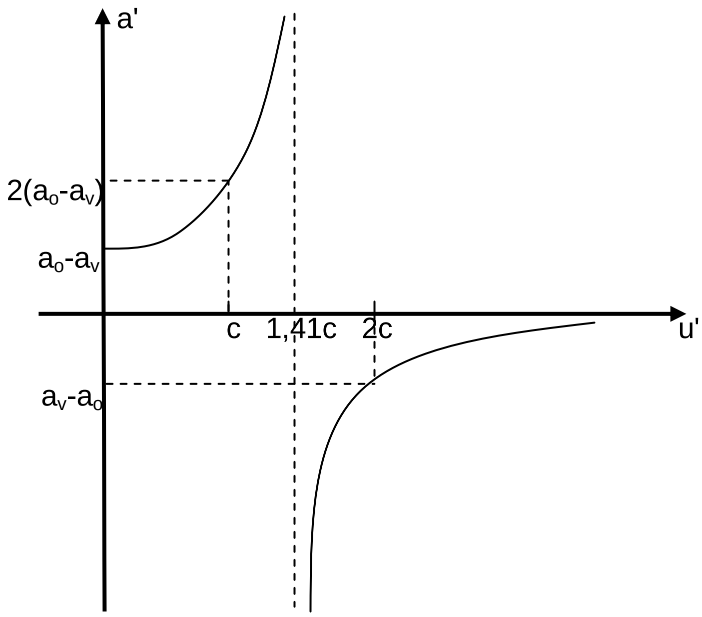


Fig.6 Diagram of acceleration of particle at changing of the speed in the field of uniform force.

## 4. The field of non-uniform force

The field of non-uniform force is a field in which considered reference frames are accelerated with non-constant acceleration through a variable force, with respect to the reference frame supposed at rest.

In that case, supposing acceleration changes with linear law $\mathbf{a}(t)=\mathbf{Y}t$ where $\mathbf{Y}[Y_x,Y_y,Y_z]$ is a vector constant, transformation equations of the Space-Time-Mass (1) become

$$\mathbf{P[}S\mathbf{]} = \mathbf{P'[}S'\mathbf{]} + \frac{\mathbf{Y}\, t^3}{6}$$

$$dt = \frac{m_o}{m'}\, dt' \qquad (23)$$

Suppose that motion happens along the axis x of S; at initial time t=0 the moving system is in the origin O with null initial speed and null initial acceleration.
In that case the graph of non-uniform accelerated motion on a two-dimensional plane (O,x,t) with origin in the point O is represented by cubic parabolas instead of quadratic parabolas as it happens in the field of uniform force, and therefore graphs are similar to those in fig.5. Similarly the relativistic physical behavior of reference frames and the relativistic behavior of physical systems into the field of non-uniform force is similar to the behavior into the field of uniform force.

## 5. Reaction forces

It is interesting to study the behavior of both reaction forces and reaction differential forces in force fields whether for mechanical systems or for electrodynamic systems.

**5a.** We know for a mechanical system with inertial mass $m_o$, in front of external resistant forces, the law of motion is given by the following general equation[1]

$$\mathbf{F}(t) = m_o \frac{d\mathbf{v}(t)}{dt} + k\mathbf{v}(t) \qquad (24)$$

where F(t) is the applied force, v(t) is the speed acquired by the mechanical system with mass $m_o$ and k is the "resistant coefficient of medium". The (24) considers whether the internal resistant force (force of inertia given by the Newton law $\mathbf{F_i}=m_o d\mathbf{v}(t)/dt$) or the external resistant force (resistant force of medium where motion happens given by $\mathbf{F_r}=k\mathbf{v}(t)$), for which we can write

$$\mathbf{F} = \mathbf{F_i} + \mathbf{F_r} \qquad (25)$$

Both forces, $\mathbf{F_i}$ and $\mathbf{F_r}$, are reaction forces besides resistant forces.
Differentiating the (25) we have

$$d\mathbf{F} = d\mathbf{F_i} + d\mathbf{F_r} \qquad (26)$$

in which $d\mathbf{F_r}$ and $d\mathbf{F_i}$ are reaction differential forces.

- If the applied force is null ($\mathbf{F}=0$) we have from (25)

$$\mathbf{F_i} = - \mathbf{F_r} \qquad (27)$$

and the two reaction forces are equal and opposite.
If also the external resistant force is null (k=0), then acceleration $\mathbf{a_o}=d\mathbf{v}(t)/dt$ is null and the speed field is an inertial field.

- If the applied force is constant ($d\mathbf{F}=0$), we have from (26)

$$d\mathbf{F_r} = - d\mathbf{F_i} \qquad (28)$$

In that case the two reaction differential forces $d\mathbf{F_r}$ and $d\mathbf{F_i}$ are equal and opposite. Precisely $d\mathbf{F_i}=m_o d\mathbf{a_o}$ is the differential force of inertia and $d\mathbf{F_r}=kd\mathbf{v}$ is the resistant differential force of medium. Through calculations we obtain

$$d\mathbf{a_o} = - \frac{k}{m_o} d\mathbf{v} \qquad (29)$$

If the resistant force is null (k=0), then $d\mathbf{a_o}=0$ and the force field is a field of uniform force.

- If the applied force isn't constant, we have

$$d\mathbf{F} = m_o d\mathbf{a_o} + kd\mathbf{v} \qquad (30)$$

that is the applied differential force is balanced by the vector sum of the two reaction differential forces. Supposing that the reaction resistant force is null (k=0), then in that case the applied differential force is balanced entirely by the differential force of inertia.

**5.b** For electrodynamic systems mass isn't constant with the speed, in fact

$$m = m_o \left(1 - \frac{v^2}{2c^2}\right) \qquad (31)$$

For simplicity of treatment we neglect the presence of external resistant forces (k=0), which is an acceptable hypothesis relative to motion of electrodynamic particles.

- If the applied force is null, from the (25) we arise that the inertial force of reaction $\mathbf{F_i}$ and acceleration are null, while electrodynamic mass changes with the speed in concordance with the (31). In these conditions electrodynamic particle has constant speed and it moves inside an inertial field.

- If the applied force is constant, always supposing that the resistant force of reaction is null (k=0), we have

$$\mathbf{F} = \mathbf{F_i} \qquad (32)$$

$$\mathbf{F} = m_o\mathbf{a_o} = m\mathbf{a} \qquad (33)$$

where m is given by the (31) and **a** is instead given by

$$\mathbf{a} = \frac{\mathbf{a_o}}{1 - \dfrac{v^2}{2c^2}} \qquad (34)$$

Because the applied force is constant (d**F**=0), while mass and acceleration are variable; differentiating the (32) we have

$$d\mathbf{F} = d\mathbf{F_i} \qquad (35)$$

$$d\mathbf{F} = md\mathbf{a} + \mathbf{a}dm \qquad (36)$$

$$md\mathbf{a} = -\mathbf{a}dm \qquad (37)$$

In the (36) $d\mathbf{F_m} = \mathbf{a}dm$ represents the differential force of inertia due to the mass variation, that produces emission of quantum electromagnetic energy, and $d\mathbf{F_a} = md\mathbf{a}$ represents the differential force of inertia due to the acceleration variation. In that case (F=constant) the (37) proves that the two differential forces of inertia are equal and opposite, that is

$$d\mathbf{F_m} = -\, d\mathbf{F_a} \qquad (38)$$

In these conditions electrodynamic particle moves inside a field of uniform force.
Doing calculations we find that the two differential forces of reaction due to the inertia are given by

$$d\mathbf{F_m} = -\, \frac{m_o \mathbf{a} v\, dv}{c^2} \qquad (39)$$

and similarly

$$d\mathbf{F_a} = \frac{m_o \mathbf{a} v\, dv}{c^2} \qquad (40)$$

- If the applied force isn't constant, then d**F**≠0 and the applied differential force is balanced simultaneously by both differential forces of inertia: the **mass differential force** that depends on the mass variation and produces quantum electromagnetic emission, and the **acceleration differential force** that depends on the acceleration variation.

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