Metric and algebraic tensors for 4-dimensional uncurved space.

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Abstract

The definition of the algebraic tensor for vector space by using the vector product of vectors from it's basis. And application it to the our 4 – dimensional space.

1) Metric tensor.

Let $\vec{e}_1, \vec{e}_2, \vec{e}_3, \vec{e}_4$ be the basis of uncurved space W. For these vectors we have :

$$(\vec{e}_1, \vec{e}_1) = 1 \quad (\vec{e}_2, \vec{e}_2) = -1 \quad (\vec{e}_3, \vec{e}_3) = -1 \quad (\vec{e}_4, \vec{e}_4) = -1 \qquad (1.1)$$

Another scalar products of these vectors equal zero. So

$$g_{11} = 1 \quad g_{22} = -1 \quad g_{33} = -1 \quad g_{44} = -1 \qquad (1.2)$$

Another components of metric tensor for W also equal zero. If we take vector

$$d\vec{R} = \vec{e}_1 \cdot i \cdot dt + \vec{e}_2 \cdot dx + \vec{e}_3 \cdot dy + \vec{e}_4 \cdot dz \qquad (1.3)$$

then we get :

$$(d\vec{R}, d\vec{R}^*) = dt^2 - dx^2 - dy^2 - dz^2 = ds^2 \qquad (1.4)$$ ds – interval.

If we take vector

$$\vec{P} = \vec{e}_1 \cdot i \cdot E + \vec{e}_2 \cdot P^x + \vec{e}_3 \cdot P^y + \vec{e}_4 \cdot P^z \qquad (1.5)$$

then we get :

$$(\vec{P}, \vec{P}^*) = E^2 - (P^x)^2 - (P^y)^2 - (P^z)^2 = m^2 \qquad (1.6)$$ m – rest mass of a particle

Here we use : $i \cdot i = -1 \qquad i^* = -i \qquad (1.7)$

2) Algebraic tensor.

Equation $[\vec{e}_\mu \times \vec{e}_\nu] = \vec{e}_\sigma \cdot F^\sigma{}_{\mu\nu} \qquad (2.1)$

defines the algebra for basis $\vec{e}_\mu$. In our space this algebra coincides with quaternion algebra :

	$\vec{e}_1$	$\vec{e}_2$	$\vec{e}_3$	$\vec{e}_4$	$\vec{e}_\nu$
$\vec{e}_1$	$\vec{e}_1$	$\vec{e}_2$	$\vec{e}_3$	$\vec{e}_4$	
$\vec{e}_2$	$\vec{e}_2$	$-\vec{e}_1$	$\vec{e}_4$	$-\vec{e}_3$	
$\vec{e}_3$	$\vec{e}_3$	$-\vec{e}_4$	$-\vec{e}_1$	$\vec{e}_2$	
$\vec{e}_4$	$\vec{e}_4$	$\vec{e}_3$	$-\vec{e}_2$	$-\vec{e}_1$	
$\vec{e}_\mu$					

Table 1

Let us take vector

$$\vec{R} = \vec{e}_1 \cdot i \cdot t + \vec{e}_2 \cdot x + \vec{e}_3 \cdot y + \vec{e}_4 \cdot z \qquad (2.2)$$

Then, using also (1.5) we derive :

$$[\vec{P} \times \vec{R}^*] = \vec{M} = \vec{e}_1 \cdot M^1 + \vec{e}_2 \cdot M^2 + \vec{e}_3 \cdot M^3 + \vec{e}_4 \cdot M^4 \quad (2.3)$$

$$M^1 = E \cdot t - P^x \cdot x - P^y \cdot y - P^z \cdot z \qquad (2.4)$$
$$M^2 = (P^y \cdot z - P^z \cdot y) + i \cdot (E \cdot x - P^x \cdot t) \qquad (2.5)$$
$$M^3 = (-P^x \cdot z + P^z \cdot x) + i \cdot (E \cdot y - P^y \cdot t) \qquad (2.6)$$
$$M^4 = (P^x \cdot y - P^y \cdot x) + i \cdot (E \cdot z - P^z \cdot t) \qquad (2.7)$$

If we demand that M^k were real at any k, then we get :

$$E \cdot x = P^x \cdot t \qquad (2.8)$$
$$E \cdot y = P^y \cdot t \qquad (2.9)$$
$$E \cdot z = P^z \cdot t \qquad (2.10)$$

Taking into account that $P^k = E \cdot V^k$ (2.11)

and $x = V^2 \cdot t$, $y = V^3 \cdot t$, $z = V^4 \cdot t$ (2.12)

we can say that equations (2.8), (2.9), (2.10) describe the motion of a particle. And real M^2, M^3, M^4 then describe orbital angular momentum of the particle.

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