A very simple formula which conducts to large primes and products of very few primes

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Abstract. Obviously, like everyone fond of arithmetic, I always dreamed to discover formulas to generate only primes; unfortunately, during the time, I dropped somewhat to find this Holy Grail. I found out that there are formulas that generate only primes, like Rowland’s formula, but often these formulas haven’t the desired impact, because, for instance, the values of the numbers used as “input” is larger than the one of the primes obtained as “output” and so on. In this paper I present a very simple formula based on Smarandache function, which, using primes of a certain form, conducts often to larger primes and products of very few primes and I also make four conjectures.

Observation:

Let p be a prime of the form 29 + 72*k, where k is positive integer; then the number n = p + 4*S(p + 1), where S is the Smarandache function, is often a prime, a square of a prime or a product of very few primes.

We have the following values of n for the first consecutive twenty-four values of p, *i.e.* 29, 101, 173, 317, 389, 461, 677, 821, 1109, 1181, 1613, 1901, 1973, 2333, 2477, 2549, 2621, 2693, 2837, 2909, 3413, 3557, 3701, 3917:

: n = 29 + 4*S(30) = 49 = 7^2;
: n = 101 + 4*S(102) = 169 = 13^2;
: n = 173 + 4*S(174) = 289 = 17^2;
: n = 317 + 4*S(318) = 529 = 23^2;
: n = 389 + 4*S(390) = 441 = 3^2*7^2;
: n = 461 + 4*S(462) = 505 = 5*101;
: n = 677 + 4*S(678) = 1129 prime;
: n = 821 + 4*S(822) = 1369 = 37^2;
: n = 1109 + 4*S(1110) = 1257 = 3*419;
: n = 1181 + 4*S(1182) = 1969 = 11*179;
: n = 1613 + 4*S(1614) = 2689 prime;
: n = 1901 + 4*S(1902) = 3169 prime;
: n = 1973 + 4*S(1974) = 2161 prime;
: n = 2333 + 4*S(2334) = 3889 prime;
: n = 2477 + 4*S(2478) = 2713 prime;
: n = 2549 + 4*S(2550) = 2617 prime;
: n = 2621 + 4*S(2622) = 2713 prime;

: n = 2693 + 4*S(2694) = 4489 = 67^2;
: n = 2837 + 4*S(2838) = 3009 = 3*17*59;
: n = 2909 + 4*S(2910) = 3297 = 3*7*157;
: n = 3413 + 4*S(3414) = 5689 prime;
: n = 3557 + 4*S(3558) = 7^2*11^2;
: n = 3701 + 4*S(3702) = 6169 = 31*199;
: n = 3917 + 4*S(3918) = 6529 prime.

Note the interesting fact that 2477 + 4*S(2478) = 2621 + 4*S(2622) = 2713.

We have the following values of n for six larger, consecutive, values of p:

: n = 720000000101 + 4*57443753 = 720229775113 prime;
: n = 720000000677 + 4*2876801 = 720011507881 = 769*936295849;
: n = 720000001037 + 4*15222631 = 720060891561 prime;
: n = 720000001901 + 4*120000000317 = 11*109*1000834031;
: n = 720000002261 + 4*120000000377 = 7*66947*2560661;
: n = 720000002333 + 4*120000000389 = 1200000003889 prime.

Note the interesting fact that the numbers (720000001901 + 1)/6, (720000002261 + 1)/6 and (720000002333 + 1)/6 are primes, respectively they are equal to 120000000317, 120000000377 and 120000000389.

Conjecture 1:
There is an infinity of primes p of the form p = 29 + 72*k, where k is positive integer.

Conjecture 2:
There is an infinity of primes q of the form q = (p + 1)/6, where p is prime of the form p = 29 + 72*k, where k is positive integer.

Conjecture 3:
There is an infinity of primes p for which the number p + 4*S(p + 1) is prime.

Example for Conjecture 3: 17 + S(18) = 23 prime.

Conjecture 4:
There is an infinity of primes p for which the number p + 4*S(p + 1) is equal to q + 4*S(q + 1), where q is a distinct prime from p and S is the Smarandache function.

Example for Conjecture 4: for p = 19 we have q = 23; indeed, 19 + 4*S(20) = 23 + 4*S(24) = 39.