Few interesting sequences obtained by recurrence and based on Smarandache function

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Abstract. In one of my previous papers, "An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function", combining two of my favorite topics of study, the recurrence relations and the Smarandache function, I discovered that the formula f(n) = S(f(n − 2)) + S(f(n − 1)), where S is the Smarandache function and f(1), f(2) are any given different non-null positive integers, seems to lead every time to a set of seven values (i.e. 11, 17, 28, 24, 11, 15, 16) which is then repeating infinitely. In this paper I show few other interesting patterns based on recurrence and Smarandache function and I define the Smarandache-Coman constants.

Conjecture 1:

The recurrent formula f(n) = S(f(n − 3)) + S(f(n − 2)) + S(f(n − 1)), where S is the Smarandache function, leads every time to the set of twelve consecutive values {31, 49, 52, 58, 56, 49, 50, 31, 55, 52, 55, 35}, set which is then repeating infinitely, for any given different non-null positive integers f(1), f(2), f(3).

Verifying the conjecture for [f(1), f(2), f(3)] = [1, 2, 3]:

: f(4) = S(1) + S(2) + S(3) = 6;
: f(5) = S(2) + S(3) + S(6) = 8;
: f(6) = S(3) + S(6) + S(8) = 10;
: f(7) = S(6) + S(8) + S(10) = 12;
: f(8) = S(8) + S(10) + S(12) = 13;
: f(9) = S(10) + S(12) + S(13) = 22;
: f(10) = S(12) + S(13) + S(22) = 28;
: f(11) = S(13) + S(22) + S(28) = **31**;
: f(12) = S(22) + S(28) + S(31) = **49**;
: f(13) = S(28) + S(31) + S(49) = **52**;
: f(14) = S(31) + S(49) + S(52) = 58;
: f(15) = S(49) + S(52) + S(58) = 56;
: f(16) = S(52) + S(58) + S(56) = 49;
: f(17) = S(58) + S(56) + S(49) = 50;
: f(18) = S(56) + S(49) + S(50) = 31;
: f(19) = S(49) + S(50) + S(31) = 55;
: f(20) = S(50) + S(31) + S(55) = 52;
: f(21) = S(31) + S(55) + S(52) = 55;

: f(22) = S(55) + S(52) + S(55) = 35;
: f(23) = S(52) + S(55) + S(35) = **31**;
: f(24) = S(55) + S(35) + S(31) = **49**;
: f(25) = S(35) + S(31) + S(49) = **52**;
(...)

Note:

It can be seen that f(26) = f(14) and the sequence becomes cyclic.

Open problems:

: Is there any exception to this apparent rule?
: Is there a finite or infinite set of exceptions?
: Is there (in case that conjecture is true) a superior limit for n such that eventually f(n) = 31, f(n + 1) = 49 and f(n + 2) = 52?

Conjecture 2:

For any k integer, k ≥ 2, the function f(n) = S(f(n − k)) + S(f(n − k + 1)) +...+ S(f(n − 2)) + S(f(n − 1)) leads to a number of k consecutive values of f(n) from which the sequence of values of f(n) is repeating infinitely, for any given different non-null positive integers f(1), f(2),..., f(k), where f(1) < f(2) < ...< f(k); we name these values the *Smarandache-Coman constants*:

: for k = 2, the set of *Smarandache-Coman constants* obviously contains two elements, i.e. 11 and 17;
: for k = 3, the set of *Smarandache-Coman constants* obviously contains three elements, i.e. 31, 49 and 52.

Open problem:

: Which is the *Smarandache-Coman set of constants* for k = 4? But for k = 5, k = 6 etc.?

Conjecture 3:

The recurrent formula f(n) = abs{S(f(n − 1)) - S(f(n − 2)) + S(f(n − 3)) - S(f(n − 4))}, where S is the Smarandache function, leads every time to f(m) = 0 for a certain m, for any given different non-null positive integers f(1), f(2), f(3), f(4), where f(1) < f(2) < f(3) < f(4).

Verifying the conjecture for [f(1), f(2), f(3), f(4)] = [1, 2, 3, 4]:

: f(5) = abs{S(4) − S(3) + S(2) - S(1)} = 2;
: f(6) = abs{S(2) - S(4) + S(3) - S(2)} = 1;
: f(7) = abs{S(1) - S(2) + S(4) - S(3)} = 0.

Verifying the conjecture for [f(1), f(2), f(3), f(4)] = [7, 11, 125, 1729]:

: f(5) = abs{S(1729) − S(125) + S(11) - S(7)} = 8;
: f(6) = abs{S(8) - S(1729) + S(125) - S(11)} = 11;
: f(7) = abs{S(11) - S(8) + S(1729) - S(125)} = 7;
: f(8) = abs{S(7) − S(11) + S(8) - S(1729)} = 19;
: f(9) = abs{S(19) - S(7) + S(11) - S(8)} = 19;
: f(10) = abs{S(19) - S(19) + S(7) - S(11)} = 4;
: f(11) = abs{S(4) − S(19) + S(19) - S(7)} = 3;
: f(12) = abs{S(3) - S(4) + S(19) - S(19)} = 1;
: f(13) = abs{S(1) - S(3) + S(4) - S(19)} = 17;
: f(14) = abs{S(17) − S(1) + S(3) - S(4)} = 15;
: f(15) = abs{S(15) - S(17) + S(1) - S(3)} = 14;
: f(16) = abs{S(14) - S(15) + S(17) - S(1)} = 18;
: f(17) = abs{S(18) − S(14) + S(15) - S(17)} = 13;
: f(18) = abs{S(13) - S(18) + S(14) - S(15)} = 9;
: f(19) = abs{S(9) - S(13) + S(18) - S(14)} = 8;
: f(20) = abs{S(8) − S(9) + S(13) - S(18)} = 5;
: f(21) = abs{S(5) - S(8) + S(9) - S(13)} = 6;
: f(22) = abs{S(6) - S(5) + S(8) - S(9)} = 4;
: f(23) = abs{S(4) − S(6) + S(5) - S(8)} = 2;
: f(24) = abs{S(2) - S(4) + S(6) - S(5)} = 4;
: f(25) = abs{S(4) - S(2) + S(4) - S(6)} = 3;
: f(26) = abs{S(3) − S(4) + S(2) - S(4)} = 3;
: f(27) = abs{S(3) - S(3) + S(4) - S(2)} = 2;
: f(28) = abs{S(2) - S(3) + S(3) - S(4)} = 2;
: f(29) = abs{S(2) − S(2) + S(3) - S(3)} = 0.

Open problems:

: Is there any exception to this apparent rule?
: Is there a finite or infinite set of exceptions?
: Is there (in case that conjecture is true) a superior limit for m such that eventually f(m) = 0?

Reference:

Coman, Marius, *An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function*, Vixra.