Two conjectures on primes and a conjecture on Fermat pseudoprimes to base two

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Abstract. I treated the 2-Poulet numbers in many papers already but they continue to be a source of inspiration for me; in this paper I make a conjecture on primes inspired by the relation between the two prime factors of a 2-Poulet number and I also make a conjecture on Fermat pseudoprimes to base two.

Conjecture 1 (on primes):

For any prime p, p ≥ 7, there exist an infinity of primes q, q > p, such that the number r = (q – 1)/(p – 1) is a natural number. In other words, for any such prime p there exist an infinity of natural numbers r such that q = r*p – p + 1 is prime.

Conjecture 2 (on primes):

For any prime p, p ≥ 7, there exist an infinity of primes q, q > p, such that the number r = (q – 1)/(p – 1) is a rational but not natural number. In other words, for any such prime p there exist an infinity of rational but not natural numbers r such that q = r*p – p + 1 is prime.

Conjecture 3 (on 2-Poulet numbers):

For any 2-Poulet number P = d_1*d_2, where $d_2 > d_1$, the following statement is true: the number r = (d_2 – 1)/(d_1 – 1) is a rational number.

Verifying the conjecture 3:

(For the first seventy-five 2-Poulet numbers)

Note:

In the column I are listed the first seventy-five 2-Poulet numbers, in the column II are listed the cases when r = (d_2 – 1)/(d_1 – 1) is a natural number (put it in other way, the cases when d_2 = r*d_1 – r + 1) and in the column III are listed the cases when r = (d_2 – 1)/(d_1 – 1) is a rational but not natural number.

I.	II.	III.
1 341 = 11*31	(d2 = 3*d1 – 2)	
2 1387 = 19*73	(d2 = 4*d1 – 3)	
3 2047 = 23*89	(d2 = 4*d1 – 3)	

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4 2701 = 37*73             (d2 = 2*d1 - 1)
5 3277 = 29*113            (d2 = 4*d1 - 3)
6 4033 = 37*109            (d2 = 2*d1 - 1)
7 4369 = 17*257            (d2 = 16*d1 - 15)
8 4681 = 31*151            (d2 = 5*d1 - 4)
9 5461 = 43*127            (d2 = 3*d1 - 2)
10 7957 = 73*109                                 (d2 - 1)/(d1 - 1) = 3/2
11 8321 = 53*157           (d2 = 3*d1 - 2)
12 10261 = 31*331          (d2 = 11*d1 - 10)
13 13747 = 59*233          (d2 = 4*d1 - 3)
14 14491 = 43*337          (d2 = 8*d1 - 7)
15 15709 = 23*683          (d2 = 31*d1 - 30)
16 18721 = 97*193          (d2 = 2*d1 - 1)
17 19951 = 71*281          (d2 = 4*d1 - 3)
18 23377 = 97*241                                (d2 - 1)/(d1 - 1) = 5/2
19 31417 = 89*353          (d2 = 4*d1 - 3)
20 31609 = 73*433          (d2 = 6*d1 - 5)
21 31621 = 103*307         (d2 = 3*d1 - 2)
22 35333 = 89*397                                (d2 - 1)/(d1 - 1) = 9/2
23 42799 = 127*337                               (d2 - 1)/(d1 - 1) = 8/3
24 49141 = 157*313         (d2 = 2*d1 - 1)
25 49981 = 151*331         (d2 = 2*d1 - 1)
26 60701 = 101*601         (d2 = 6*d1 - 5)
27 60787 = 89*683                                (d2 - 1)/(d1 - 1) = 31/4
28 65077 = 59*1103         (d2 = 19*d1 - 18)
29 65281 = 97*673          (d2 = 7*d1 - 6)
30 80581 = 61*1321         (d2 = 22*d1 - 21)
31 83333 = 167*499         (d2 = 3*d1 - 2)
32 85489 = 53*1613         (d2 = 31*d1 - 30)
33 88357 = 149*593         (d2 = 4*d1 - 3)
34 90751 = 151*601         (d2 = 4*d1 - 3)
35 104653 = 229*457        (d2 = 2*d1 - 1)
36 123251 = 59*2089        (d2 = 36*d1 - 35)
37 129889 = 193*673                              (d2 - 1)/(d1 - 1) = 7/2
38 130561 = 137*953        (d2 = 7*d1 - 6)
39 150851 = 251*601                              (d2 - 1)/(d1 - 1) = 12/5
40 162193 = 241*673                              (d2 - 1)/(d1 - 1) = 14/5
41 164737 = 257*641                              (d2 - 1)/(d1 - 1) = 5/2
42 181901 = 101*1801       (d2 = 18*d1 - 17)
43 188057 = 89*2113        (d2 = 24*d1 - 23)
44 194221 = 167*1163       (d2 = 7*d1 - 6)
45 196093 = 157*1249       (d2 = 8*d1 - 7)
46 215749 = 79*2731        (d2 = 35*d1 - 34)
47 219781 = 271*811        (d2 = 3*d1 - 2)
48 220729 = 103*2143       (d2 = 21*d1 - 20)
49 226801 = 337*673        (d2 = 2*d1 - 1)
50 233017 = 43*5419        (d2 = 129*d1 - 128)
51 241001 = 401*601                              (d2 - 1)/(d1 - 1) = 3/2
52 249841 = 433*577                              (d2 - 1)/(d1 - 1) = 4/3
53 253241 = 157*1613                             (d2 - 1)/(d1 - 1) = 31/3
54 256999 = 233*1103                             (d2 - 1)/(d1 - 1) = 19/4
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55	264773 = 149*1777	(d2 = 12*d1 - 11)	
56	271951 = 151*1801	(d2 = 12*d1 - 11)	
57	275887 = 263*1049	(d2 = 4*d1 - 3)	
58	280601 = 277*1013		(d2 - 1)/(d1 - 1) = 11/3
59	282133 = 307*919	(d2 = 3*d1 - 2)	
60	294271 = 103*2857	(d2 = 28*d1 - 27)	
61	318361 = 241*1321		(d2 - 1)/(d1 - 1) = 11/2
62	357761 = 131*2731	(d2 = 21*d1 - 20)	
63	390937 = 313*1249	(d2 = 4*d1 - 3)	
64	396271 = 223*1777	(d2 = 8*d1 - 7)	
65	422659 = 163*2593	(d2 = 16*d1 - 15)	
66	435671 = 191*2281	(d2 = 12*d1 - 11)	
67	443719 = 167*2657	(d2 = 16*d1 - 15)	
68	452051 = 251*1801		(d2 - 1)/(d1 - 1) = 36/5
69	458989 = 277*1657	(d2 = 6*d1 - 5)	
70	481573 = 337*1429		(d2 - 1)/(d1 - 1) = 17/4
71	486737 = 233*2089	(d2 = 9*d1 - 8)	
72	489997 = 157*3121	(d2 = 20*d1 - 19)	
73	513629 = 293*1753	(d2 = 6*d1 - 5)	
74	514447 = 359*1433	(d2 = 4*d1 - 3)	
75	556169 = 457*1217		(d2 - 1)/(d1 - 1) = 8/3

Comment:

It can be seen that are already outlined few subsets of 2-Poulet numbers, such the following ones:

: 2-Poulet numbers $P = d_1*d_2$ for which $r = (d_2 - 1)/(d_1 - 1)$ is of the form r = p^m/2^n, where p odd prime and m, n positive integers; such numbers are: 7957, 23377, 35333, 60787, 129889, 164737, 241001, 256999, 318361, 481573 (...);

: 2-Poulet numbers $P = d_1*d_2$ for which $r = (d_2 - 1)/(d_1 - 1)$ is of the form r = n/3, where n positive integer; such numbers are: 42799, 249841, 253241, 280601, 556169 (...);

: 2-Poulet numbers $P = d_1*d_2$ for which $r = (d_2 - 1)/(d_1 - 1)$ is of the form r = n/5, where n positive integer; such numbers are: 150851, 162193, 452051 (...).