Electrodynamics and the Lagrange function

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By the Lagrange function or Lagrangian in the mechanics is understood the difference between the kinetic and potential energy of the system ofin question

$$L = W_k(t) - W_p(t).$$

If we integrate Lagrangian with respect to the time, then we will obtain the first Gamilton function, called action. Since in the general case kinetic energy depends on speeds, and potential - from the coordinates, action can be recorded as

$$S = \int_{t_1}^{t_2} L(x_i, v_i)dt$$

With the condition of the conservatism of this system Lagrange formalism assumes least-action principle, when system during its motion selects the way, with which the action is minimum.

In the electrodynamics Lagrangian of the charged particle, which is moved with the relativistic speed, is written as follows [1]:

$$L = -mc^2\sqrt{1-\frac{v^2}{c^2}} - e\left(\varphi + \mu_0(\vec{v}\vec{A}_H)\right), \qquad (1)$$

where e, m and $\vec{v}$ - charge mass and the velocity of particle, c - the speed of light, μ_0 - magnetic permeability of vacuum, φ the scalar potential of electric field, $\vec{A}_H$ - the vector potential of magnetic field, in which it moves with particle.

In the work [2] are located misunderstanding, on page 279 read: “Therefore even in the relativistic approximation Lagrange's function in the electromagnetic field cannot be represented in the form differences in the kinetic and potential energy” (end of the quotation).

In relationship (1), the author confuses the member, who contains the scalar product of the charge rate and vector potential, and he does not know, to what form of energy it to carry.

Among other things, this uncertainty is not over, and Landau works [3]. The introduction of the Lagrange function and moving charge in this work on paragraphs 16 and 17. With the introduction of these concepts in paragraph 16 is done the following observation: “The following below assertions it is necessary to examine to a considerable degree as the result of experimental data. The form of action for the particle in the electromagnetic field cannot be established on the basis only of general considerations, such as the requirement of relativistic invariance. (latter it would allow in action also the member of form integral of Ads, where A scalar function)” (end of the quotation).

But with the further consideration of this question of any experimental data the author does not give and it is not completely understandable, on what bases the Lagrange function introduces in the form (1). It is further - it is still worse. Without understanding the physical essence of Lagrangian the author immediately includes the potential part (the scalar product of speed and vector potential) in generalized momentum, and then for finding the force is differentiated on the coordinate of Lagrangian, calculating gradient from this value (see relationship after equality (18.1) [3]). But, finding gradient from the work indicated, the author thus recognizes his potential status.

Strictly speaking, the record of Lagrangian in the form (1) does not satisfy the condition of the conservatism of system. This is connected with the fact that the vector potential, entering this relationship, it is connected with the motion of the strange charges, with which interacts the moving charge. A change in the charge rate, for which is located Lagrangian, will involve a change in the speed of these charges, and energy of the moving charge will be spent to this. In order to ensure the conservatism of system, it is necessary to know interaction energy of the moving charge with all strange charges, including with those, on which depends vector potential. This can be made a way of using the scalar-vector potential.

The scalar potential $\varphi(r)$ at the point of the presence of charge is determined by all surrounding charges g_j and is determined by the relationship:

$$\varphi(r)=\sum_j \frac{1}{4\pi\varepsilon}\frac{g_j}{r_j}$$

The potential creates each moving charge at the observation point

$$\varphi'(r,v_\perp)=\varphi(r)ch\frac{v_\perp}{c},$$

If some quantity of moving and fixed charges surrounds this point of space, then for finding the scalar potential in the given one to point it is necessary to produce the summing up of their potentials:

$$\varphi'(r)=\sum_j \varphi(r_j)ch\frac{v_{j\perp}}{c}=\sum_j \frac{1}{4\pi\varepsilon}\frac{g_j}{r_j}ch\frac{v_{j\perp}}{c}$$

Taking into account this circumstance Lagrangian of the charge e, which is found in the environment of the fixed and moving strange charges can be written down as follows:

$$L=-e\sum_j \frac{1}{4\pi\varepsilon}\frac{g_j}{r_j}ch\frac{v_{j\perp}}{c}\ , \qquad (2)$$

If the charge is moving relative to the selected ISM speed v then its Lagrangian is determined by the ratio (2), except that as speeds are $v_{j\perp}$ relative velocities of charges in relation to the charge and adds a member that defines the kinetic energy of the charge. Lagrangian in this case takes the form:

$$L = -mc^2\sqrt{1-\frac{v^2}{c^2}} - e\sum_j \frac{1}{4\pi\varepsilon}\frac{g_j}{r_j}ch\frac{v_{j\perp}}{c}$$

[1]. Фейнман Р., Лейтон Р., Сэндс М. Фейнмановские лекции по физике. М: Мир, 1977.

[2]. Левич В. Г. Курс теоретической физики. М: Физматгиз, 1962. – 696 с.

[3]. Ландау Л. Д., Лифшиц Е. М. Теория поля, том 2. М: Наука, 1982.- 620 с.