Leptonic SU_5 and O_{10} Unification Incorporating Family SU_3

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Abstract

We study the representations and basic multiplets of the O_{10} algebra in terms of the SU_5 tensorial elements, and construct the coupling of the 45 gauge bosons to the 16 Weyl fermions. After exhibiting the coupling terms corresponding to a single generation of quarks and leptons, pertaining to the usual grand unified theory with electroweak $SU_2 \times U_1$ and color SU_3 components, we propose a different approach to the underlying grand symmetry as corresponding to a variety of leptonic particles (electron-like and neutrino-like), and where the decomposition of SU_5 proceeds via a family SU_3 symmetry. We discuss the implications of such an SU_3 family symmetry for the structure of the vector boson spectrum in high-energy collider phenomenology. On the other hand, our scheme promotes the idea that the hadronic constituents, rather than being fractionally charged confined quarks, may turn out to be nothing other than leptonic varieties with integral electric charges. The existence of hadrons as extended objects may find explanation in solitonic solutions of the underlying nonlinear gauge theory. We propose the further incorporation of the theory in a 14-dimensional gravidynamic framework.

1 Introduction

The current view about fundamental particles and their interactions is based on the recognition of three replicated generations of leptons and quarks, with electromagnetic and weak (electroweak) interactions for both, the leptons and the quarks, and strong interactions only for the quarks. The leptons are the three charged particles, namely, the electron, the muon, and the tau, each having an associated neutrino, and their antiparticles. The quarks are proposed[1], [2], [3] as the constituents of a host of strongly interacting hadrons (the fermionic baryons with the proton and the neutron as lightest members, and the mesons, with the pions as lightest members), and comprise the up and down quarks (being constituents of the proton and the neutron), the charm and strange quarks, and the top and bottom quarks. The electroweak interactions of leptons and quarks may be described[4], [5], [6], [7] by an SU_2 gauge symmetry involving Weyl doublets (for instance, the neutrino and the electron, or the upquark and the downquark, etc. and not their antiparticles) and a U_1 gauge symmetry involving both, the particles and the antiparticles. The strong interactions of quarks may be described by an SU_3

gauge symmetry[8], [9], [10], [11] corresponding to the 3-fold color degree of freedom that was proposed for the quarks in order to explain their composite baryonic spectrum correctly. Each generation of leptons and quarks, and their antiparticles, consists of 15 Weyl fermions (16 if the neutrino has mass, which case requires the existence of the antineutrino). The consolidation of the $SU_2 \times U_1$ electroweak symmetry and the SU_3 color symmetry of a single generation of quarks and leptons can take shape in the framework of an SU_5 gauge symmetry[12], or further in an O_{10} extension[13], [14]. The extension of such socalled grand unification theory to $SU_{N>5}$ counterparts, unifying several lepton-quark generations[15], [16], [17], and possibly to their orthogonal extensions, and even to higher-dimensional gravidynamic schemes[18], have also been contemplated.

Part of the work to be presented in this article is to study the O_{10} algebra in the notation that pertains to its maximal SU_5 subalgebra. This includes the study of the O_{10} fundamental multiplet representations that can describe the fermionic particles and the gauge bosons that mediate their interactions. A bonus of this algebraic work is the writing out of the various boson-fermion couplings that are described by the O_{10} grand unified theory for leptons and quarks.

On the other hand, whereas many years of research concerning the phenomenology of the hadronic decays, and interactions, have strongly supported the constituent quark picture, this picture remains quite enigmatic and full of mystery. The dogmatic view that the fractionally charged colored quarks ($\frac{2}{3}$ for upquarks and $-\frac{1}{3}$ for downquarks, in units of the proton charge), that have never been observed directly, would never need to be seen as free particles is a concept that is *hard to comprehend in realistic physical theory*. As a matter of fact, the spectrum of hadrons can very well be described by utilizing leptonic varieties[19], [20] of particles with integral electric charges (electron-like and neutrino-like). The only problem regarding the use of leptonic particles to describe hadronic structure is the fact that familiar leptons do not show any sign of strong interactions. However, this does not imply that other varieties of leptons could not participate in hadronic composition. Besides, there is the possibility that *solitonic or magnetic-like* interactions of the underlying gauge theoretical structure of a leptonic theory may provide a framework to derive a hadronic-like spectrum and the associated strong interactive features. The main part of the work to be presented in this article is to describe an O_{10} *unification model for leptonic particles*. We shall examine the resulting fermionic and bosonic spectrum and the associated gauge couplings. We shall discuss the implications of such a model for high-energy collider phenomenology, and also the possible further consolidation in a gravidynamic 14-dimensional framework.

In the following section, we shall present the O_{10} algebra in terms of an SU_5 covariant formalism. This will be followed by sections treating the adjoint multiplet to which the gauge bosons would belong, and the fundamental spinorial multiplet to which the Weyl fermionic particles would belong. In a succeeding section, the SU_5 invariant terms that comprise the coupling of the bosonic to the fermionic particles are written out. Two subsequent sections would deal with splitting the SU_5 invariant terms into terms that either exhibit a *color* SU_3 symmetry, as in the grand unified lepton-quark theory, or into terms that exhibit a *family* SU_3 symmetry, as in our proposed lepton unification scheme. Other issues will be discussed in the final section.

2 The O_{10} Algebra in Terms of SU_5

The O_{10} Lie algebra has 45 generators. In terms of its SU_5 maximal subalgebra, the 45 generators may decommpose into such elements as $\{J, J_a{}^b, Q_{ab}, Q^{ab}\}$. Here J is a U_1 generator, $J_a{}^b$ are the 24 generators of SU_5, with the tracelessness condition $J_a{}^a = 0$. The indices $(a, b, c, \cdots)$ correspond to the fundamental 5-plet of SU_5, and a summation over repeated indices is always implied. The conjugate generators Q_{ab} and Q^{ab} are antisymmetric SU_5 tensors, each having 10 components.

Whereas the U_1 generator J commutes with the SU_5 generators $J_a{}^b$, it would have the following commutators with Q_{ab} and Q^{ab},

$$\begin{cases} [J, Q_{ab}] = \frac{2}{\sqrt{5}} Q_{ab} \\ \left[J, Q^{ab}\right] = -\frac{2}{\sqrt{5}} Q^{ab} \end{cases} \tag{1}$$

The commutators of $J_a{}^b$ with the Q's are

$$\begin{cases} \left[J_a{}^b, Q_{cd}\right] = \left(\delta_c{}^b Q_{ad} - \delta_d{}^b Q_{ac} - \frac{2}{5}\delta_a{}^b Q_{cd}\right) \\ \left[J_a{}^b, Q^{cd}\right] = -\left(\delta_a{}^c Q^{bd} - \delta_a{}^d Q^{bc} - \frac{2}{5}\delta_a{}^b Q^{cd}\right) \end{cases} \tag{2}$$

The commutators of the Q's among themselves are

$$\begin{cases} [Q_{ab}, Q_{cd}] = 0 \\ \left[Q^{ab}, Q^{cd}\right] = 0 \\ \left[Q_{ab}, Q^{cd}\right] = \begin{pmatrix} \frac{2}{\sqrt{5}}\left(\delta_a{}^c \delta_b{}^d - \delta_a{}^d \delta_b{}^c\right) J + \\ \left(\delta_a{}^c J_b{}^d - \delta_a{}^d J_b{}^c + \delta_b{}^d J_a{}^c - \delta_b{}^c J_a{}^d\right) \end{pmatrix} \end{cases} \tag{3}$$

We can verify that the Jacobi identities involving any three of the generators J, $J_a{}^b$, Q_{ab}, or Q^{ab}, are all satisfied, and that any of the latter generators would commute the the following quadratic (Casimir) operator:

$$J^2 + J_a{}^b J_b{}^a + \frac{1}{2} Q_{ab} Q^{ab} + \frac{1}{2} Q^{ab} Q_{ab} \tag{4}$$

3 The Adjoint Multiplet of O_{10}

An adjoint multiplet of O_{10} can be introduced via a module of the form

$$\mathcal{A} = AJ + A_a{}^b J_b{}^a + \frac{1}{2} A_{ab} Q^{ab} + \frac{1}{2} A^{ab} Q_{ab} \tag{5}$$

Like the corresponding generators, the components ${A_a}^b$ satisfy the condition ${A_a}^a = 0$, and the conjugate components A_{ab} are antisymmetric in their SU_5 indices. Introducing another adjoint module $\mathcal{B}$ in a like manner, we can compute the commutator $[\mathcal{A}, \mathcal{B}]$. The result would be a new adjoint module $\mathcal{F}$ whose components are composed as follows:

$$F = \frac{1}{\sqrt{5}} \left(A^{ab} B_{ab} - A_{ab} B^{ab} \right) \tag{6}$$

$$F_a{}^b = \left\{ \begin{array}{l} -\left(A_a{}^c B_c{}^b - A_c{}^b B_a{}^c \right) \\ \\ -\left(A_{ac} B^{bc} - A^{bc} B_{ac} - \frac{1}{5}\delta_a{}^b A_{cd} B^{cd} + \frac{1}{5}\delta_a{}^b A^{cd} B_{cd} \right) \end{array} \right\} \tag{7}$$

$$F_{ab} = -\frac{2}{\sqrt{5}} A B_{ab} + A_a{}^c B_{bc} + A_{ac} B_b{}^c - (a \leftrightarrow b) \tag{8}$$

$$F^{ab} = \frac{2}{\sqrt{5}} A B^{ab} - A_c{}^a B^{bc} - A^{ac} B_c{}^b - (a \leftrightarrow b) \tag{9}$$

The above equations can be used to deduce the infinitesimal O_{10} transformations of an adjoint multiplet $\mathcal{B}$ simply by substituting Ω for A. Hence with an adjoint parameter multiplet with components Ω, $\Omega_a{}^b$, Ω_{ab}, and Ω^{ab}, we have the following transformations that pertain to the module $\mathcal{B}$:

$$\delta B = \frac{1}{\sqrt{5}} \left(\Omega^{ab} B_{ab} - \Omega_{ab} B^{ab} \right) \tag{10}$$

$$\delta B_a{}^b = \left\{ \begin{array}{l} -\left(\Omega_a{}^c B_c{}^b - \Omega_c{}^b B_a{}^c \right) \\ \\ -\left(\Omega_{ac} B^{bc} - \Omega^{bc} B_{ac} - \frac{1}{5}\delta_a{}^b \Omega_{cd} B^{cd} + \frac{1}{5}\delta_a{}^b \Omega^{cd} B_{cd} \right) \end{array} \right\} \tag{11}$$

$$\delta B_{ab} = -\frac{2}{\sqrt{5}} \Omega B_{ab} + \Omega_a{}^c B_{bc} + \Omega_{ac} B_b{}^c - (a \leftrightarrow b) \tag{12}$$

$$\delta B^{ab} = \frac{2}{\sqrt{5}} \Omega B^{ab} - \Omega_c{}^a B^{bc} - \Omega^{ac} B_c{}^b - (a \leftrightarrow b) \tag{13}$$

For two adjoint multiplets $\mathcal{A}$ and $\mathcal{B}$ whose components transform according to the above equations, we can verify the invariance of the following bilinear form:

$$\mathcal{A} \cdot \mathcal{B} = AB + A_a^b B_b^a + \frac{1}{2} A_{ab} B^{ab} + \frac{1}{2} A^{ab} B_{ab} \tag{14}$$

4 The Fundamental Spinorial O_{10} Multiplet

The fundamental spinorial multiplet of O_{10} has 16 components. This comes about since a spinor in 10 dimensions has $2^5 = 32$ components. These can split covariantly into two conjugate chiral parts with 16 components each. With respect to SU_5, we can have the decomposition $\mathbf{16} = \mathbf{1} + \mathbf{5} + \mathbf{10}^*$. Accordingly, in order to construct an operator representation of the fundamental 16-plet of O_{10}, we shall introduce the SU_5

covariant operators $\{K, K_a, K^{ab}\}$, where K_a is a vector and K^{ab} is an antisymmetric tensor. For the conjugate representation **16***, we would have the set $\{K^*, K^a, K_{ab}\}$. In the followings, we shall write down the commutators of the foregoing operators with the generators J, $J_a{}^b$, Q_{ab}, and Q^{ab} of the O_{10} algebra, all being manifestly covariant with respect to the SU_5 subalgebra. We shall start with the **16** representation, to be followed by the **16*** counterpart.

For the commutators with the U_1 generator J, we write

$$\begin{cases} [J, K] = \frac{\sqrt{5}}{2} K \\ [J, K_a] = -\frac{3}{2\sqrt{5}} K_a \\ \left[J, K^{ab}\right] = \frac{1}{2\sqrt{5}} K^{ab} \end{cases} \tag{15}$$

Whereas the generators $J_a{}^b$ of SU_5 commute with K, we have

$$\left[J_a{}^b, K_c\right] = \left(\delta_c{}^b K_a - \frac{1}{5}\delta_a{}^b K_c\right) \tag{16}$$

$$\left[J_a{}^b, K^{cd}\right] = -\left(\delta_a{}^c K^{bd} - \delta_a{}^d K^{bc} - \frac{2}{5}\delta_a{}^b K^{cd}\right) \tag{17}$$

The generators Q_{ab} commute with K, and we have

$$[Q_{ab}, K_c] = -\frac{1}{2}\epsilon_{abcde} K^{de} \tag{18}$$

$$\left[Q_{ab}, K^{cd}\right] = \left(\delta_a{}^c \delta_b{}^d - \delta_a{}^d \delta_b{}^c\right) K \tag{19}$$

The generators Q^{ab} would commute with K_c, and we have

$$\left[Q^{ab}, K\right] = K^{ab} \tag{20}$$

$$\left[Q^{ab}, K^{cd}\right] = -\epsilon^{abcde} K_e \tag{21}$$

We can verify that the Jacobi identities that involve any two of the O_{10} generators J, $J_a{}^b$, Q_{ab}, or Q^{ab} with any operator K, K_a, or K^{ab}, are all satisfied.

Moving to the conjugate set $\{K^*, K^a, K_{ab}\}$, for the commutators of the U_1 generator J, we have

$$\begin{cases} [J, K^*] = -\frac{\sqrt{5}}{2} K^* \\ [J, K^a] = \frac{3}{2\sqrt{5}} K^a \\ [J, K_{ab}] = -\frac{1}{2\sqrt{5}} K_{ab} \end{cases} \tag{22}$$

Whereas the generators $J_a{}^b$ of SU_5 commute with K^*, we have

$$\left[J_a{}^b, K^c\right] = -\left(\delta_a{}^c K^b - \frac{1}{5}\delta_a{}^b K^c\right) \tag{23}$$

$$\left[J_a{}^b, K_{cd}\right] = \left(\delta_c{}^b K_{ad} - \delta_d{}^b K_{ac} - \frac{2}{5}\delta_a{}^b K_{cd}\right) \tag{24}$$

The generators Q_{ab} would commute with K^c, and we have

$$[Q_{ab}, K^*] = -K_{ab} \tag{25}$$

$$[Q_{ab}, K_{cd}] = \epsilon_{abcde} K^e \tag{26}$$

The generators Q^{ab} would commute with K^*, and we have

$$\left[Q^{ab}, K^c\right] = \frac{1}{2}\epsilon^{abcde} K_{de} \tag{27}$$

$$\left[Q^{ab}, K_{cd}\right] = -\left(\delta_c{}^a \delta_d{}^b - \delta_c{}^b \delta_d{}^a\right) K^* \tag{28}$$

Again, we can verify that the Jacobi identities that involve any two of the O_{10} generators J, $J_a{}^b$, Q_{ab}, or Q^{ab} with any operator K^*, K^a, or K_{ab}, are all satisfied.

Having written the commutators of the O_{10} generators with the set of operators K, K_a, and K^{ab} and the conjugate set K^*, K^a, and K_{ab}, we can also verify than any of the O_{10} generators J, $J_a{}^b$, Q_{ab}, or Q^{ab} would commute with a quadratic operator of the form

$$K^* K + K^a K_a + \frac{1}{2} K_{ab} K^{ab} \tag{29}$$

We are now ready to construct a modular representation of the fundamental spinorial O_{10} multiplet and its conjugate. This is done by introducing two conjugate modules of the form:

$$\begin{cases} \mathcal{B} = BK^* + B_a K^a + \frac{1}{2} B^{ab} K_{ab} \\ \mathcal{B}^* = B^* K + B^a K_a + \frac{1}{2} B_{ab} K^{ab} \end{cases} \tag{30}$$

With an adjoint module

$$\mathcal{A} = AJ + A_a{}^b J_b{}^a + \frac{1}{2} A_{ab} Q^{ab} + \frac{1}{2} A^{ab} Q_{ab} \tag{31}$$

we can compute the commutators $[\mathcal{A}, \mathcal{B}]$ and $[\mathcal{A}, \mathcal{B}^*]$, and the resulting fundamental modules $\mathcal{F}$ and $\mathcal{F}^*$ would have components that are composites. For the module $\mathcal{F}$, we have

$$\begin{cases} F = -\frac{\sqrt{5}}{2} AB - \frac{1}{2} A_{ab} B^{ab} \\ F_a = \frac{3}{2\sqrt{5}} AB_a - A_a{}^b B_b + \frac{1}{4}\epsilon_{abcde} A^{bc} B^{de} \\ F^{ab} = \begin{pmatrix} -\frac{1}{2\sqrt{5}} AB^{ab} - \left(A_e{}^a B^{be} - A_e{}^b B^{ae}\right) \\ -A^{ab} B + \frac{1}{2}\epsilon^{abcde} A_{cd} B_e \end{pmatrix} \end{cases} \tag{32}$$

For the module $\mathcal{F}^*$, we have

$$\begin{cases} F^* = \frac{\sqrt{5}}{2}AB^* + \frac{1}{2}A^{ab}B_{ab} \\ F^a = -\frac{3}{2\sqrt{5}}AB^a + A_b{}^a B^b - \frac{1}{4}\epsilon^{abcde}A_{bc}B_{de} \\ F_{ab} = \begin{pmatrix} \frac{1}{2\sqrt{5}}AB_{ab} + (A_a{}^c B_{bc} - A_b{}^c B_{ac}) \\ +A_{ab}B^* - \frac{1}{2}\epsilon_{abcde}A^{cd}B^e \end{pmatrix} \end{cases} \tag{33}$$

We can now write the infinitesimal O_{10} transformations that pertain to the components of a fundamental multiplet. This is easily done by introducing an adjoint parameter multiplet with components Ω, $\Omega_a{}^b$, Ω_{ab}, and Ω^{ab}, and replacing A by Ω in the foregoing expressions. For the fundamental module $\mathcal{B}$, we have

$$\begin{cases} \delta B = -\frac{\sqrt{5}}{2}\Omega B - \frac{1}{2}\Omega_{ab}B^{ab} \\ \delta B_a = \frac{3}{2\sqrt{5}}\Omega B_a - \Omega_a{}^b B_b + \frac{1}{4}\epsilon_{abcde}\Omega^{bc}B^{de} \\ \delta B^{ab} = \begin{pmatrix} -\frac{1}{2\sqrt{5}}\Omega B^{ab} - \left(\Omega_e{}^a B^{be} - \Omega_e{}^b B^{ae}\right) \\ -\Omega^{ab}B + \frac{1}{2}\epsilon^{abcde}\Omega_{cd}B_e \end{pmatrix} \end{cases} \tag{34}$$

and for the conjugate module $\mathcal{B}^*$, we have

$$\begin{cases} \delta B^* = \frac{\sqrt{5}}{2}\Omega B^* + \frac{1}{2}\Omega^{ab}B_{ab} \\ \delta B^a = -\frac{3}{2\sqrt{5}}\Omega B^a + \Omega_b{}^a B^b - \frac{1}{4}\epsilon^{abcde}\Omega_{bc}B_{de} \\ \delta B_{ab} = \begin{pmatrix} \frac{1}{2\sqrt{5}}\Omega B_{ab} + (\Omega_a{}^c B_{bc} - \Omega_b{}^c B_{ac}) \\ +\Omega_{ab}B^* - \frac{1}{2}\epsilon_{abcde}\Omega^{cd}B^e \end{pmatrix} \end{cases} \tag{35}$$

With two fundamental modules $\mathcal{A}$ and $\mathcal{B}$, and their conjugate $\mathcal{A}^*$ and $\mathcal{B}^*$, whose components do transform according to the above equations, we can verify the invariance of the following two forms of bilinears:

$$\mathcal{A}^* \cdot \mathcal{B} = A^*B + A^aB_a + \frac{1}{2}A_{ab}B^{ab} \tag{36}$$

$$\mathcal{B}^* \cdot \mathcal{A} = B^*A + B^aA_a + \frac{1}{2}B_{ab}A^{ab} \tag{37}$$

With the above algebraic technology, we can proceed to write the O_{10} invariant coupling of a fundamental fermionic multiplet to the adjoint gauge multiplet, and doing this in an SU_5 covariant manner.

5 The O_{10} Invariant Boson-Fermion Coupling

We introduce a fundamental fermionic 16-plet Ψ with SU_5 components $\{\psi, \psi_a, \psi^{ab}\}$, all being Weyl fermions, and also the Dirac conjugate $\bar{\Psi}$, being a conjugate 16-plet with components $\{\bar{\psi}, \bar{\psi}_a, \bar{\psi}^{ab}\}$. We also introduce the vector gauge bosons via an adjoint module $\mathcal{V}$, and respective components V, ${V_a}^b$, V_{ab}, and V^{ab}, where we shall suppress the vectorial spacetime index. The gauge-invariant fermionic Lagrangian that includes the gauge coupling would take the form

$$\bar{\Psi}(i\gamma\cdot\nabla)\Psi = \bar{\Psi}(i\gamma\cdot\partial)\Psi + \bar{\Psi}\gamma\cdot[\mathcal{V},\Psi] \tag{38}$$

where it should be clear that we must compute the invariant bilinears, as well as the commutators. The kinetic bilinear would take the following SU_5 form:

$$\bar{\Psi}(i\gamma\cdot\partial)\Psi \Rightarrow \bar{\psi}(i\gamma\cdot\partial)\psi + \bar{\psi}^a(i\gamma\cdot\partial)\psi_a + \frac{1}{2}\bar{\psi}_{ab}(i\gamma\cdot\partial)\psi^{ab} \tag{39}$$

Using the algebraic methods of the preceding section, we obtain for the coupling terms

$$\bar{\Psi}\gamma\cdot[\mathcal{V},\Psi] \Rightarrow \left\{\begin{array}{l} -\frac{\sqrt{5}}{2}A\bar{\psi}\gamma\psi + \frac{3}{2\sqrt{5}}A\bar{\psi}^a\gamma\psi_a - \frac{1}{4\sqrt{5}}A\bar{\psi}_{ab}\gamma\psi^{ab} \\ \\ -{A_a}^b\bar{\psi}^a\gamma\psi_b + {A_a}^b\bar{\psi}_{bc}\gamma\psi^{ac} \\ \\ -\frac{1}{2}A_{ab}\bar{\psi}\gamma\psi^{ab} + \frac{1}{4}\epsilon^{abcde}A_{ab}\bar{\psi}_{cd}\gamma\psi_e \\ \\ -\frac{1}{2}A^{ab}\bar{\psi}_{ab}\gamma\psi + \frac{1}{4}\epsilon_{abcde}A^{ab}\bar{\psi}^c\gamma\psi^{de} \end{array}\right\} \tag{40}$$

The foregoing fermionic kinetic terms, as well as the boson-fermion coupling terms, are exhibited in a manifestly covariant SU_5 tensorial notaion. In the following section, we shall decompose these terms with respect to color SU_3 thus displaying the quark-lepton content of the underlying grand unification symmetry. That will be followed by a section where we shall decompose the terms with respect to a family SU_3 regarding the model as purely leptonic.

6 Boson-Fermion Couplings in Grand Unified O_{10}

In a single-generation quark-lepton O_{10} unification scheme the electric charge operator is embedded as a diagonal generator of SU_5 with the eigenvalues

$$\{0, -1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\} \tag{41}$$

Here, the eigenvalue 0 corresponds to a neutrino, the -1 to an electron, and the three values $\frac{1}{3}$ to an anti downquark, hence exhibiting the SU_3 color symmetry. Accordingly, we shall specify the components of the fundamental 16-plet of Weyl fermions Ψ as follows. The SU_5 single ψ component will be denoted by an antineutrino ν^*, and its

Dirac conjugate $\bar{\psi}$ by $\bar{\nu^*}$. The components of the 5-plet ψ_a, and the components of its Dirac conjugate $\bar{\psi}^a$, will be denoted according to

$$\begin{cases} \psi_1 \to \nu & \bar{\psi}^1 \to \bar{\nu} \\ \psi_2 \to e & \bar{\psi}^2 \to \bar{e} \\ \psi_i \to d^*_i & \bar{\psi}^i \to \bar{d^*}^i \end{cases} \tag{42}$$

Notice that we use the indices $(i, j, k, \cdots)$ to correspond to those of color SU_3, and the antiparticles are marked with a star (*). Hence, in the above, we have introduced the left-handed Weyl fermions corresponding to the neutrino ν, the electron e, and the colored anti downquark d^*_i

For the 10 components ψ^{ab}, being antisymmetric in SU_5 indices, we have

$$\begin{cases} \psi^{12} \to e^* & \bar{\psi}_{12} \to \bar{e^*} \\ \psi^{1i} \to d^i & \bar{\psi}_{1i} \to \bar{d}_i \\ \psi^{2i} \to u^i & \bar{\psi}_{2i} \to \bar{u}_i \\ \psi^{ij} \to \epsilon^{ijk} u^*_k & \bar{\psi}_{ij} \to \epsilon_{ijk} \bar{u^*}^k \end{cases} \tag{43}$$

Hence, we have introduced the Weyl components for the positron e^*, the downquark d^i, the upquark u^i, and the anti upquark u^*_i.

Moving to the vector bosons $V_a{}^b$, satisfying the tracelessness condition $V_a{}^a = 0$, we make the following assignments:

$$\begin{cases} V_1{}^1 \to \mathcal{Z} & V_1{}^2 \to \mathcal{W}^* & V_1{}^i \to \mathcal{Y}^i \\ V_2{}^1 \to \mathcal{W} & V_2{}^2 \to -\mathcal{A} - \frac{1}{4}\mathcal{Z} & V_2{}^i \to \mathcal{X}^i \\ V_i{}^1 \to \mathcal{Y}_i & V_i{}^2 \to \mathcal{X}_i & V_i{}^j \to \mathcal{G}_i{}^j + \frac{1}{3}\delta_i{}^j \mathcal{A} - \frac{1}{4}\delta_i{}^j \mathcal{Z} \end{cases} \tag{44}$$

Here, we have introduced the usual vector bosons of grand unified SU_5, where $\mathcal{A}$ would represent the photon, $\mathcal{Z}$ the neutral weak boson, $\mathcal{G}_i{}^j$ the octet of vector gluons that correspond to color SU_3, $(\mathcal{W}, \mathcal{W}^*)$ the charge ± 1 weak bosons, while the $(\mathcal{X}_i, \mathcal{X}^i)$ and the $(\mathcal{Y}_i, \mathcal{Y}^i)$ are the colored leptoquark bosons of charges $\pm\frac{4}{3}$ and $\pm\frac{1}{3}$, respectively.

For the antisymmetric tensors V_{ab}, and their conjugates V^{ab}, we make the assignments:

$$\begin{cases} V_{12} \to \mathcal{E} & V^{12} \to \mathcal{E}^* \\ V_{1i} \to \mathcal{D}_i & V^{1i} \to \mathcal{D}^i \\ V_{2i} \to \mathcal{U}_i & V^{2i} \to \mathcal{U}^i \\ V_{ij} \to \epsilon_{ijk} \mathcal{P}^k & V^{1ij} \to \epsilon^{ijk} \mathcal{P}_k \end{cases} \tag{45}$$

Here we have introduced the charge ±1 vector bosons $(\mathcal{E}, \mathcal{E}^*)$, the colored vector bosons $(\mathcal{D}_i, \mathcal{D}^i)$ of charge $\pm\frac{1}{3}$, the colored vector bosons $(\mathcal{U}_i, \mathcal{U}^i)$ of charge $\pm\frac{2}{3}$ that are similar to $(\mathcal{P}_i, \mathcal{P}^i)$. Notice that some terminology reflects the fact that the corresponding bosons are electron-like, downquark-like, and upquark-like.

We still have an SU_5 singlet vector boson V, corresponding to the U_1 generator within O_{10} but outside SU_5. We shall denote that by $\mathcal{N}$, being neutral.

With the above specifications for fermionic and bosonic components, we can split the SU_5 indices of the Lagrangian terms given in the preceding section. In the following subsections, we shall present the results in terms of the vector boson to which the fermions (quarks and leptons) do couple. It should be remarked that the vector boson fields may still *need to be rescaled* in practice, when their invariant kinetic terms will be written in canonical forms.

6.1 Coupling to the Neutral Boson $\mathcal{N}$

Here we give the couplings of all fundamental fermions to the neutral gauge boson $\mathcal{N}$ that corresponds to the U_1 symmetry outside SU_5.

$$\mathcal{N} \times \frac{1}{2\sqrt{5}} \begin{pmatrix} -3\bar{\nu}\gamma\nu + 5\bar{\nu^*}\gamma\nu^* - 3\bar{e}\gamma e + \bar{e^*}\gamma e^* \\ +\bar{u}_i\gamma u^i + \bar{u^*}^i\gamma u_i^* + \bar{d}_i\gamma d^i - 3\bar{d^*}^i\gamma d_i^* \end{pmatrix} \tag{46}$$

Notice that all particles, namely, the neutrino, the electron, the upquark, and the downquark, and their antiparticles, do participate in the above couplings.

6.2 Coupling to the Photon $\mathcal{A}$

Here we give the couplings of the fundamental fermionic particles to the photon,

$$\mathcal{A} \times \left(-\bar{e}\gamma e + \bar{e^*}\gamma e^* + \frac{2}{3}\bar{u}_i\gamma u^i - \frac{2}{3}\bar{u^*}^i\gamma u_i^* - \frac{1}{3}\bar{d}_i\gamma d^i + \frac{1}{3}\bar{d^*}^i\gamma d_i^* \right) \tag{47}$$

Of course, the neutrino and its antiparticle are absent from the above, only the charged particles would participate in these couplings.

6.3 Coupling to the $\mathcal{Z}$ Boson

Here we give the couplings of the fundamental fermionic particles to the neutral weak boson $\mathcal{Z}$,

$$\mathcal{Z} \times \frac{1}{4} \begin{pmatrix} 4\bar{\nu}\gamma\nu - \bar{e}\gamma e - 3\bar{e^*}\gamma e^* \\ \\ -3\bar{d}_i\gamma d^i - \bar{d^*}\gamma d_i^* + 2\bar{u}_i\gamma u^i + 2\bar{u^*}^i\gamma u_i^* \end{pmatrix} \tag{48}$$

Notice that the antineutrino ν^* does not participate in the above couplings, since it is outside SU_5.

6.4 Coupling to the $\mathcal{W}$ Bosons

Here we give the couplings of the fundamental fermions to the $(\mathcal{W}, \mathcal{W}^*)$ weak bosons which have ± 1 electric charges,

$$\begin{pmatrix} \mathcal{W} \times \left(\bar{e}\gamma\nu - \bar{d}_i \gamma u^i \right) \\ \\ +\mathcal{W}^* \times \left(\bar{\nu}\gamma e - \bar{u}_i \gamma d^i \right) \end{pmatrix} \tag{49}$$

You should be able to tell from the above expressions that $\mathcal{W}^*$ is the boson with charge $+1$, and that $\mathcal{W}$ is the one with charge -1.

6.5 Coupling to the Color Gluons $\mathcal{G}$

Here we give the couplings of the colored fermions (the quarks and the antiquarks) to the vector gluons ${\mathcal{G}_i}^j$ of color SU_3,

$$ {\mathcal{G}_i}^j \times \left(-\bar{u}_j \gamma u^i + \bar{u^*}^i \gamma u^*_j - \bar{d}_j \gamma d^i + \bar{d^*}^i \gamma d^*_j \right) \tag{50}$$

6.6 Coupling to the $\mathcal{X}$ Bosons

Here we give the couplings of the fundamental fermions to the $(\mathcal{X}_i, \mathcal{X}^i)$ vector bosons. The latter are colored particles with $\pm\frac{4}{3}$ as electric charges.

$$\begin{pmatrix} \mathcal{X}_i \times \left(\bar{d^*}^i \gamma e - \bar{e^*} \gamma d^i - \epsilon^{ijk} \bar{u}_j \gamma u^*_k \right) \\ \\ +\mathcal{X}^i \times \left(\bar{e}\gamma d^*_i - \bar{d}_i \gamma e^* + \epsilon_{ijk} \bar{u^*}^j \gamma u^k \right) \end{pmatrix} \tag{51}$$

Notice that the neutrinos do not participate in the above couplings, simply because there are no charge $\pm\frac{4}{3}$ fermions to participate with.

6.7 Coupling to the $\mathcal{Y}$ Bosons

Here we give the couplings of the fundamental fermions to the $(\mathcal{Y}_i, \mathcal{Y}^i)$ vector bosons. The latter are colored particles with $\pm\frac{1}{3}$ as electric charges.

$$\begin{pmatrix} \mathcal{Y}_i \times \left(\bar{d^*}^i \gamma \nu + \bar{e^*} \gamma u^i - \epsilon^{ijk} \bar{d}_i \gamma u^*_k \right) \\ \\ +\mathcal{Y}^i \times \left(\bar{\nu}\gamma d^*_i + \bar{u}_i \gamma e^* + \epsilon_{ijk} \bar{u^*}^j \gamma d^k \right) \end{pmatrix} \tag{52}$$

6.8 Coupling to the $\mathcal{E}$ Bosons

Here we give the couplings of the fundamental fermions to the $(\mathcal{E}, \mathcal{E}^*)$ vector bosons. The latter are particles with ± 1 as electric charges.

$$\begin{pmatrix} \mathcal{E}^* \times \left(\bar{e}^* \gamma \nu^* - \bar{d^*}^i \gamma u_i^* \right) \\ \\ +\mathcal{E} \times \left(\bar{\nu^*} \gamma e^* - \bar{u^*}^i \gamma d_i^* \right) \end{pmatrix} \tag{53}$$

Notice the participation of the antineutrino ν^* in the above couplings, since both the antineutrino and the $\mathcal{E}$ bosons are outside SU_5.

6.9 Coupling to the $\mathcal{D}$ Bosons

Here we give the couplings of the fundamental fermions to the $(\mathcal{D}^i, \mathcal{D}_i)$ vector bosons. The latter are colored particles with $\pm\frac{1}{3}$ as electric charges.

$$\begin{pmatrix} \mathcal{D}^i \times \left(\bar{d}_i \gamma \nu^* + \bar{e} \gamma u_i^* - \epsilon_{ijk} \bar{d^*}^j \gamma u^k \right) \\ \\ +\mathcal{D}_i \times \left(\bar{\nu^*} \gamma d^i + \bar{d^*}^j \gamma e - \epsilon^{ijk} \bar{u}_j \gamma d_k^* \right) \end{pmatrix} \tag{54}$$

Again, notice the participation of the antineutrino ν^* in the above couplings, since both the antineutrino and the $\mathcal{D}$ bosons are outside SU_5.

6.10 Coupling to the $\mathcal{U}$ Bosons

Here we give the couplings of the fundamental fermions to the $(\mathcal{U}^i, \mathcal{U}_i)$ vector bosons. The latter are colored particles with $\pm\frac{2}{3}$ as electric charges.

$$\begin{pmatrix} \mathcal{U}^i \times \left(-\bar{\nu} \gamma u_i^* + \bar{u}_i \gamma \nu^* + \epsilon_{ijk} \bar{d^*}^j \gamma d^k \right) \\ \\ +\mathcal{U}_i \times \left(-\bar{u^*}^i \gamma \nu + \bar{\nu^*} \gamma u^i - \epsilon^{ijk} \bar{d}_j \gamma d_k^* \right) \end{pmatrix} \tag{55}$$

Again, notice the participation of the antineutrino ν^* in the above couplings, since both the antineutrino and the $\mathcal{U}$ bosons are outside SU_5.

6.11 Coupling to the $\mathcal{P}$ Bosons

Here we give the couplings of the fundamental fermions to the $(\mathcal{P}^i, \mathcal{P}_i)$ vector bosons. The latter are colored particles with $\pm\frac{2}{3}$ as electric charges.

$$\begin{pmatrix} \mathcal{P}^i \times \left(-\bar{u}_i \gamma \nu + \bar{\nu^*} \gamma u_i^* + \bar{d}_i \gamma e - \bar{e^*} \gamma d_i^* \right) \\ \\ +\mathcal{P}_i \times \left(-\bar{\nu} \gamma u^i + \bar{u^*}^i \gamma \nu^* + \bar{e} \gamma d^i - \bar{d^*}^i \gamma e^* \right) \end{pmatrix} \tag{56}$$

Again, notice the participation of the antineutrino ν^* in the above couplings, since both the antineutrino and the $\mathcal{P}$ bosons are outside SU_5.

7 Boson-Fermion Couplings in Leptonic O_{10} Incorporating Family SU_3

In our leptonic SU_5 and O_{10} unification schemes that would incorporate an SU_3 family symmetry, we shall imbed the electric charge operator as a diagonal SU_5 generator, with the following eigenvalues

$$\{+1, -1, 0, 0, 0\} \tag{57}$$

Here, the first eigenvalue +1 corresponds to a positron (or a positron-like particle), the −1 to the electron (or an electron-like particle), and the three 0 values to neutrinos (in fact antineutrinos in the 5-plet of what follows). Hence the SU_3 associated with the three neutrinos is just the *family* symmetry to be incorporated. Notice that the above SU_5 scheme is an extension of the leptonic SU_3 model[23] of electroweak interactions, the minimal theory which consolidates the electroweak symmetries of the electron, the positron, and a single neutrino.

According to the above embedding of electric charge, we shall specify the components of the O_{10} fundamental 16-plet of Weyl fermions Ψ. In general, we shall denote neutral (or neutrino-like) fermions by the symbol ν, and electron-like particles (having electric charge −1) by the symbol e. Indices $(r, s, t, \ldots)$ will label particles in the triplet representation of family SU_3. Antiparticles will carry a star (like ν^* for the antineutrino and e^* for the positron).

First, the SU_5 singlet Weyl fermion ψ will be denoted by an antineutrino ν^*, and its Dirac conjugate $\bar{\psi}$ by $\bar{\nu}^*$.

The components of the 5-plet ψ_a, and the components of its Dirac conjugate $\bar{\psi}^a$, will be denoted according to

$$\begin{cases} \psi_1 \to e^* & \bar{\psi}^1 \to \bar{e^*} \\ \psi_2 \to e & \bar{\psi}^2 \to \bar{e} \\ \psi_r \to \nu^*_r & \bar{\psi}^r \to \bar{\nu^*}^r \end{cases} \tag{58}$$

Notice that we have introduced a positron-like particle e^*, and an electron-like particle e, and a family triplet of antineutrinos ν^*_r, $(r = 1, 2, 3)$.

For the 10-component antisymmetric tensor ψ^{ab}, and Dirac conjugate $\bar{\psi}_{ab}$, we make the following assignments,

$$\begin{cases} \psi^{12} \to \nu & \bar{\psi}_{12} \to \bar{\nu} \\ \psi^{1r} \to e^r & \bar{\psi}_{1r} \to \bar{e}_r \\ \psi^{2r} \to (e^*)^r & \bar{\psi}_{2r} \to (\bar{e^*})_r \\ \psi^{rs} \to \epsilon^{rst}\nu_t & \bar{\psi}_{rs} \to \epsilon_{rst}\bar{\nu}^t \end{cases} \tag{59}$$

Notice that we have introduced an SU_3 singlet neutrino ν. This would correspond to the SU_5 singlet antineutrino ν^*, introduced before. We have introduced a family SU_3 triplet of electron-like particles e^r, a corresponding triplet of positron-like particles $(e^*)^r$, and a triplet of neutrinos ν_r. The latter triplet of neutrinos would correspond to the triplet of antineutrinos ν_r^* introduced earlier.

Hence our fundamental fermionic particles consist of singlets and triplets, with respect to family SU_3, of neutrino-like and electron-like particles, and their antiparticles. Would the triplets correspond to *the actually observed three leptons (electron, muon, and tau) and their associated neutrinos*, and would the singlets correspond to some yet unobserved 4th generation of leptons?

Moving to the vector bosons, we begin with the SU_5 traceless tensor ${V_a}^b$, and make the following component assignments:

$$\begin{cases} {V_1}^1 \to \mathcal{A} + \frac{1}{2}\mathcal{Z} & {V_1}^2 \to \mathcal{X}^* & {V_1}^r \to \mathcal{W}^r \\ {V_2}^1 \to \mathcal{X} & {V_2}^2 \to -\mathcal{A} + \frac{1}{2}\mathcal{Z} & {V_2}^r \to \mathcal{Y}^r \\ {V_r}^1 \to \mathcal{W}_r & {V_r}^2 \to \mathcal{Y}_r & {V_r}^s \to {\mathcal{H}_r}^s - \frac{1}{3}{\delta_r}^s\mathcal{Z} \end{cases} \tag{60}$$

Here $\mathcal{A}$ corresponds to the photon, $\mathcal{Z}$ to the massive neutral boson of electroweak theory, and ${\mathcal{H}_r}^s$, being traceless, to an octet of neutral particles that correspond to family (or horizontal) SU_3. Notice that we have introduced, as well, a triplet of particles $(\mathcal{W}^r, \mathcal{W}_r)$. These would carry ± 1 as electric charges, and correspond to the $W^\pm$ vector bosons of electroweak theory. We also have the particles $(\mathcal{X}, \mathcal{X}^*)$ that carry ± 2 as electric charges, and the triplets $(\mathcal{Y}_r, \mathcal{Y}^r)$ that carry ± 1 as electric charges. Particles like the present $\mathcal{X}$'s and $\mathcal{Y}$'s have already been predicted[23] long time ago, in connection with the SU_3 electroweak model.

For the antisymmetric SU_5 tensor V_{ab}, and the conjugate V^{ab}, we make the following component assignments:

$$\begin{cases} V_{12} \to \mathcal{N} & V^{12} \to \mathcal{N}^* \\ V_{1r} \to \mathcal{F}_r & V^{1r} \to \mathcal{F}^r \\ V_{2r} \to \mathcal{E}_r & V^{2r} \to \mathcal{E}^r \\ V_{rs} \to \epsilon_{rst}\mathcal{N}^t & V^{rs} \to \epsilon^{rst}\mathcal{N}_t \end{cases} \tag{61}$$

Here we have introduced the conjugate neutral singlets $(\mathcal{N}, \mathcal{N}^*)$, and the conjugate neutral triplets $(\mathcal{N}_t, \mathcal{N}^t)$. We have also introduced the charge ± 1 conjugate triplets $(\mathcal{E}_r, \mathcal{E}^r)$, and the charge ± 1 conjugate triplets $(\mathcal{F}_r, \mathcal{F}^r)$. These vector bosons correspond to the coset of O_{10} over SU_5.

The remaining neutral vector boson V that is an SU_5 singlet will be denoted by $\mathcal{V}$.

With the above specifications for fermionic and bosonic components, we can split the SU_5 indices of the Lagrangian terms given much earlier, in §5. In the following subsections, we shall present the results in terms of the vector boson to which the fermions (only leptons) do couple. Again, it should be remarked that the vector boson fields may still need to be rescaled in practice, when their invariant kinetic terms will be written in canonical forms.

7.1 Coupling to the $\mathcal{V}$ Boson

Here we give the couplings of the fundamental leptons to the $\mathcal{V}$ vector boson. The latter is a neutral particle corresponding to the U_1 symmetry outside SU_5.

$$\mathcal{V} \times \frac{1}{2\sqrt{5}} \left(\begin{array}{l} \bar{\nu}\gamma\nu + 5\bar{\nu^*}\gamma\nu^* - 3\bar{e}\gamma e - 3\bar{e^*}\gamma e^* \\ \\ +\bar{\nu}^r\gamma\nu_r - 3\bar{\nu^*}^r\gamma\nu^*_r + \bar{e}_r\gamma e^r + \bar{e^*}_r\gamma e^{*r} \end{array} \right) \tag{62}$$

Notice that all 16 Weyl leptons do participate in the above couplings, and that the index r labels the leptons that are triplets with respect to family SU_3.

7.2 Coupling to the Photon $\mathcal{A}$

Here we give the expected couplings of the charged leptons to the photon $\mathcal{A}$,

$$\mathcal{A} \times (-\bar{e}\gamma e + \bar{e^*}\gamma e^* - \bar{e}_r\gamma e^r + \bar{e^*}_r\gamma e^{*r}) \tag{63}$$

Notice that we have four electron-like particles (e, e^r), classified as a singlet and a triplet with respect to family SU_3, and their antiparticles (e^*, e^{*r}).

7.3 Coupling to the $\mathcal{Z}$ Boson

Here we give the couplings of the fundamental leptons to $\mathcal{Z}$, the massive neutral vector boson of electroweak theory,

$$\mathcal{Z} \times \left(\begin{array}{l} \frac{1}{2}\bar{e}\gamma e + \frac{1}{2}\bar{e^*}\gamma e^* - \frac{1}{6}\bar{e}_r\gamma e^r - \frac{1}{6}\bar{e^*}_r\gamma e^{*r} \\ \\ -\bar{\nu}\gamma\nu + \frac{2}{3}\bar{\nu}^r\gamma\nu_r - \frac{1}{3}\bar{\nu^*}^r\gamma\nu^*_r \end{array} \right) \tag{64}$$

Notice that the SU_5 singlet antineutrino does not participate in the above couplings, and that the coupling to a singlet particle is stronger than the corresponding coupling to a triplet of the same charge.

7.4 Coupling to the $\mathcal{W}$ Bosons

Here we give the couplings of the fundamental leptons to the $(\mathcal{W}^r, \mathcal{W}_r)$ vector bosons. The latter are charge ± 1 particles like the $W^{\pm}$ of electroweak theory, however, being triplets with respect to family SU_3,

$$\begin{cases} \mathcal{W}^r \times \left(\bar{e^*}\gamma\nu^*_r + \bar{e^*}_r\gamma\nu + \epsilon_{rst}\bar{\nu}^s\gamma e^t\right) \\ +\mathcal{W}_r \times \left(\bar{\nu}\gamma e^{*r} + \bar{\nu^*}^r\gamma e^* - \epsilon^{rst}\bar{e}_s\gamma\nu_t\right) \end{cases} \tag{65}$$

Again, notice that the SU_5 singlet antineutrino would not participate in the above couplings. It is important to notice, as well, how the triplet of $\mathcal{W}$ does exchange triplets of neutrinos and triplets of charged leptons via the *mechanism of family alternation*, manifested in the use of the epsilon symbol. On the other hand, singlet neutrinos are exchanged with triplets of charged antileptons, and singlets of charged antileptons are exchanged with triplets of antineutrinos.

7.5 Coupling to the $\mathcal{H}$ Bosons

Here we give the couplings of the fundamental leptons to the ${\mathcal{H}_r}^s$ vector bosons. The latter are an octet of neutral gauge particles that correspond to the family (or horizontal) SU_3 symmetry,

$${\mathcal{H}_r}^s \times \left(\bar{\nu}^r\gamma\nu_s + \bar{\nu^*}^r\gamma\nu^*_s - \bar{e}_s\gamma e^r - \bar{e^*}_s\gamma e^{*r}\right) \tag{66}$$

Notice that only the neutrinos and the charged leptons, and their antiparticles, that are triplets with respect to family SU_3 do participate in the above couplings.

7.6 Coupling to the $\mathcal{X}$ Bosons

Here we give the couplings of the fundamental leptons to the $(\mathcal{X}, \mathcal{X}^*)$ vector bosons, the latter being charge ± 2 particles,

$$\mathcal{X} \times \left(\bar{e}\gamma e^* - \bar{e}_r\gamma e^{*r}\right) + \mathcal{X}^* \times \left(\bar{e^*}\gamma e - \bar{e^*}_r\gamma e^r\right) \tag{67}$$

Notice that the $(\mathcal{X}, \mathcal{X}^*)$ vector bosons would exchange the charged leptons, whether singlets or triplets, with their respective antiparticles.

7.7 Coupling to the $\mathcal{Y}$ Bosons

Here we give the couplings of the fundamental leptons to the $(\mathcal{Y}^r, \mathcal{Y}_r)$ vector bosons. The latter are charge ± 1 particles that are triplets with respect to the family SU_3 symmetry,

$$\begin{cases} \mathcal{Y}^r \times \left(\bar{e}\gamma\nu^*_r - \bar{e}_r\gamma\nu + \epsilon_{rst}\bar{\nu}^s\gamma e^{*t}\right) \\ +\mathcal{Y}_r \times \left(\bar{\nu^*}^r\gamma e - \bar{\nu}\gamma e^r - \epsilon^{rst}\bar{e^*}_s\gamma\nu_t\right) \end{cases} \tag{68}$$

Again, notice that the SU_5 singlet antineutrino would not participate in the above couplings. It is important to notice, as well, how the triplet of $\mathcal{Y}$ does exchange triplets of neutrinos with triplets of charged antileptons (in contrast with the case of the triplet of $\mathcal{W}$ vector bosons where charged leptons are involved) via the mechanism of family alternation, manifested with the use of the epsilon symbol. On the other hand, singlet neutrinos are exchanged with triplets of charged leptons (antileptons in the case of $\mathcal{W}$'s), and singlets of charged leptons (antileptons in the case of $\mathcal{W}$'s) are exchanged with triplets of antineutrinos.

7.8 Coupling to the $\mathcal{N}$ Bosons

Here we give the couplings of the fundamental leptons to the neutral vector bosons $\mathcal{N}$, $\mathcal{N}^*$, $\mathcal{N}_r$, and $\mathcal{N}^r$, being singlets and triplets with respect to family SU_3.

$$\begin{cases} \mathcal{N} \times (\bar{\nu^*}\gamma\nu - \bar{\nu}^r\gamma\nu_r^*) \\ +\mathcal{N}^* \times (\bar{\nu}\gamma\nu^* - \bar{\nu^*}^r\gamma\nu_r) \\ +\mathcal{N}_r \times (\bar{\nu}^r\gamma\nu^* - \bar{\nu^*}^r\gamma\nu + \bar{e}\gamma e^r - \bar{e^*}\gamma e^{*r}) \\ +\mathcal{N}^r \times (-\bar{\nu}\gamma\nu_r^* + \bar{\nu^*}\gamma\nu_r + \bar{e}_r\gamma e - \bar{e^*}_r\gamma e^*) \end{cases} \tag{69}$$

7.9 Coupling to the $\mathcal{F}$ Bosons

Here we give the couplings of the fundamental leptons to the $(\mathcal{F}_r, \mathcal{F}^r)$ vector bosons. The latter are charge ± 1 particles.

$$\begin{cases} \mathcal{F}_r \times (\bar{\nu^*}\gamma e^r + \bar{\nu}^r\gamma e + \epsilon^{rst}\bar{e^*}_s\gamma\nu_t^*) \\ +\mathcal{F}^r \times \left(\bar{e}\gamma\nu_r + \bar{e}_r\gamma\nu^* - \epsilon_{rst}\bar{\nu^*}^s\gamma e^{*t}\right) \end{cases} \tag{70}$$

7.10 Coupling to the $\mathcal{E}$ Bosons

Here we give the couplings of the fundamental leptons to the $(\mathcal{E}_r, \mathcal{E}^r)$ vector bosons. The latter are charge ± 1 particles.

$$\begin{cases} \mathcal{E}_r \times (\bar{\nu^*}\gamma e^{*r} - \bar{\nu}^r\gamma e^* - \epsilon^{rst}\bar{e}_s\gamma\nu_t^*) \\ +\mathcal{E}^r \times (-\bar{e^*}\gamma\nu_r + \bar{e^*}_r\gamma\nu^* + \epsilon_{rst}\bar{\nu^*}^s\gamma e^t) \end{cases} \tag{71}$$

8 Discussion

Apart from presenting the structure of the presumably well-known quark-lepton O_{10} model of grand unification, we have proposed in this paper an O_{10} model that unifies the symmetries of several purely leptonic (electron-like and neutrino-like) particles. In fact, this model is a consolidation of an SU_5 symmetry, this being an extension that includes a *family* SU_3, of the minimal model that unifies the electron, the positron, and the neutrino (three Weyl fermions) in an SU_3 electroweak gauge theory[21], [22], [23]. With the inclusion of a family SU_3, the theory exhibits the fundamental particles (both, the leptons and the vector bosons) either as singlets or as triplets. As we have remarked in discussing the SU_3 family symmetry that appears in an SU_7 or O_{14} unification model[24] for quarks and leptons, the important question, in this regard, is *whether the observed three generations of leptons would correspond to this triplet structure*, and whether the heavier 4th leptons, do exist. Whether one can predict the masses of the heavier leptons, and the mass splitting within the triplet, is a problem that awaits the completion of *a truly predictive effective theory of symmetry breaking in quantum field theory*, "perhaps different from the deficient Higgs mechanism that seems to lack any real predictive power."

The O_{10} unification model of this paper should be embedded in a 14-dimensional gravidynamic extension, in the same manner that an O_{14} unification model was embedded[25] in the 18-dimensional gravidynamic model. We shall present the associated algebraic work in another article. However, we should point out here that such an embedding *would duplicate the number of fundamental leptons*, hence giving more possibilities for some *not yet observed leptonic varieties* to play the role of hadronic constituents. On the other hand, the geometrical structure of the extra-dimensional theory might also play its part in understanding hadronic physics.

Again, an important implication that should be relevant to current high-energy collider phenomenology is the possible observation of a triplet of $W^{\pm}$-like particles. Apart from the possible existence of other (heavier) vector bosons predicted by the theory, the observation of the extra W's *should be the nearest and most imminent possibility.* An important particle that appears in this theory, and was already predicted in the SU_3 electroweak gauge theory[21], is the charge ± 2 vector boson (the $\mathcal{X}$ boson in this article). This gauge particle would exchange an electron-like particle with a positron-like particle. Whereas, most high energy collider searches use electron-positron or proton-antiproton collisions, it seems to us that *using highly energetic electron-electron or proton-proton collisions*, if feasible, might lead to the production of the $\mathcal{X}$ particles.

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