General Relativity as curvature of space

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Abstract

With the Planck 'constants' length, time, mass and acceleration will be shown, that a Quantum Gravity of the cosmos exists. This paper shows how Einstein's Field Equations in Friedman Robertson Walker Metric solves the Planck Era context.

1 The Planck 'constants'

Planck length $\Delta x = \sqrt{\frac{Gh}{c^3}}$

Planck time $\Delta t = \sqrt{\frac{Gh}{c^5}}$

Planck mass $\Delta m = \sqrt{\frac{hc}{G}}$

Planck acceleration $\Delta a = \frac{c}{\Delta t} = \sqrt{\frac{c^7}{hG}}$

2 Modern Cosmology

Within modern cosmology the Einstein's Field Equations would be written with cosmological term Λ as follows (see [2] and [3]):

$$R_{ik} - \frac{1}{2} g_{ik} R - \Lambda g_{ik} = \frac{8\pi G}{c^4} T_{ik} \quad (2.0)$$

The solutions of the Field Equations in Friedman-Roberson-Walker-Metric are:

FRW Equation (I)

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3} + \frac{\Lambda c^2}{3} - \frac{kc^2}{R^2} \quad (2.1)$$

FRW Equation (II)

$$\frac{\ddot{R}}{R} = \frac{\Lambda c^2}{3} - \frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) \quad (2.2)$$

Einstein abandoned the cosmological term Λ as his "greatest blunder" after Hubble's 1928 discovery that the distant galaxies are expanding away from each other. Within a Universe with ideal Quantum gas and without cosmological term Λ and geometry factor $k = 0$ the equation (2.1) and (2.2) will become:

FRW Equation (I)

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3} \quad (2.3)$$

and FRW Equation (II)

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) \quad (2.4)$$

With the relation $p = \frac{\rho c^2}{3}$ (Quantum gas) will change (2.4) as follows:

$$\frac{\ddot{R}}{R} = -\frac{8\pi G\rho}{3} \quad (2.5)$$

Within Thermodynamics we assume $dE = TdS - pdV$ and for an adiabatic process it holds $TdS = 0$. We become $d(\epsilon V) = d\epsilon V + \epsilon dV = -pdV$ and it follows with $d\epsilon = -(\epsilon + p)\frac{dV}{V}$ and for the Radiation Dominated (RD) Universe $p = \frac{\epsilon}{3}$ we assume:

$$\frac{d\epsilon}{\epsilon} = -\frac{4dV}{3V} \; or \; \epsilon \sim V^{-4/3} \sim R^{-4} \quad (2.6)$$

If we assume $R = ct_0$ and $\dot{R} = c$ (with t_0 = Age of Universe = 13.8[Gyr] = $4.3549e^{17}$[s]) we receive from (2.3):

$$\frac{c^2}{R^2} = \frac{8\pi G\rho}{3} \quad (2.7)$$

We multiply (2.7) with $\frac{1}{c^2R^2}$ and with energy density $\epsilon = \rho c^2$ (RD) we get:

$$\frac{1}{R^4} = \frac{8\pi G\epsilon}{3c^4R^2} \quad (2.8)$$

In the context of (2.6) $\epsilon \sim R^{-4}$ follows that $\frac{G}{R^2} = constant$ (see [5] section 7.8).

The equivalence between Radiation Dominated (RD) and Matter Dominated (MD) Universe is given by:

$$z_{eq} = \frac{\frac{3c^2}{8\pi G t_0^2}}{\tilde{a} T_{CMB}^4} = 20317$$

Cosmic Microwave Background Temperature $T_{CMB} = 2.725$ [K]

The Radiation constant is $\tilde{a} = 7.5657e^{-16}$ $[\frac{J}{Km^3}]$

3 Planck-Era

In Planck-Era the following 6 relations are valid:

$\Delta m\ \Delta x = \frac{h}{c}$ (3.1) (Heisenberg uncertainty principle $\Delta p\ \Delta x = h$)

$\Delta m\ \Delta t = \frac{h}{c^2}$ (3.2) (Heisenberg uncertainty principle $\Delta E\ \Delta t = h$)

$\frac{\Delta m}{\Delta a} = \frac{h}{c^3}$ (3.3) (Quantum Gravity $\Delta a = \frac{\Delta m c^3}{h}$)

$\frac{\Delta m}{\Delta x} = \frac{c^2}{G}$ (3.4)

$\frac{\Delta m}{\Delta t} = \frac{c^3}{G}$ (3.5)

With Planck-Force $\Delta F = \Delta m\ \Delta a = \frac{c^4}{G}$ (3.6)

The Planck Temperature is $\Delta T = (\frac{3c^7}{8\pi h G^2 \tilde{a}})^{1/4} = 5.8404e^{31}$ $[K]$

The Planck Energy $\Delta E = \Delta m c^2 = 6.08088\ k\Delta T = \sqrt{\frac{hc^5}{G}}$

4 Gravitation as curvature of space

We assume in macroscopic scale the equations (3.1) til (3.6) as:

(Entropy constant $\zeta_r = \sqrt{\frac{R_{eq}}{\Delta x}} = 1.5036e^{29}$) $R_{eq} = \frac{ct_0}{z_{eq}} = 6.426e^{21}$ [m]

For the Radiation Dominated (RD) Universe we get: $G_{RD} = \frac{c^3}{\zeta_r^4 h} R_{RD}^2$

For the Matter Dominated (MD) Universe we get: $G_{MD} = \frac{G}{ct_0} R_{MD}$

For Matter and Radiation Equivalence we get: $G_{eq} = \frac{c^3}{\zeta_r^4 h} R_{eq}^2 = \frac{G}{ct_0} R_{eq} = 3.2848e^{-15}[\frac{m^3}{s^2\ kg}]$

Planck length at equivalence $\Delta x_{eq} = \sqrt{\frac{G_{eq} h}{c^3}} = 2.8422e^{-37}[m]$

Planck time at equivalence $\Delta t_{eq} = \sqrt{\frac{G_{eq} h}{c^5}} = 9.4805e^{-46}[s]$

Planck mass at equivalence $\Delta m_{eq} = \sqrt{\frac{hc}{G_{eq}}} = 7.7765e^{-6}[kg]$

$M_{eq} = M_{MD} = \zeta_r^2 \Delta m_{eq} = 1.7582e^{53}[kg]$

$M_{eq} R_{eq} = \zeta_r^4 \frac{h}{c} \quad (4.1)$

$M_{eq} t_{eq} = \zeta_r^4 \frac{h}{c^2} \quad (4.2)$

$\frac{M_{eq}}{a_{eq}} = \zeta_r^4 \frac{h}{c^3} \quad (4.3)$ with $a_{eq} = \frac{c}{t_{eq}} = \frac{M_{eq} c^3}{\zeta_r^4 h} \Rightarrow M_{eq} t_{eq} = \zeta_r^4 \frac{h}{c^2}$

$\frac{M_{eq}}{R_{eq}} = \frac{c^2}{G_{eq}} \quad (4.4)$

$\frac{M_{eq}}{t_{eq}} = \frac{c^3}{G_{eq}} \quad (4.5)$

With Planck-Force $\Delta F_{eq} = M_{eq} a = \frac{c^4}{G_{eq}} \quad (4.6)$

With the Acceleration $a_{eq} = \frac{c^4}{M_{eq} G_{eq}} = \frac{c^2}{R_{eq}} = \frac{G_{eq} M_{eq}}{R_{eq}^2} = \frac{M_{eq} c^3}{\zeta_r^4 h}$ it follows:

$$\frac{G_{eq}}{R_{eq}^2} = \frac{c^3}{\zeta_r^4 h} \quad (4.7)$$

For $\dot{R}^2 = \frac{G_{eq}M_{eq}}{R_{eq}}$ is with (4.7): $\dot{R}^2 = M_{eq}R_{eq}\frac{c^3}{\zeta_r^4 h} = c^2$

The FRW Equation (I) (2.3) is with $\dot{R} = c$ as follows:

$$\frac{c^2}{R_{eq}^2} = \frac{8\pi G_{eq}\rho}{3}$$

or with (2.8) and (4.7):

$$\frac{1}{R_{eq}^4} = \frac{8\pi\epsilon}{3\zeta_r^4 hc}$$

We become the R^4 dependency of (2.6) as follows:
$\epsilon_{eq} = \frac{3\zeta_r^4 hc}{8\pi R_{eq}^4} = \tilde{a}T_{eq}$ with $T_{eq} = T_{CMB}z_{eq}$

5 The universe as a black hole

(Entropy constant $\zeta = \sqrt{\frac{R}{\Delta x}} = 1.7952e^{30}$) $R = ct_0 = 1.3056e^{26}$ [m]
For a Virtual Photon we assume: $E_\gamma = h\nu = m\ a\ x$

The Radius is $R = \frac{GM}{c^2}$ and Acceleration $a = \frac{GM}{R^2} = \frac{c^2}{R}$

We get: $E_\gamma = \frac{h\nu}{c^2}\frac{c^2}{R}\frac{c}{\nu} = \frac{hc}{R} = kT_{BH} \Rightarrow$ The Bekenstein-Hawking-Temperature is: $T_{BH} = \frac{hc}{kR}$

The Bekenstein-Hawking Planck-Temperature is $\Delta T_{BH} = \frac{\Delta E}{k} = \frac{hc}{k\Delta x}$

With $R = \zeta^2\Delta x$ follows: $T_{BH} = \frac{hc}{k\zeta^2\Delta x} = \frac{\Delta T_{BH}}{\zeta^2}$

The Entropy is $S = -k\ ln\ P = -k\ ln(e^{-\frac{R^2}{\Delta x^2}}) = k\zeta^4$ (P = Normaldistribution)

The total Energy is $E = ST_{BH} = \zeta^2 k\Delta T_{BH} = \zeta^2\Delta E = M_{MD}c^2$

6 Consequences

There is no problem with the Singularty at Zero because:

$$G_{RD}(R) = \frac{R^2c^3}{\zeta^4 h} = G_{RD}(t) = \frac{t^2c^5}{\zeta^4 h} \quad (5.1)$$

For the Planck Era we receive:

Planck length $\Delta x(G) = \sqrt{\frac{G(R)h}{c^3}} = \frac{R}{\zeta^2}$

Planck time $\Delta t(G) = \sqrt{\frac{G(t)h}{c^5}} = \frac{t}{\zeta^2}$

Planck mass $\Delta m(G) = \sqrt{\frac{hc}{G(R)}} = \frac{h\zeta_r^2}{cR} \Rightarrow \Delta m(G)\Delta x(G) = \frac{h}{c}$

Planck acceleration $\Delta a(G) = \sqrt{\frac{c^7}{hG(R)}} = \frac{c^2\zeta_r^2}{R} \Rightarrow \Delta a(G)\Delta x(G) = c^2$

Density $\rho = \frac{3c^2}{8\pi G R^2} = \frac{3}{8\pi G t^2} = \rho_c \Rightarrow k = 0$

General Relativity: $\frac{\dot{R}^2}{R^2} = \frac{8\pi G\rho}{3} = \frac{c^2}{R^2} = \frac{1}{t^2} \Rightarrow R = ct$

7 References

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