A recurrent formula inspired by Rowland's formula and based on Smarandache function which might be a criterion for primality

Abstract. Studying the two well known recurrent relations with the exceptional property that they generate only values which are equal to 1 or are odd primes, id est the formula which belongs to Eric Rowland and the one that belongs to Benoit Cloitre, I managed to discover a formula based on Smarandache function, from the same family of recurrent relations, which, instead to give a prime value for any input, seems to give the same value, 2, if and only if the value of the input is an odd prime; also, for any value of input different from 1 and different from an odd prime, the value of output is equal to n + 1. I name this relation the Coman-Smarandache criterion for primality and the exceptions from this rule, if they exist, Coman-Smarandache pseudoprimes.

Introduction

The Rowland's formula was first noticed in 2003 summer camp NKS (New Kind of Science) organized by Wolfram Science and was subsequently proved to be true (transformed in theorem) by one of the participants in this camp, Eric Rowlands, who also conjectured that all odd primes can be generated by this formula. This formula (theorem) is:
: Let be the following recurrence relation: $f(1) = 7$, and, for $n \geq 2$, $f(n) = f(n - 1) + \gcd[n, f(n - 1)]$; then, the formula $g(n) = f(n) - f(n - 1)$ has the exceptional property that it's result can be only a value which is equal to 1 or to an odd prime. The first values of $g(n)$ are (see the sequence A132199 in OEIS): 1, 1, 1, 5, 3, 1, 1, 1, 1, 11, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 23, 3, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 (...). The first primes resulting from this formula are (see the sequence A137613 in OEIS): 5, 3, 11, 3, 23, 3, 47, 3, 5, 3, 101, 3, 7, 11, 3, 13, 233, 3, 467, 3, 5, 3, 941, 3, 7, 1889 (...).

French mathematician Benoit Cloitre further found a similar formula:
: Let $f(1) = 1$, and, for $n \geq 2$, $f(n) = f(n - 1) + \mathrm{lcm}[n, f(n - 1)]$; then, the formula $g(n) = f(n)/f(n - 1) - 1$ has also, as result, only a value which is equal to 1 or to an odd prime.

Conjecture 1

Let $f(1) = 1$ and $f(n) = S(f(n - 1)) + \mathrm{lcm}[n, S(f(n - 1))]$, where S is the Smarandache function and lcm the least common multiple. Then the value of the function g(n) = f(n)/S(f(n − 1)) is equal to 2 if and only if n is an odd prime.

Conjecture 2

The value of the function g(n), defined in Conjecture 1, is g(n) = n + 1 for n different from 1 and n different from odd primes.

Verifying the conjectures
(up to n = 17)

: f(2) = 1 + lcm[2, 1] = 3; then g(2) = 3/1 = 3;
: **f(3)** = 3 + lcm[3, 3] = 6; then g(3) = 6/3 = **2**;
: f(4) = 3 + lcm[4, 3] = 15; then g(4) = 15/3 = 5;
: **f(5)** = 5 + lcm[5, 5] = 10; then g(5) = 10/5 = **2**;
: f(6) = 5 + lcm[6, 5] = 35; then g(6) = 35/5 = 7;
: **f(7)** = 7 + lcm[7, 7] = 14; then g(7) = 14/7 = **2**;
: f(8) = 7 + lcm[8, 7] = 63; then g(8) = 63/7 = 9;
: f(9) = 7 + lcm[9, 7] = 70; then g(9) = 70/7 = 10;
: f(10) = 7 + lcm[10, 7] = 77; then g(10) = 77/7 = 11;
: **f(11)** = 11 + lcm[11, 11] = 22; then g(11) = 22/11 = **2**;
: f(12) = 11 + lcm[12, 11] = 143; then g(12) = 143/11 = 13;
: **f(13)** = 13 + lcm[13, 13] = 26; then g(13) = 26/13 = **2**;
: f(14) = 13 + lcm[14, 13] = 195; then g(14) = 195/13 = 15;
: f(15) = 13 + lcm[15, 13] = 208; then g(15) = 208/13 = 16;
: f(16) = 13 + lcm[16, 13] = 221; then g(16) = 221/13 = 17;
: **f(17)** = 17 + lcm[17, 17] = 17; then g(17) = 34/17 = **2**.

Note

The function g(n) = f(n)/S(f(n − 1)) − 1, where f(n) = f(n − 1) + lcm[n, f(n − 1)] might also be interesting to study as a prime generating formula, as it gives prime values (i.e. 5, 17, 23, 191, 383) for the following consecutive values of n: 4, 5, 6, 7, 8; however, for n = 9 the value obtained is a semiprime and for n = 10 is not even obtained an integer value, because m is not always divisible by S(m) so f(n), which is always divisible by f(n − 1), is not always divisible by S(f(n − 1)).

References:

1. Rowland, Eric, *A simple prime-generating recurrence*;
2. Peterson, Ivars, *A new formula for generating primes*;
3. Shevelev, Vladimir, *Generalizations of the Rowland Theorem*.