Is the Natario warp drive a valid candidate for an interstellar voyage to the star system Gliese 667C(GJ 667C)??

Fernando Loup *
Residencia de Estudantes Universitas Lisboa Portugal

January 28, 2014

Abstract

Warp Drives are solutions of the Einstein Field Equations that allows superluminal travel within the framework of General Relativity. There are at the present moment two known solutions: The Alcubierre warp drive discovered in 1994 and the Natario warp drive discovered in 2001. The major drawback concerning warp drives is the huge amount of negative energy able to sustain the warp bubble.In order to perform an interstellar space travel to a "nearby" star at 22 light-years away with 3 potential habitable exo-planets(Gliese 667C) at superluminal speeds in a reasonable amount of time a ship must attain a speed of about 200 times faster than light.However the negative energy density at such a speed is directly proportional to the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!! We introduce here a shape function that defines the Natario warp drive as an excellent candidate to low the negative energy density requirements from 10^{48} to affordable levels.We also discuss other warp drive drawbacks:collisions with hazardous interstellar matter(asteroids or comets) that may happen in a real interstellar travel from Earth to Gliese 667C and Horizons(causally disconnected portions of spacetime).We terminate this work with a description of the star system Gliese 667C

*spacetimeshortcut@yahoo.com

1 Introduction

The Warp Drive as a solution of the Einstein Field Equations of General Relativity that allows superluminal travel appeared first in 1994 due to the work of Alcubierre.([1]) The warp drive as conceived by Alcubierre worked with an expansion of the spacetime behind an object and contraction of the spacetime in front.The departure point is being moved away from the object and the destination point is being moved closer to the object.The object do not moves at all[1].It remains at the rest inside the so called warp bubble but an external observer would see the object passing by him at superluminal speeds(pg 8 in [1])(pg 1 in [2])(pg 34 in [5]).

Later on in 2001 another warp drive appeared due to the work of Natario.([2]).This do not expands or contracts spacetime but deals with the spacetime as a "strain" tensor of Fluid Mechanics(pg 5 in [2]). Imagine the object being a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed but inside the aquarium the fish is at the rest with respect to his local neighborhoods.

However there are 3 major drawbacks that compromises the warp drive physical integrity as a viable tool for superluminal interstellar travel.

The first drawback is the quest of large negative energy requirements enough to sustain the warp bubble. In order to travel to a "nearby" star at 20 light-years at superluminal speeds in a reasonable amount of time a ship must attain a speed of about 200 times faster than light.However the negative energy density at such a speed is directly proportional to the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!

Another drawback that affects the warp drive is the quest of the interstellar navigation:Interstellar space is not empty and from a real point of view a ship at superluminal speeds would impact asteroids,comets,interstellar space dust and photons.

The last drawback raised against the warp drive is the fact that inside the warp bubble an astronaut cannot send signals with the speed of the light to control the front of the bubble because an Horizon(causally disconnected portion of spacetime)is established between the astronaut and the warp bubble.

We can demonstrate that the Natario warp drive can "easily" overcome these obstacles as a valid candidate for superluminal interstellar travel.

In June 2013 a team of astronomers from *ESO* (European Southern Observatory)La Silla Chile published a discovery that is considered a major breakthrough in exo-planetary astronomy:The discovery of the star system $Gliese667C$ at 22 light-years from Earth with three planets in the Habitable Zone.It was the first time a planetary system was found with more than one planet in the Habitable Zone.$Gliese667C$ is a red dwarf star class M and according with these astronomers M stars tend to form systems with multiple planets in the Habitable Zone unlike G stars like the Sun that may form or may not form more than one planet in the Habitable Zone.And there are 260 billions of red dwarfs M stars in our galaxy more than twice the 100 billions of G stars like our Sun.

[1]do not violates Relativity

If M stars have the tendency to have multiple planets in their Habitable Zones and if there are 260 billions of them in our galaxy then the possibility to find life outside Earth raised exponentially.

A spaceship equipped with a Natario warp drive could travel from Earth to $Gliese667C$ in a month and half at a speed of 200 times faster then light.

It was the affirmation above that gave to ourselves the idea to write this work.

Warp drives are often regarded as academic curiosities or mathematical tools to teach General Relativity with no practical applications so we decided to write a work about the warp drive theory not for the people already familiarized with it since the material we present here is not new .

We decided to focus ourselves on people with mathematical background but without precious contact with the warp drive theory and we wrote this work that introduces the Natario warp drive and all its mathematical foundations in order to promote the theory among other important communities of scientists outside the mainstream of the warp drive researchers and we mean specially the community of exo-planetary astronomers.

In this work we cover only the Natario warp drive and we avoid comparisons between the differences of the models proposed by Alcubierre and Natario since these differences were already deeply covered by the existing available literature.However we use the Alcubierre shape function to define its Natario counterpart.

The mathematics presented here is only a compilation of work already done by Natario without nothing new but perhaps with a more pedagogical presentation in order to captive the interest of non-familiarized readers.Anyone with basic knowledge in $3+1$ General Relativity should be able to read this work.

We adopt here the Geometrized system of units in which $c = G = 1$ for geometric purposes and the International System of units for energetic purposes

This work is organized as follows:

- Section 2)-Introduces the Natario warp drive spacetime for people familiarized with basic $3+1$ General Relativity but without knowledge in Differential Forms in a pedagogical way.
- Section 3)-Outlines the problems of the immense magnitude in negative energy density when a ship travels with a speed of 200 times faster than light. The negative energy density for such a speed is directly proportional to the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!..
- Section 4)-Covers the problem of interstellar navigation in the Natario warp drive spacetime and how collisions with hazardous objects in interstellar space(asteroids or comets) can be avoided
- Section 5)-We introduce a shape function that defines the Natario warp drive spacetime being this function an excellent candidate to lower the energy density requirements in the Natario warp drive to affordable levels completely obliterating the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!..

- Section 6)-Outlines the possibility of how to overcome the Horizon problem from an original point of view of General Relativity.

- Section 7)-We introduce a pedagogical description of the star system $Gliese667C$.

Although this work was designed to be an independent and self-contained it can be regarded as a companion of our works [9],[10] and [11]

2 Warp Drive with Zero Expansion:

In 2001 Natario introduced a new warp drive spacetime defined using the canonical basis of the Hodge star in spherical coordinates defined as follows(pg 4 in [2])[2]:

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r \sin\theta d\varphi) \tag{1}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial \theta} \sim rd\theta \sim (r \sin\theta d\varphi) \wedge dr \tag{2}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi} \sim r \sin\theta d\varphi \sim dr \wedge (rd\theta) \tag{3}$$

We can introduce now the Natario definition for the warp drive:
(pg 4 in [2]):

- 1)-Any Natario vector nX generates a warp drive spacetime if $nX = 0$ for a small value of $|x|$ defined by Natario as the interior of the warp bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ for a large value of $|x|$ defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble and $X = vs$ or $X = -vs$ being the shift vector embedded inside the Natario vector.[3]
- 2)-The Natario vector nX is defined by the shift vector X as $nX = Xdx$ being $X = vs$ or $X = -vs$

Back again to the concept of the fish and the aquarium;

Imagine a fish inside an aquarium and the aquarium is floating in the surface of a river but carried out by the river stream.The warp bubble in this case is the aquarium whose walls do not expand or contract. An observer in the margin of the river would see the aquarium passing by him at a large speed(a Natario vector $nX = vsdx$ and a shift vector $X = vs$ being vs the speed of the warp bubble) but inside the aquarium the fish is at the rest with respect to his local neighborhoods(a Natario vector $nX = 0$ and a shift vector $X = 0$ meaning a speed $vs = 0$ inside the aquarium because inside the aquarium the fish is stopped.The fish is at the rest with respect to its local neighborhoods which means to say the water inside the aquarium that is being carried out with the fish at the same time and the water is acting as a local coordinates frame).[4]

The spaceship is the fish and the aquarium is the warp bubble.

An observer in outer space at the rest with respect to a local coordinates frame outside the bubble of course(the vacuum of space surrounding him) sees a warp bubble passing by him at a large speed vs but inside the bubble the spaceship is at the rest with respect to a local coordinates frame inside the bubble(the vacuum of space surrounding the spaceship).

[2]See Appendix C on Hodge stars and differential forms
[3]The mathematical relations between the shift vector and the Natario vector are given in the Appendix B
[4]See Appendix G for an artistic presentation of a spaceship inside a Natario warp bubble

Applying the Natario equivalence between spherical and cartezian coordinates as shown below (pg 5 in [2])[5]:

$$\frac{\partial}{\partial x} \sim dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \sim r^2\sin\theta\cos\theta d\theta\wedge d\varphi + r\sin^2\theta dr\wedge d\varphi = d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \quad (4)$$

And for dx we have(pg 5 in [2]) :

$$dx = d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \quad (5)$$

Inserting all this stuff in the Natario vector $nX = Xdx$ we would get the following expression for the corresponding shift vectors $X = vs$ or $X = -vs$(pg 5 in [2])

$$nX = Xdx = vs(t)d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \quad (6)$$

$$nX = Xdx = -vs(t)d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \quad (7)$$

Rewriting the Natario vector as(pg 5 in [2])

Note that we replaced $\frac{1}{2}$ by $f(r)$ in order to obtain(pg 5 in [2]):

$$nX = vs(t)d\left(f(r)r^2\sin^2\theta d\varphi\right) \quad (8)$$

$$nX = -vs(t)d\left(f(r)r^2\sin^2\theta d\varphi\right) \quad (9)$$

We get the final expressions for the Natario vector nX:(pg 5 in [2])

$$nX = v_s(t)d\left[f(r)r^2\sin^2\theta d\varphi\right] \sim 2v_sf(r)\cos\theta\mathbf{e}_r - v_s(2f(r)+rf'(r))\sin\theta\mathbf{e}_\theta \quad (10)$$

$$nX = -v_s(t)d\left[f(r)r^2\sin^2\theta d\varphi\right] \sim -2v_sf(r)\cos\theta\mathbf{e}_r + v_s(2f(r)+rf'(r))\sin\theta\mathbf{e}_\theta \quad (11)$$

From now on we will use this pedagogical approach that gives results practically similar the ones depicted in the original Natario vector shown above:

$$nX = v_s(t)d\left[f(r)r^2\sin^2\theta d\varphi\right] \sim 2v_sf(r)\cos\theta dr - v_s(2f(r)+rf'(r))r\sin\theta d\theta \quad (12)$$

$$nX = -v_s(t)d\left[f(r)r^2\sin^2\theta d\varphi\right] \sim -2v_sf(r)\cos\theta dr + v_s(2f(r)+rf'(r))r\sin\theta d\theta \quad (13)$$

In order to make the Natario warp drive holds true we need for the Natario vector nX a continuous Natario shape function being $f(r) = \frac{1}{2}$ for large r(outside the warp bubble) and $f(r) = 0$ for small r(inside the warp bubble) while being $0 < f(r) < \frac{1}{2}$ in the walls of the warp bubble(pg 5 in [2])

[5]The mathematical demonstration of this expression will be given in the Appendix C on Hodge stars and differential forms

In order to avoid confusion with the Alcubierre shape function $f(rs)$ we will redefine the Natario shape function as $n(r)$ and the Natario vector as shown below

$$nX = v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim 2v_s n(r)\cos\theta dr - v_s(2n(r) + rn'(r))r\sin\theta d\theta \tag{14}$$

$$nX = -v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s n(r)\cos\theta dr + v_s(2n(r) + rn'(r))r\sin\theta d\theta \tag{15}$$

Examining the following Natario vector $nX = Xdx$ or $nX = vsdx$ with a shift vector $X = vs$ using the Natario continuous shape function $n(r)$ being $n(r) = \frac{1}{2}$ for large r(outside the warp bubble) and $n(r) = 0$ for small r(inside the warp bubble) while being $0 < n(r) < \frac{1}{2}$ in the walls of the warp bubble(Natario warped region)(pg 5 in [2]):

$$nX = v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim 2v_s n(r)\cos\theta dr - v_s(2n(r) + rn'(r))r\sin\theta d\theta \tag{16}$$

We must study the behavior of the Natario vector nX for the Natario shape function $n(r)$ with a shift vector $X = vs(t)$ and a warp bubble speed $vs = 200$ with respect to a distant observer.Two hundred times faster than light.

According to Natario we have 3 possible values for the Natario shape function(pg 5 in [2])::

- 1)-Inside the warp bubble $n(r) = 0$
- 2)-Outside the warp bubble $n(r) = \frac{1}{2}$
- 3)-In the warp bubble walls(Natario warped region) $0 < n(r) < \frac{1}{2}$

Lets see what happens with the Natario vector nX for each one of these conditions:

- A)-Inside the warp bubble $n(r) = 0$

  Inside the warp bubble $n(r) = 0$ as a constant value.Then the derivatives of $n(r)$ vanishes and we can rewrite the Natario vector nX as follows:

$$nX = 2v_s n(r)\cos\theta dr - v_s(2n(r) + r[\frac{dn(r)}{dr}])r\sin\theta d\theta \tag{17}$$

$$nX = 2v_s[0]\cos\theta dr - v_s(2[0])r\sin\theta d\theta \tag{18}$$

$$nX = 0!!! \tag{19}$$

  No motion at all!!!.The observer(the spaceship) inside the Natario warp bubble is completely at the rest with respect to its local spacetime neighborhoods and this observer dont feel any acceleration or any g-forces..Then for this observer $\frac{\partial}{\partial x} \sim dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta = 0$.Remember that the Natario vector is still $nX = vsdx$.But the shift vector is still $X = vs$.The Natario warp bubble still moves with a speed $vs = 200$ with respect to a distant observer.Two hundred times faster than light but the internal observer inside the warp bubble is completely at the rest and completely in safe from the g-forces that would kill him moving ar such hyper-fast velocities.The spaceship remains at the rest inside the warp bubble however the bubble moves with a speed 200 times faster than light with respect to distant external observers outside the bubble.

- B)-Outside the warp bubble $n(r) = \frac{1}{2}$

  Outside the warp bubble $n(r) = \frac{1}{2}$ as a constant value.Then the derivatives of $n(r)$ vanishes and we can rewrite the Natario vector nX as follows:

$$nX \simeq 2v_s n(r)\cos\theta dr - v_s(2n(r) + r[\frac{dn(r)}{dr}])r\sin\theta d\theta \tag{20}$$

$$nX \simeq 2v_s\frac{1}{2}\cos\theta dr - v_s(2\frac{1}{2})r\sin\theta d\theta \tag{21}$$

$$nX \simeq v_s\cos\theta dr - v_s r\sin\theta d\theta \tag{22}$$

  Remember that in this case we have the Natario vector as still being $nX = vsdx$ with the shift vector defined as $X = vs$.But now we have $\frac{\partial}{\partial x} \sim dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \neq 0$.The Natario vector do not vanishes and an external observer would see the Natario warp bubble passing by him at a $vs = 200$ two hundred times faster than light due to the shift vector $X = vs$

- C)-In the warp bubble walls(Natario warped region) $0 < n(r) < \frac{1}{2}$

  This is the region where the walls of the Natario warp bubble resides.Following our example of the fish inside the aquarium then the Natario warped region would correspond to the glass of the aquarium walls.It is not a good idea to place an observer here because the energy needed to distort the spacetime generating the warp drive is placed in this region.The Natario vector nX would then be:

$$nX = v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim 2v_s n(r)\cos\theta dr - v_s(2n(r) + r[\frac{dn(r)}{dr}])r\sin\theta d\theta \tag{23}$$

Now the reader can understand the point of view of the idea outlined by Natario in 2001.The Natario vector $nX = vsdx = 0$ vanishes inside the warp bubble because inside the warp bubble there are no motion at all because $dx = 0$ while being $nX = vsdx \neq 0$ not vanishing outside the warp bubble because an external observer sees the warp bubble passing by him with a speed defined by the shift vector $X = vs$

According to pg 8 in [2] photons still outside the warp bubble but approaching the warp bubble from the front appears with the frequency highly Doppler Blueshifted when seen by the observer in the center of the bubble.Actually these photons are moving in the direction of the bubble towards it with a speed c but the bubble is moving in the direction of the photons and towards these with a speed $vs >> c$.Collisions between highly energetic photons and warp bubble walls are a major concern in warp drive science.We will address this issue later.

If the warp bubble moves with a speed vs then a source of photons(a star) is being seen by the observer in the center of the bubble with a relative approximation speed vs

Computing the Blueshift using the classical Doppler-Fizeau formula for a photon approaching the ship from the front we have:[6]

$$f = f_0 \frac{c + va}{c - vb} \tag{24}$$

The terms above are:

- 1)-f is the photon frequency seen by an observer
- 2)-f_0 is the original frequency of the emitted photon
- 3)-c is the light speed.in our case $c = 1$
- 4)-va is the relative speed of the light source approaching the observer.In our case is vs
- 5)-vb in the relative speed of the light source moving away from the observer. In our case because the photon is coming to the observer $vb = 0$

Rewriting the Doppler-Fizeau expression for an incoming photon approaching the warp bubble from the front we should expect for(see pg 8 in [2])[7]:

$$f = f_0(1 + vs) \tag{25}$$

$$f = f_0(1 + X) \tag{26}$$

Energy E is Planck Constsnt $\hbar$ multiplied by frequency so for the energy we would have:

$$E = E_0(1 + vs) \tag{27}$$

Note that as larger is $vs > 1$ as large is the energy of the photon and hence the Blueshift.The warp bubble walls would collide with these highly energetic photons.[8]This is a serious obstacle that compromises the physical feasibility of the warp drive.Again we will address this issue later.

[6]Remember that the warp drive do not obey Lorentz transformations so the classical formula can be applied to get the results of [2]

[7]Remember that $X = vs$ is the shift vector

[8]See again Appendix G.The spaceship is inside the bubble but the warp bubble is travelling in interstellar space filled with photons.Imagine a photon coming to the warp bubble walls:the blue region

Notice that outside the warp bubble the shift vector $X = vs$ with $vs > 1$ and we have (pg 6 in [2]).See also fig 1 pg 7 in [2].

$$\sin\alpha = \frac{1}{v_s} \tag{28}$$

According to Natario this is the familiar expression for the Mach cone angle[9].For superluminal(warp) speed scenario we prefer to redefine this as the Natario cone angle.See fig 3 pg 9 in [2]

But if $vs > 1$ then $\sin\alpha < 1$ and $\alpha < \frac{\pi}{2}$ and as higher are the values of vs then $\sin\alpha \simeq 0$ and also $\alpha \simeq 0$

As higher are the values of vs the Natario cone angle becomes zero so we can see that vs defines the inclination of the Natario cone angle. For "low" superluminal(warp) speeds the inclination is close to $\frac{\pi}{2}$ because although $vs > 1$ vs stands close to 1 while for "high" superluminal(warp) speed the Natario cone angle gets closer to zero because $vs >> 1$.Then according to pg 6 in [2]) we are left with 3 possible scenarios for the inclination of the Natario cone angle:

- 1)-$\|\mathbf{X}\| = 1$ and $vs = 1$.
- 2)- $X = vs$ and $vs > 1$.
- 3)-$X = vs$ and $vs >> 1$

We must examine each one of these possible scenarios:

- 1)-$\|\mathbf{X}\| = 1$ and $vs = 1$.

$$\sin\alpha = \frac{1}{\|\mathbf{X}\|} \tag{29}$$

$$\sin\alpha = 1 \dashrightarrow \alpha = \frac{\pi}{2} \dashrightarrow \|\mathbf{X}\| = 1 \tag{30}$$

This situation happens when the shift vector $X = vs$ and $vs = 1$.The warp drive reaches luminal speed.This means to say that the the warp drive reaches the Horizon[10].The Natario cone angle similar to the Mach cone angle appears(see pg 15 in [4]) with an inclination of $\frac{\pi}{2}$ because $sin\alpha = 1$ which means to say that the Natario cone angle appears perpendicular to the direction of motion.

This is a very important issue concerning warp drive science:we know that a jet when achieves the speed of the sound produces a sonic "boom" or a Mach "boom" that can break the glass of windows if the jet is flying at low altitude over a metropolitan area or can even damage the human hear if the jet is flying at a very low altitude over a populated area.

[9]See the graphical presentation of the Mach cone angle in the Appendix L.Compare this with fig 1 pg 7 in [2].Although we consider $\frac{1}{v_s}$ because we make $c = 1$ the expression really means $\frac{c}{v_s}$

[10]We will address the issue of Horizons later in this work

With the warp drive the same thing happens:although the interstellar space possesses a very low density when compared to the air it is not completely empty.The interstellar vacuum pure and simply do not exists:Interstellar space is filled with photons emitted by stars,space dust,asteroids comets or meteorites,ionized clouds of gas,supernova remnants etc.And remember that the light speed is much higher than the speed of the sound so in the end the result is the same.

When the jet reaches the speed of the sound the jet collides with the molecules of the gases of the atmosphere,water vapor,dust from the pollution etc and produces the sonic "boom" or the Mach "boom".

When the starship reaches the light speed it collides with the interstellar matter in front of the bubble producing the luminal "boom" or the Natario "boom".So to stay near a spaceship when it is close to reach the light speed is a very dangerous option.The ship is in safe inside the Natario warp bubble completely at the rest but the walls of the bubble are experiencing highly energetic collisions against the matter of the interstellar medium.Like the Doppler Blueshifts mentioned earlier this is a major concern to warp drive science.We will address this issue later.

This means to say that like a jet must take-off at subsonic speeds and move away from metropolitan or populated areas still at subsonic speeds gaining altitude in order to break the sound barrier at a safe distance or safe altitude a spaceship cannot break the light speed barrier near a planet.The collisions between the walls of the warp bubble and Doppler Blueshifted photons for a factor $vs = 200$ would release an immense amount of energy that could damage the surface of the planet[11].The ship must leave the planet at subluminal speeds and cross the entire planetary system still at subluminal speeds in order to reach its borders.Only outside the planetary system and faraway from any planet the ship can break the light speed barrier.[12] [13]

- 2)- $X = vs$ and $vs > 1$.

$$\sin\alpha = \frac{1}{v_s} \tag{31}$$

$$\sin\alpha < 1 \dashrightarrow \alpha < \frac{\pi}{2} \dashrightarrow \|\mathbf{X}\| > 1 \dashrightarrow \sin\alpha \simeq 1 \dashrightarrow \alpha \simeq \frac{\pi}{2} \tag{32}$$

This situation happens when $X = vs$ and $vs > 1$ and the warp drive reaches "low" superluminal(warp) Speed.The Natario cone angle similar to the Mach cone angle appears(see pg 15 in [4]) now with an inclination of less than $\frac{\pi}{2}$ but still close to it because vs is still close to 1.Example for a $vs = 2$(two times light speed) then $\sin\alpha = \frac{1}{2}$ and $\alpha = \frac{\pi}{6}$.[14]

[11]Think for a moment on hard X-rays from cosmic rays or gamma radiation also emitted by stars.And think on how many of these energetic photons exists per cubic centimeter of space.Now imagine a collision between each one of these photons with the energy multiplied by 200 with the walls of the warp bubble in the neighborhoods of a planet

[12]see the artistical presentations of this scenario in the Appendix I

[13]see also the artistical presentations of a Natario warp drive in real interstellar space Appendix M

[14]see the artistical presentations of this scenario in the Appendix J

- 3)-$X = vs$ and $vs >> 1$

$$\sin\alpha = \frac{1}{v_s} \tag{33}$$

$$\sin\alpha << 1 \dashrightarrow \alpha << \frac{\pi}{2} \dashrightarrow \|\mathbf{X}\| >> 1 \dashrightarrow \sin\alpha \simeq 0 \dashrightarrow \alpha \simeq 0 \tag{34}$$

This situation happens when $X = vs$ and $vs >> 1$ and the warp drive reaches "high" superluminal(warp) speed.The Natario cone angle similar to the Mach cone angle appears(see pg 15 in [4]) now with an inclination of much less than $\frac{\pi}{2}$ and highly inclined almost parallel to the direction of motion because $\sin\alpha \simeq 0$ so $\alpha \simeq 0$ too.Example for a $vs = 200$(two hundred times light speed) then $\sin\alpha = \frac{1}{200} \simeq 0$.[15]

Redefining the Natario vector nX as being the rate-of-strain tensor of Fluid Mechanics as shown below(pg 5 in [2]):

$$nX = X^r\mathbf{e}_r + X^\theta\mathbf{e}_\theta + X^\varphi\mathbf{e}_\varphi \tag{35}$$

$$nX = X^r dr + X^\theta r d\theta + X^\varphi r\sin\theta d\varphi \tag{36}$$

Remember that X^r,X^θ and X^φ are the corresponding shift vectors embedded in the Natario vector nX.[16]

Bur we already know that the Natario vector is given by:

$$nX = v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim 2v_s n(r)\cos\theta dr - v_s(2n(r) + rn'(r))r\sin\theta d\theta \tag{37}$$

$$nX = -v_s(t)d\left[n(r)r^2\sin^2\theta d\varphi\right] \sim -2v_s n(r)\cos\theta dr + v_s(2n(r) + rn'(r))r\sin\theta d\theta \tag{38}$$

Note that in our expressions for the Natatio vector we do not have the term $X^\varphi r\sin\theta d\varphi$ and the Natario vector can be simply rewritten as:

$$nX = X^{rs}drs + X^\theta rsd\theta \tag{39}$$

Hence we should expect for the shift vectors X^r,X^θ the following expressions:

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{40}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{41}$$

$$X^\theta = v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{42}$$

$$X^\theta = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{43}$$

[15] see the artistical presentations of this scenario in the Appendix K

[16] Again see the Appendices B and C for the relations between the shift vector,Natario vector and differential forms and Hodge stars

Applying the extrinsic curvature tensor for the shift vectors contained in the Natario vector nX we would get the following results(pg 5 in [2]):

$$K_{rr} = \frac{\partial X^r}{\partial r} = -2v_s n'(r)\cos\theta \tag{44}$$

$$K_{\theta\theta} = \frac{1}{r}\frac{\partial X^\theta}{\partial\theta} + \frac{X^r}{r} = v_s n'(r)\cos\theta; \tag{45}$$

$$K_{\varphi\varphi} = \frac{1}{r\sin\theta}\frac{\partial X^\varphi}{\partial\varphi} + \frac{X^r}{r} + \frac{X^\theta\cot\theta}{r} = v_s n'(r)\cos\theta \tag{46}$$

$$K_{r\theta} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{X^\theta}{r}\right) + \frac{1}{r}\frac{\partial X^r}{\partial\theta}\right] = v_s\sin\theta\left(n'(r) + \frac{r}{2}n''(r)\right) \tag{47}$$

$$K_{r\varphi} = \frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{X^\varphi}{r}\right) + \frac{1}{r\sin\theta}\frac{\partial X^r}{\partial\varphi}\right] = 0 \tag{48}$$

$$K_{\theta\varphi} = \frac{1}{2}\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{X^\varphi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial X^\theta}{\partial\varphi}\right] = 0 \tag{49}$$

Examining the first three results we can clearly see that(pg 5 in [2]):

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \tag{50}$$

This is the trace of the extrinsic curvature tensor that defines the expansion of the normal volume elements and in the Natario warp drive this is zero.The Natario warp drive do not expands or contracts the spacetime.

A warp drive with zero expansion.

The observer(spaceship) is still immersed in the interior of the warp bubble and this bubble is carried out by the spacetime "stream" at faster than light velocities with the observer at the rest with respect to its local neighborhoods inside the bubble feeling no g-forces and no accelerations. Imagine an aquarium floating in the course of a river with a fish inside it...the walls of the aquarium are the walls of the warp bubble...Imagine that this river is a "rapid" and the aquarium is being carried out by the river stream...the aquarium walls do not expand or contract...an observer in the margin of the river would see the aquarium passing by him at an arbitrarily large speed but inside the aquarium the fish would be protected from g-forces or accelerations generated by the stream...because the fish would be at the rest with respect to its local spacetime neighborhoods inside the aquarium.The Natario warp drive is being carried out by the spacetime "stream" like a fish in the stream of a river due to the resemblances between the Natario vector nX and the rate-of-strain tensor of Fluid Mechanics

In the Natario warp drive the spacetime contraction in one direction(radial) is balanced by the spacetime expansion in the remaining direction(perpendicular)(pg 5 in [2]).[17]

[17] See Appendix H the artistic presentation of the Natario warp drive where the expansion in one direction is counter-balanced by a contraction in the opposite directions

The energy density in the Natario warp drive is given by the following expression(pg 5 in [2]):

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(r))^2\cos^2\theta + \left(n'(r) + \frac{r}{2}n''(r)\right)^2\sin^2\theta\right]. \tag{51}$$

$$\rho = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(\frac{dn(r)}{dr})^2\cos^2\theta + \left(\frac{dn(r)}{dr} + \frac{r}{2}\frac{d^2n(r)}{dr^2}\right)^2\sin^2\theta\right]. \tag{52}$$

This energy density is negative and depends of the Natario shape function $n(r)$ and its derivatives.The Natario shape function must be analytical[18] in every point of the warp bubble.In order to generate the warp drive as a dynamical spacetime large outputs of energy are needed and this is a critical issue that will be solved perhaps by a real theory of Quantum Gravity but everything depends on the form or the behavior of the Natario shape function. We will address the question of the energy requirements later in this work and now we will introduce here the Natario warp drive continuous shape function $n(r)$.

Introducing here $f(rs)$ as the Alcubierre shape function that defines the Alcubierre warp drive spacetime we can define the Natario shape function $n(rs)$ that defines the Natario warp drive spacetime using its Alcubierre counterpart.Below is presented the equation of the Alcubierre shape function.[19].

$$f(rs) = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{53}$$

$$rs = \sqrt{(x - xs)^2 + y^2 + z^2} \tag{54}$$

According with Alcubierre any function $f(rs)$ that gives 1 inside the bubble and 0 outside the bubble while being $1 > f(rs) > 0$ in the Alcubierre warped region is a valid shape function for the Alcubierre warp drive.(see eqs 6 and 7 pg 4 in [1] or top of pg 4 in [2]).

In the Alcubierre shape function xs is the center of the warp bubble where the ship resides. R is the radius of the warp bubble and @ is the Alcubierre dimensionless parameter[20] related to the thickness.According to Alcubierre these can have arbitrary values.We outline here the fact that according to pg 4 in [1] the parameter @ can have arbitrary values.rs is the path of the so-called Eulerian observer that starts at the center of the bubble $xs = R = rs = 0$ and ends up outside the warp bubble $rs > R$.

According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

The Natario warp drive continuous shape function can be defined by:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{55}$$

$$n(rs) = \frac{1}{2}[1 - [\frac{1}{2}[1 - tanh[@(rs - R)]]]] \tag{56}$$

[18] Must be continuous and differentiable in every point of its trajectory
[19] $tanh[@(rs + R)] = 1$,$tanh(@R) = 1$ for very high values of the Alcubierre thickness parameter $@ >> |R|$
[20] see pg 6 in [10],pg 7 and 10 in [9]

This shape function gives the result of $n(rs) = 0$ inside the warp bubble and $n(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region.

Note that the Alcubierre shape function is being used to define its Natario shape function counterpart.

For the Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $n(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario bubble).

3 The Problem of the Negative Energy in the Natario Warp Drive Spacetime-The Unphysical Nature of Warp Drive

The negative energy density for the Natario warp drive is given by(see pg 5 in [2])

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{57}$$

Converting from the Geometrized System of Units to the International System we should expect for the following expression:

$$\rho = -\frac{c^2}{G}\frac{vs^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{rs}{2}n''(rs)\right)^2\sin^2\theta\right]. \tag{58}$$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for[21]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{59}$$

In the equatorial plane($1+1$ dimensional spacetime with $rs = x - xs$,$y = 0$ and center of the bubble $xs = 0$):

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{60}$$

Note that in the above expressions the warp drive speed vs appears raised to a power of 2. Considering our Natario warp drive moving with $vs = 200$ which means to say 200 times light speed in order to make a round trip from Earth to a nearby star at 20 light-years away in a reasonable amount of time(in months not in years) we would get in the expression of the negative energy the factor $c^2 = (3\times10^8)^2 = 9\times10^{16}$ being divided by $6,67\times10^{-11}$ giving $1,35\times10^{27}$ and this is multiplied by $(6\times10^{10})^2 = 36\times10^{20}$ coming from the term $vs = 200$ giving $1,35\times10^{27}\times36\times10^{20} = 1,35\times10^{27}\times3,6\times10^{21} = 4,86\times10^{48}$!!!

A number with 48 zeros!!!The planet Earth have a mass[22] of about $6\times10^{24}kg$

This term is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!or better:The amount of negative energy density needed to sustain a warp bubble at a speed of 200 times faster than light requires the magnitude of the masses of 1.000.000.000.000.000.000.000.000 planet Earths for both Alcubierre and Natario cases!!!!

And multiplying the mass of Earth by c^2 in order to get the total positive energy "stored" in the Earth according to the Einstein equation $E = mc^2$ we would find the value of $54\times10^{40} = 5,4\times10^{41}Joules$.

Earth have a positive energy of $10^{41}Joules$ and dividing this by the volume of the Earth(radius R_{Earth} = 6300 km approximately) we would find the positive energy density of the Earth.Taking the cube of the Earth radius $(6300000m = 6,3\times10^6)^3 = 2,5\times10^{20}$ and dividing $5,4\times10^{41}$ by $(4/3)\pi R_{Earth}^3$ we would find the value of $4,77\times10^{20}\frac{Joules}{m^3}$. So Earth have a positive energy density of $4,77\times10^{20}\frac{Joules}{m^3}$ and we are talking about negative energy densities with a factor of 10^{48} for the warp drive while the quantum theory allows only microscopical amounts of negative energy density.

[21]see Appendix A

[22]see Wikipedia:The free Encyclopedia

So we would need to generate in order to maintain a warp drive with 200 times light speed the negative energy density equivalent to the positive energy density of 10^{28} Earths!!!!

A number with 28 zeros!!!.Unfortunately we must agree with the major part of the scientific community that says:"Warp Drive is impossible and unphysical!!".

However looking better to the expression of the negative energy density in the equatorial plane of the Natario warp drive:

$$\rho = T_{\mu\nu} u^{\mu} u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{61}$$

We can see that a very low derivative and hence its square can perhaps obliterate the huge factor of 10^{48} ameliorating the negative energy requirements to sustain the warp drive.

In section 5 we will introduce another Natario shape function that defines the Natario warp drive spacetime and this function allows the reduction of the negative energy density requirements to arbitrary low values completely obliterating the factor 10^{48} which is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!...

Why we cannot integrate the Natario shape function $n(rs)$ defined in the previous section as:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{62}$$

With $f(rs)$ the Alcubierre shape function being:[23].

$$f(rs) = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{63}$$

In order to get the total energy requirements needed to sustain the Natario warp drive??

The square of the derivative of the Alcubierre shape function is given by:

$$f'(rs)^2 = \frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{64}$$

In the equatorial plane $y = 0$ and we can neglect the second order derivative of the Natario shape function and consequently its square.The square of the first order derivative is then given by:

$$n'(rs)^2 = \frac{1}{4}f'(rs)^2 \tag{65}$$

$$n'(rs)^2 = \frac{1}{4}(\frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}]) \tag{66}$$

$$n'(rs)^2 = \frac{1}{16}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{67}$$

[23] $tanh[@(rs + R)] = 1$,$tanh(@R) = 1$ for very high values of the Alcubierre thickness parameter $@ >> |R|$

An interesting feature is the fact that the square of the derivative of the Natario shape function in the equatorial plane is 4 times lower than its Alcubierre counterpart.

Back again to the negative energy density in the Natario warp drive:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{68}$$

The total energy needed to sustain the Natario warp bubble is obtained by integrating the negative energy density ρ over the volume of the Natario warped region.
Since we are in the equatorial plane then only the term in rs accounts for and the total energy integral can be given by:

$$E = \int(\rho)drs = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\int(3(n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs \tag{69}$$

Above we placed the constant terms c,G and vs^2 outside the integral. But

$$\int((n'(rs))^2)drs = \int(\frac{1}{16}[\frac{@^2}{cosh^4[@(rs-R)]}])drs = \frac{@^2}{16}\int([\frac{1}{cosh^4[@(rs-R)]}])drs \tag{70}$$

Since @ ia also a constant.Then the total energy integral for the Natario warp drive is given by:

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@^2}{16}\int([\frac{1}{cosh^4[@(rs-R)]}])drs \tag{71}$$

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@^2}{16}\int([\frac{drs}{cosh^4[@(rs-R)]}]) \tag{72}$$

The result of the integral is given by:[24]

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@}{16}\frac{tanh(@(rs-R))}{3}[\frac{1}{cosh^2(@(rs-R))}+2] \tag{73}$$

Now we must discuss a little bit of warp drive basics:

- 1)-According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

Then inside the bubble and outside the bubble the Natario shape function have always constant or fixed values(0 inside the bubble and $\frac{1}{2}$ outside the bubble) and its derivative is zero.Hence the region where the Natario shape function vary its values resulting in non-null derivatives is the Natario warped region($0 < n(rs) < \frac{1}{2}$) which means to say the walls of the warp bubble.

- 2)-Since the negative energy density depends on non-null derivatives of the Natario shape function this means to say that the negative energy density resides in the Natario warped region(warp bubble walls)[25]

[24]See Appehdix N
[25]See Appendix G for a graphical presentation of the warp bubble walls

Then the region where

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs \neq 0 \tag{74}$$

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@}{16}\frac{tanh(@(rs-R))}{3}[\frac{1}{cosh^2(@(rs-R))}+2] \neq 0 \tag{75}$$

is the Natario warped region$(0 < n(rs) < \frac{1}{2})$ with $n'(rs))^2 \neq 0$

Lets define the beginning of the Natario warped region where $n(rs)$ ceases to be zero as the point a and the end of the Natario warped region where $n(rs)$ is about to reach the value of $\frac{1}{2}$ as the point b.Remember that rs is the path of the so-called Eulerian observer that starts at the center of the bubble $xs = R = rs = 0$ and ends up outside the warp bubble $rs > R$.So we have a certain value for rs in the beginning of the Natario warped region which is a and another value for rs in the end of the Natario warped region which is b.

The difference $b - a$ is the width of the Natario warped region and we must integrate the negative energy density over the width.So a and b are the integration limits of the negative energy definite integral and we should expect for:

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int_a^b((n'(rs))^2)drs \tag{76}$$

Now making:

$$\int((n'(rs))^2)drs = F(rs) \tag{77}$$

By the rules of definite integration we should expect for:

$$\int_a^b((n'(rs))^2)drs = F(b) - F(a) \tag{78}$$

Note that if the difference between b and a is very small close to zero then $b - a \cong 0$ meaning a warped region of very small width or very small thickness.Hence we have $F(b) - F(a) \cong 0$

From the previous section we know that @ is the Alcubierre dimensionless parameter related to the thickness of the bubble which can possesses arbitrary values and as large @ is as thicker or thinner the bubble becomes.Then for a very small thickness or width we must have a thickness parameter $@ >> |R|$ which means to say a very large value for @.

Examining again the negative energy density definite integral with the limits of integration in its proper places:

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int_a^b((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@}{16}|_a^b\frac{tanh(@(rs-R))}{3}[\frac{1}{cosh^2(@(rs-R))}+2] \neq 0 \tag{79}$$

Note that a large @ multiplied by $\frac{c^2}{G}\frac{v_s^2}{8\pi}$(which is 10^{48} for 200 times faster than light) will make the negative energy requirements even worst.The shape function introduced in the previous section is not suitable for a real Natario warp drive spacetime.In section 5 we will present a better function.

4 Horizons,Infinite Doppler Blueshifts and Interstellar Navigation in the Natario Warp Drive Spacetime in a $1+1$ Dimensional Spacetime

The equation of the Natario warp drive spacetime is given by[26]:

$$ds^2 = [1 - (X^{rs})^2 - (X^{\theta})^2]dt^2 + 2[X^{rs}drs + X^{\theta}rsd\theta]dt - drs^2 - rs^2 d\theta^2 \tag{80}$$

The expressions for X^{rs} and X^{θ} in the equation above are given by:(see pg 5 in [2])

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{81}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{82}$$

$$X^{\theta} = v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{83}$$

$$X^{\theta} = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{84}$$

We are interested in the two-dimensional $1+1$ version of the Natario warp drive in the dimensions rs and t (motion over the $x-axis$ only with $\theta = 0$ $cos\theta = 1$ and $\sin\theta = 0.)$ given by:

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 \tag{85}$$

With X^{rs} being given by:

$$X^{rs} = 2v_s n(rs) \tag{86}$$

According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

A Natario warp drive valid shape function can be given by:

$$n(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{87}$$

Its derivative square is :

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{88}$$

The shape function above gives the result of $n(rs) = 0$ inside the warp bubble and $n(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region(see pg 5 in [2]).

Note that the Alcubierre shape function is being used to define its Natario shape function counterpart. The term WF in the Natario shape function is dimensionless too:it is the warp factor.For a while it is important to outline only that the warp factor $WF >> |R|$ is much greater then the modulus of the bubble radius.

[26]see Appendices B and C

For the Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $n(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario bubble).

An Horizon occurs every time an observer in the center of the warp bubble send a photon towards the front of the bubble.The photon will stop somewhere in the Natario warped region never reaching the outermost layers of the Natario warped region which are causally disconnected with respect to the observer in the center of the bubble.This means to say that the observer in the center of the bubble cannot signal or control the outermost regions of the warp bubble[27].

The motion of a photon being sent to the front of the bubble obeys the null-like geodesics ($ds^2 = 0$)

Inserting the condition of the null-like geodesics ($ds^2 = 0$) in the equation of the Natario warp drive spacetime in $1 + 1$ dimensions we have:

$$ds^2 = 0 \dashrightarrow [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 = 0 \tag{89}$$

Solving the quadratic form above for $\frac{dx}{dx}$ being the speed of the photon being sent to the front(or the rear) of the warp bubble we can see that we have two solutions:one for the photon sent towards the front of the ship($-$ sign) and another for the photon sent to the rear of the ship($+$ sign). [28]

$$\frac{drs_{front}}{dt} = X^{rs} - 1 \tag{90}$$

$$\frac{drs_{rear}}{dt} = X^{rs} + 1 \tag{91}$$

$$X^{rs} = 2v_s n(rs) \tag{92}$$

We are interested in the result of the photon being sent to the front.

The Natario shape function $n(rs)$ is defined as being 0 inside the warp bubble and $\frac{1}{2}$ outside the warp bubble while being $\frac{1}{2} > n(rs) > 0$ in the Natario warped region according to pg 5 in [2].

Solving the Natario warp drive equation in a $1 + 1$ spacetime for a null-like interval $ds^2 = 0$ we will have the following equation for the motion of the photon sent to the front:

$$\frac{dx}{dt} = 2vsn(rs) - 1 \tag{93}$$

Inside the Natario warp bubble $n(rs) = 0$ and $2vsn(rs) = 0$.
Outside the Natario warp bubble $n(rs) = \frac{1}{2}$ and $2vsn(rs) = vs$.

Somewhere inside the Natario warped region when $n(rs)$ starts to increase from 0 to $\frac{1}{2}$ making the term $2vsn(rs)$ increases from 0 to vs and assuming a continuous behavior then in a given point $2vsn(rs) = 1$ and $\frac{dx}{dt} = 0$.The photon stops,A Horizon is established

[27] at least using photons.
[28] See Appendix D

The value of the Natario shape function when or where the photon stops is given by[29]:

$$n(rs) = \frac{1}{2vs} \tag{94}$$

Note that for superluminal speeds $(vs > 1)(c = 1)$ as higher the speed as close to zero $n(rs)$ becomes.So as higher the speed $(vs >>> 1)(vs- > +\infty)$ the point where the photon stops. approaches zero[30][31]

$$\lim_{vs->+\infty}(\frac{1}{2vs}) = 0 \tag{95}$$

The exact position rs of the so-called Eulerian observer where(or when) the photon stops in the Horizon of the $1+1$ Natario warp drive spacetime is given by:

$$rs = R + \frac{1}{@} arctanh(1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) \tag{96}$$

As higher the thickness parameter @ becomes and also as higher the warp factor WF becomes as thicker or thinner the warp bubble becomes too.(see pgs 8,10 and 11 in [9])

The term $(\frac{1}{@} \simeq 0)$ as long as $(@ \simeq +\infty)$ and the terms $(\sqrt[WF]{\frac{1}{vs}} \simeq 0)$ and $(\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}} \simeq 1)$ giving $(2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}} \simeq 2)$ as long as $(vs \simeq +\infty)$

But $(\sqrt[WF]{\frac{1}{vs}} > 0)$[32] and $(1 - \sqrt[WF]{\frac{1}{vs}} \simeq 1)$ although we will always have the condition $(1 - \sqrt[WF]{\frac{1}{vs}} < 1)$ implying in $(\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}} < 1)$ and in consequence $(2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}} < 2)$ however we can clearly see that $(2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}} > 1)$ as long as $vs >> 1$ superluminal.

From the conclusions given above we can see that $(1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}} < 0)$

So we can expect the following results concerning the position of the Eulerian observer in the Horizon of the $1+1$ Natario warp drive spacetime:

$$\frac{1}{@} \simeq 0 \tag{97}$$

$$arctanh(1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) < 0 \tag{98}$$

$$\frac{1}{@} arctanh(1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) << 0 \dashrightarrow \frac{1}{@} arctanh(1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) \simeq 0 \tag{99}$$

[29] See Appendix E

[30] value of $n(rs)$ inside the bubble

[31] although $(vs- > +\infty)$ we will always have the condition $(vs < +\infty)$ then vs will never reach (∞) so $(\frac{1}{2vs} - > 0)$ but will never reaches zero and the condition $(\frac{1}{2} > \frac{1}{2vs} > 0)$ will always remains valid keeping the Horizon inside the Natario warped region

[32] again $(vs- > +\infty)$ and $(vs < +\infty)$ then vs will never reach (∞)

$$rs = R + \frac{1}{@} arctanh(1 - 2 \sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) < R \dashrightarrow rs = R + \frac{1}{@} arctanh(1 - 2 \sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) \simeq R \quad (100)$$

The photon never stops outside the bubble ($rs > R$) and not over the bubble radius ($rs = R$).It stops somewhere inside the Natario warped region where ($\frac{1}{2} > n(rs) > 0$) because ($n(rs) = \frac{1}{2vs}$) being ($0 < \frac{1}{2vs} < \frac{1}{2}$) as long as ($vs >> 1$).

-)-For a while we will neglect the geometrical distribution of negative energy density in the the $1+1$ dimensional Natario warp drive spacetime when a photon is sent to the front of the bubble:

We are ready now to examine the topic of the Infinite Doppler Blueshifts in the Horizon suffered by photons sent towards the front of the warp bubble.(See pg 6 and 8 in [2]) [33]

The unit vector n defined below represents the direction of the corresponding light ray from the point of view of the Eulerian observer outside the warp bubble.(See pg 6 in [2])

$$n = \frac{dx}{dx} - 2vsn(rs) \quad (101)$$

In the Horizon ($\frac{dx}{dx} = 0$) and ($2vsn(rs) = 1$)
Then we have:

$$n = \frac{dx}{dx} - 2vsn(rs) = 0 - 1 = -1 \quad (102)$$

The equation of the observed energy is given by(See pg 8 in [2]):

$$E_0 = E(1 + n.vsf(rs)) \quad (103)$$

So when a photon reaches the Horizon we have the following conditions: $n = -1$ and $2vsn(rs) = 1$.Inserting these values in the equation of the energy(pg 8 in [2]) we have:

$$E = \frac{E_0}{(1 + n.vsf(rs))} = \frac{E_0}{(1 + -1.1)} = \frac{E_0}{0} \quad (104)$$

And then we have the Infinite Doppler Blueshift in the Horizon as mentioned by Natario in pg 8 in [2].

-)-Considering now the geometrical distribution of negative energy density in the $1+1$ dimensional Natario warp drive spacetime when a photon is sent to the front of the bubble:

The infinite Doppler Blueshift do not happens in the Natario warp drive spacetime.This means to say that the Natario warp drive is stable and perfectly physically possible to be achieved.

[33]See Appendix F

Consider again the negative energy density in the Natario warp drive spacetime(see pg 5 in [2])[34,35]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{105}$$

In pg 6 in [2] a warp drive with a x-axis only (a $1+1$ dimensional warp drive)is considered.In this case $[y^2 + z^2] = 0$ but the Natario energy density is not zero and given by:.

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2\right] \tag{106}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{107}$$

Note that in front of the ship in the Natario warp drive spacetime even in the $1+1$ dimensional case there exists negative energy density in the front of the ship.This negative energy density deflects the photon before the point of the Horizon.

The amount of negative energy density in the Horizon can be computed inserting the value of the rs in the Horizon into the negative energy densxity equation

$$rs = R + \frac{1}{@}arctanh(1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) \tag{108}$$

And the negative energy density that exists in the Natario warped region before the Horizon point will deflect the photon before the Horizon point.

So Infinite Doppler Blueshifts can generate instabilities but the these instabilities never happens in the Natario warp drive spacetime because the photon never reaches the Horizon.The photon is deflected by the negative energy in front of the ship.

Still according with Natario in pg 7 before section 5.2 in [3] negative energy density means a negative mass density and a negative mass density generates a repulsive gravitational field that deflects the hazardous interstellar matter in the front of the warp bubble. This repulsive gravitational field in front of the ship in the Natario warp drive spacetime protects the ship from impacts with the interstellar matter and also protects the ship from photons highly Doppler Blueshifted approaching the ship from the front.So in the Natario warp drive the problem of the collisions with dangerous interstellar matter never happens.[36]

[34] $n(rs)$ is the Natario shape function.Equation written the Geometrized System of Units $c = G = 1$
[35] Equation written in Cartezian Coordinates.See Appendix A
[36] See Appendix M

Adapted from the negative mass in Wikipedia:The free Encyclopedia:

"if we have a small object with equal inertial and passive gravitational masses falling in the gravitational field of an object with negative active gravitational mass (a small mass dropped above a negative-mass planet, say), then the acceleration of the small object is proportional to the negative active gravitational mass creating a negative gravitational field and the small object would actually accelerate away from the negative-mass object rather than towards it."

5 Reducing the Negative Energy Density Requirements in the Natario Warp Drive in a $1+1$ Dimensional Spacetime

Now we are ready to demonstrate how the negative energy density requirements can be greatly reduced for the Natario warp drive in a $1+1$ dimensional spacetime:

We already know the form of the equation of the Natario warp drive in a $1+1$ dimensional spacetime:[37]

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 \tag{109}$$

$$X^{rs} = 2v_s n(rs) \tag{110}$$

According to Natario(pg 5 in [2]) any function that gives 0 inside the bubble and $\frac{1}{2}$ outside the bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region is a valid shape function for the Natario warp drive.

A Natario warp drive valid shape function can be given by:

$$n(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{111}$$

Its derivative square is :

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{112}$$

The shape function above gives the result of $n(rs) = 0$ inside the warp bubble and $n(rs) = \frac{1}{2}$ outside the warp bubble while being $0 < n(rs) < \frac{1}{2}$ in the Natario warped region(see pg 5 in [2]).

Note that the Alcubierre shape function $f(rs)$ is being used to define its Natario shape function counterpart. The term WF in the Natario shape function is dimensionless too:it is the warp factor that will squeeze the region where the derivatives of the Natario shape function are different than 0.The warp factor is always a fixed integer number directly proportional to the modulus of the bubble radius.$WF > |R|$.

For the Natario shape function introduced above it is easy to figure out when $f(rs) = 1$(interior of the Alcubierre bubble) then $n(rs) = 0$(interior of the Natario bubble) and when $f(rs) = 0$(exterior of the Alcubierre bubble)then $n(rs) = \frac{1}{2}$(exterior of the Natario bubble).

[37] See Appendix B

We must analyze the differences between this new Natario shape function with warp factors compared to the original Natario shape function presented in Section 2 and mainly the differences between their derivative squares essential to low the negative energy density requirements in the $1+1$ Natario warp drive spacetime.In order to do so we need to use the Alcubierre shape function.

- 1)-Alcubierre shape function and its derivative square:[38].

$$f(rs) = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{113}$$

$$f'(rs)^2 = \frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{114}$$

- 2)-original Natario shape function and its derivative square:

$$n(rs) = \frac{1}{2}[1 - f(rs)] \tag{115}$$

$$n'(rs)^2 = \frac{1}{16}[\frac{@^2}{cosh^4[@(rs - R)]}] \tag{116}$$

- 3)-Natario shape function with warp factors and its derivative square:

$$n(rs) = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{117}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}]f'(rs)^2 \tag{118}$$

$$n'(rs)^2 = [\frac{1}{4}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{1}{4}[\frac{@^2}{cosh^4[@(rs - R)]}]] \tag{119}$$

$$n'(rs)^2 = [\frac{1}{16}]WF^4[1 - f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs - R)]}] \tag{120}$$

- 4)-negative energy density in the $1+1$ Natario warp drive spacetime:

$$\rho = T_{\mu\nu}u^\mu u^\nu = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{121}$$

We already know that the region where the negative energy density is concentrated is the warped region in both Alcubierre ($1 > f(rs) > 0$) and Natario ($0 < n(rs) < \frac{1}{2}$) cases.

And we also know that for a speed of 200 times light speed the negative energy density is directly proportional to 10^{48} resulting from the term $\frac{c^2}{G}\frac{v_s^2}{8\pi}$.

So in order to get a physically feasible Natario warp drive the derivative of the Natario shape function must obliterate the factor 10^{48}.

[38] $tanh[@(rs + R)] = 1$,$tanh(@R) = 1$ for very high values of the Alcubierre thickness parameter $@ >> |R|$

Examining first the negative energy density from the original Natario shape function:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{122}$$

$$n'(rs)^2 = \frac{1}{16}[\frac{@^2}{cosh^4[@(rs-R)]}] \tag{123}$$

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[\frac{3}{16}[\frac{@^2}{cosh^4[@(rs-R)]}]\right] \tag{124}$$

We already know from section 3 that @ is the Alcubierre parameter related to the thickness of the bubble and a large $@ > |R|$ means a bubble of very small thickness.On the other hand a small value of $@ < |R|$ means a bubble of large thickness.But @ cannot be zero and cannot be $@ << |R|$ so independently of the value of @ the factor $\frac{c^2}{G}\frac{v_s^2}{8\pi}$ still remains with the factor 10^{48} from 200 times light speed which is being multiplied by @'2 making the negative energy density requirements even worst!!

Examining now the negative energy density from the Natario shape function with warp factors:

$$n'(rs)^2 = [\frac{1}{16}]WF^4[1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs-R)]}] \tag{125}$$

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[[\frac{3}{16}]WF^4[1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs-R)]}]\right] \tag{126}$$

Comparing both negative energy densities we can clearly see that the differences between the equations is the term resulting from the warp factor which is:

$$WF^4[1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}] \tag{127}$$

Inside the bubble $f(rs) = 1$ and $[1-f(rs)^{WF}]^{2(WF-1)} = 0$ resulting in a $n'(rs)^2 = 0$.This is the reason why the Natario shape function with warp factors do not have derivatives inside the bubble.

Outside the bubble $f(rs) = 0$ and $[f(rs)^{2(WF-1)}] = 0$ resulting also in a $n'(rs)^2 = 0$.This is the reason why the Natario shape function with warp factors do not have derivatives outside the bubble.

Using the Alcubierre warped region we have:

In the Alcubierre warped region $1 > f(rs) > 0$.In this region the derivatives of the Natario shape function do not vanish because if $f(rs) < 1$ then $f(rs)^{WF} << 1$ resulting in an $[1-f(rs)^{WF}]^{2(WF-1)} << 1$ .Also if $f(rs) < 1$ then $[f(rs)^{2(WF-1)}] << 1$ too if we have a warp factor $WF > |R|$.

Note that if $[1-f(rs)^{WF}]^{2(WF-1)} << 1$ and $[f(rs)^{2(WF-1)}] << 1$ their product
$[1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}] <<<< 1$

Note that inside the Alcubierre warped region $1 > f(rs) > 0$ when $f(rs)$ approaches 1 $n'(rs)^2$ approaches 0 due to the factor $[1-f(rs)^{WF}]^{2(WF-1)}$ and when $f(rs)$ approaches 0 $n'(rs)^2$ approaches 0 again due to the factor $[f(rs)^{2(WF-1)}]$.

Back again to the negative energy density using the Natario shape function with warp factors:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{c^2}{G}\frac{v_s^2}{8\pi}\left[[\frac{3}{16}]WF^4[1-f(rs)^{WF}]^{2(WF-1)}[f(rs)^{2(WF-1)}][\frac{@^2}{cosh^4[@(rs-R)]}]\right] \tag{128}$$

Independently of the thickness parameter @ or the bubble radius R for a warp factor $WF = 500$ we have the following situations considering the Alcubierre warped region $1 > f(rs) > 0$:

- 1)-in the beginning of the Alcubierre warped region when $f(rs) = 0,9$ then $[f(rs)^{2(WF-1)}] = [(0,9)^{2(500-1)}] = (0,9)^{2(499)} = (0,9)^{998} = 2,157865742868E-046$
- 2)-in the middle of the Alcubierre warped region when $f(rs) = 0,5$ then $[f(rs)^{2(WF-1)}] = [(0,5)^{2(500-1)}] = (0,5)^{2(499)} = (0,5)^{998} = 3,733054474013E-301$
- 3)-in the end of the Alcubierre warped region when $f(rs) = 0,1$ then $[f(rs)^{2(WF-1)}] = [(0,1)^{2(500-1)}] = (0,1)^{2(499)} = (0,1)^{998} = 0,000000000000E+000$

Note that the Natario shape function with warp factors completely obliterated the term $\frac{c^2}{G}\frac{v_s^2}{8\pi}$ with the factor 10^{48} from 200 times light speed making the negative energy density requirements physically feasible!!

And remember that 10^{48} is 1.000.000.000.000.000.000.000.000 times bigger in magnitude than the mass of the planet Earth!!!

6 A Causally Connected Superluminal Natario Warp Drive Spacetime using Micro Warp Bubbles

In 2002 Gauthier,Gravel and Melanson appeared with the idea of the micro warp bubbles.([7],[8])

According to them,microscopical particle-sized warp bubbles may have formed spontaneously immediately after the Big Bang and these warp bubbles could be used to transmit information at superluminal speeds.These micro warp bubbles may exist today.(see abs of [8])

A micro warp bubble with a radius of 10^{-10} meters could be used to transport an elementary particle like the electron whose Compton wavelength is 2.43×10^{-12} meters at a superluminal speed.These micro warp bubbles may have formed when the Universe had an age between the Planck time and the time we assume that Inflation started.(see pg 306 of [7])

Following the ideas of Gauthier,Gravel and Melanson ([7],[8]) a micro warp bubble can send information or particles at superluminal speeds.(abs of [8],pg 306 in [7]).

The idea of Gauthier,Gravel and Melanson ([7],[8]) to send information at superluminal speeds using micro warp bubbles is very interesting and as a matter of fact shows to us how to solve the Horizon problem. Imagine that we are inside a large superluminal warp bubble and we want to send information to the front.Photons sent from inside the bubble to the front would stop in the Horizon but we also know that incoming photons from outside would reach the bubble.[39] The external observer outside the bubble have all the bubble causally connected while the internal observer is causally connected to the point before the Horizon.Then the external observer can create the bubble while the internal observer cannot. This was also outlined by Everett-Roman in pg 3 in [6]. Unless we find a way to overcome the Horizon problem. We inside the large warp bubble could create and send one of these micro warp bubbles to the front of the large warp bubble but with a superluminal speed $vs2$ larger than the large bubble speed $X = 2vsn(rs)$.Then $vs2 >> X$ or $vs2 >> 2vsn(rs)$ and this would allow ourselves to keep all the warp bubble causally connected from inside overcoming the Horizon problem without the need of the "tachyonic" matter.

- 1)- Superluminal micro warp bubble sent towards the front of the large superluminal warp bubble $vs2 = \frac{dx}{dt} > X - 1 > vs - 1 \dashrightarrow X = 2vsn(rs)$

From above it easy to see that a micro warp bubble with a superluminal speed $vs2$ maintains a large superluminal warp bubble with speed vs causally connected from inside if $vs2 > vs$

[39] not true for the Natario warp drive because the negative energy density in the front with repulsive gravitational behavior would deflect all the photons sent from inside and outside the bubble effectively shielding the Horizon from the photon avoiding the catastrophical Infinite Doppler Blueshift

From the point of view of the astronaut inside the large warp bubble he is the internal observer with respect to the large warp bubble but he is the external observer from the point of view of the micro warp bubble so he keeps all the light-cone of the micro warp bubble causally connected to him so he can use it to send superluminal signals to the large warp bubble from inside.(Everett-Roman in pg 3 in [6]).

According with Natario in pg 7 before section 5.2 in [3] negative energy density means a negative mass density and a negative mass generates a repulsive gravitational field that deflect photons or positive mass-density particles from the interstellar medium or particles sent to the bubble walls by the astronaut inside the bubble.

However while the negative mass deflects the positive mass or photons[40] a negative mass always attracts another negative mass so the astronaut cannot send positive particles or photons to the large warp bubble but by sending micro warp bubbles these also possesses negative masses that will be attracted by the negative mass of the large warp bubble effectively being a useful way to send signals.

[40]by negative gravitational bending of light

7 The Star System Gliese 667C(GJ 667C)

In June 2013 a team of astronomers from *ESO* (European Southern Observatory) La Silla Chile announced a major breakthrough in exo-planetary astronomy. They discovered 3 planets of the class Super Earth[41] orbiting the Habitable Zone(HZ)[42] of the star system $Gliese667C$ ($GJ667C$).(see fig 15 pg 19 in [12],fig 15 pg 18 in [13])located at 22 light years from Earth in the constellation of Scorpion ($6,8$ parsecs see pg 2 in [12],[13])

It was the first time a planetary system was spotted with 3 planets in the HZ of a star and all of them are larger and heavier than Earth[43] and the *ESO* team suspect that a fourth planet still lies within the limits of the HZ(see abs of [12],[13])[44].So each one of them have gravity enough to retain atmospheres richer in oxygen and carbon dioxide able to sustain life as we know.

How remarkable is this discovery?

$Gliese667C$ is a red dwarf spectral class M and according to the *ESO* team there exists in our galaxy a large population of M stars with multiple planets in their HZ (see abs and pg 2 in [12],[13],see also pg 19 in [13],pg 20 in [12]) unlike or Sun which is a G spectral class with only one planet in its HZ(Earth) and more than 90 percent of the planetary mass of our star system is concentrated on Jupiter(1200 Earth masses) and Saturn(700 Earth masses) well outside the HZ of the Sun.According to the *ESO* team M stars are better candidates than G stars to look for planets in the HZ. Our galaxy (Milky Way) have nearly 400 billions of stars where 260 billions are red dwarfs class M 100 billion are yellow stars class G like the Sun and the remaining 40 billions are red giants,blue giants,white dwarfs,brown dwarfs,neutron stars,quark stars,black holes etc.The population of M stars in our galaxy is more than twice the population of G stars so if the M stars are the larger number population in our galaxy and the great majority of them have multiple planets in their HZ then the possibility or probability to find life outside Earth raised drastically and exponentially:Think on 260 billions of M stars with 3 to 4 planets in the HZ of each star.Still according to the *ESO* team they hope $GJ667C$ is the first of many systems around M stars with multiple HZ planets still waiting to be discovered. (see also pg 19 in [13],pg 20 in [12]).

Lets illustrate how important is the discovery of $Gliese667C$

Consider a star with only one planet in its HZ(eg the Sun).The probability of existence of liquid water in such a planet is 50 percent.

In short!!!:Or water exists or simply do not exists!!!.So the probability is "fifty-fifty".

Fortunately in the early Earth history the water appeared here due to happy circumstances:Water coming from vapors from lava of ancient volcanoes.So for the Earth the "fifty-fifty" worked with positive results and happy results for ourselves!!!!.

[41] Heavier than Earth but lighter than Neptune

[42] The zone of a star not too hot and not too cold where water can exists in liquid state able to sustain life as we know.The HZ is sometimes called ecosphere and HZ planets are often called "goldilocks" planets.

[43] See Appendix P

[44] See Appendix O

But imagine that the vapors from the lava of the ancient Earth volcanoes were of sulphuric acid for example like the lava from the volcanoes of the Jupiter moon Io.Earth would possesses by now oceans of sulphuric acid and life as we know would be impossible.This scenario would depict a "fifty-fifty" with negative results and unhappy results for ourselves!!!!!.

Now consider $Gliese667C$:since there are 3 candidates in the HZ there are three opportunities of planets with water so there are 3 opportunities for the "fifty-fifty".Three opportunities are better than a single one and we would appreciate positive "fifty-fifty" results in all of them in an optimistic perspective of course but perhaps at least one of them may have resulted positive.

And what is most attractive:$Gliese667C$ lies "only" at 22 light-years of distance so a spaceship equipped with a Natario warp drive travelling at 200 times light speed would reach these planets in a month and half.

We wrote this work on the Natario warp drive theory with all the required mathematical techniques that although familiar to people acquainted with warp drives these techniques are required to get a better understanding of the theory in order to be read by people with mathematical background but not acquainted with warp drives and we mean specially exo-planetary astronomers perhaps to promote the warp drive theory among this important community of scientists and mainly to captive their interest by the Natario warp drive theory

8 Conclusion

In this work we described the Natario warp drive theory in an explanation destined to scientists with mathematical background but not familiarized with warp drives in order to promote the Natario warp drive theory among important communities of scientists outside the scope of those already familiarized with warp drives and we mean specially exo-planetary astronomers.

We introduced all the mathematical techniques needed to fully understand the Natario warp drive to transmit a solid, consistent and well-established theory.These techniques are not new and as a matter of fact this work is a compilation of work already done by Natario in order to provide here a clear and large coverage of this topic.

In order to illustrate how a spaceship inside a warp bubble is completely at the rest with respect to a frame in its local neighborhoods but it is seen by a remote distant observer faraway from the bubble with a speed of 200 times faster than light we used the metaphorical example of the fish and the aquarium.The warp bubble is the aquarium and the bubble walls are the glass of the aquarium floating in the stream of a river.An observer in the margin would see the aquarium passing by him at a large speed being carried out by the stream but inside the aquarium the fish is at the rest free from g-forces or accelerations.The warp drive distortion of the spacetime act like a river stream in the Natario model carrying the warp bubble with the spaceship inside at the rest but a distant observer sees the bubble passing by him at a large speed and in our case 200 times light speed.

We discussed the 3 major physical problems needed to be solved in order to make the Natario warp drive a suitable way for superluminal interstellar travel and we presented some theoretical solutions that may accomplish our goal.

For the negative energy density needed to travel at 200 times light speed we lowered the total amount from 10^{48} which is 1.000.000.000.000.000.000.000.000 the mass of the Earth to arbitrary low levels using a Natario shape function with warp factors derived from the modulus of the bubble radius.

Also we demonstrated that the problems of interstellar navigation and impacts with hazardous objects(comets asteroids,photons) that exists in interstellar space can easily be circumvented in the Natario warp drive.

Horizons and causally disconnected portions of spacetime will need to wait for a theory that encompasses both General Relativity and the Non-Locality of Quantum Mechanics in order to be solved but perhaps General Relativity can by itself solve the problem if we really discover how to assemble warp bubbles from a negative energy density concentration.

Lastly we presented the star system $Gliese667C$ with 3 exo-planets in the Habitable Zone (HZ).It was the first time a planetary system of this kind was ever found and we hope that $Gliese667C$ is the first of many more systems still waiting to be discovered $Gliese667C$ is a red dwarf class M and M stars accounts for 260 billions of stars in our galaxy each one perhaps with multiple exo-planets in their HZ so the probability or possibility of life outside Earth raised dramatically and exponentially.[45]

[45] See Appendices O and Q

This work was an attempt to make a "bridge" [46] between warp drives from the point of view of General Relativity and Exo-Planetary Astronomy.Why would we need warp drives to travel at 200 times light speed if not to visit planets able to sustain life orbiting other stars ???

But unfortunately we still dont know how to generate the negative energy density and negative mass and above everything else we dont know how to generate the shape function that distorts the spacetime geometry creating the warp drive effect.So unfortunately all the discussions about warp drives are still under the domain of the mathematical conjectures.

However we are confident to affirm that the Natario warp drive will survive the passage of the Century XXI and will arrive to the Future.The Natario warp drive as a valid candidate for faster than light interstellar space travel will arrive to the the Century $XXIV$ on-board the future starships up there in the middle of the stars helping the human race to give his first steps in the exploration of our Galaxy

The title of this work is a question:

Is the Natario warp drive a valid candidate for an interstellar voyage to the star system $Gliese667C(GJ667C)$??

Our answer is:certainly yes!!

Live Long And Prosper

[46] warp drives and exo-planetary astronomy are "brother" sciences and "brothers" must walk together and with given hands

9 Appendix A:The Natario Warp Drive Negative Energy Density in Cartezian Coordinates

The negative energy density according to Natario is given by(see pg 5 in [2])[47]:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\cos^2\theta + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2\sin^2\theta\right] \tag{129}$$

In the bottom of pg 4 in [2] Natario defined the x-axis as the polar axis.In the top of page 5 we can see that $x = rs\cos(\theta)$ implying in $\cos(\theta) = \frac{x}{rs}$ and in $\sin(\theta) = \frac{y}{rs}$

Rewriting the Natario negative energy density in cartezian coordinates we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2(\frac{x}{rs})^2 + \left(n'(rs) + \frac{r}{2}n''(rs)\right)^2(\frac{y}{rs})^2\right] \tag{130}$$

Considering motion in the equatorial plane of the Natario warp bubble (x-axis only) then $[y^2 + z^2] = 0$ and $rs^2 = [(x - xs)^2]$ and making $xs = 0$ the center of the bubble as the origin of the coordinate frame for the motion of the Eulerian observer then $rs^2 = x^2$ because in the equatorial plane $y = z = 0$.

Rewriting the Natario negative energy density in cartezian coordinates in the equatorial plane we should expect for:

$$\rho = T_{\mu\nu}u^{\mu}u^{\nu} = -\frac{1}{16\pi}K_{ij}K^{ij} = -\frac{v_s^2}{8\pi}\left[3(n'(rs))^2\right] \tag{131}$$

[47] $n(rs)$ is the Natario shape function.Equation written in the Geometrized System of Units $c = G = 1$

10 Appendix B:Mathematical Demonstration of the Natario Warp Drive Equation using the Natario Vector nX for a constant speed vs

The warp drive spacetime according to Natario is defined by the following equation but we changed the metric signature from $(-,+,+,+)$ to $(+,-,-,-)$(pg 2 in [2])

$$ds^2 = dt^2 - \sum_{i=1}^{3}(dx^i - X^i dt)^2 \tag{132}$$

where X^i is the so-called shift vector.This shift vector is the responsible for the warp drive behavior defined as follows(pg 2 in [2]):

$$X^i = X, Y, Z \curvearrowright i = 1, 2, 3 \tag{133}$$

The warp drive spacetime is completely generated by the Natario vector nX(pg 2 in [2])

$$nX = X^i \frac{\partial}{\partial x^i} = X\frac{\partial}{\partial x} + Y\frac{\partial}{\partial y} + Z\frac{\partial}{\partial z}, \tag{134}$$

Defined using the canonical basis of the Hodge Star in spherical coordinates as follows(pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r\sin\theta d\varphi) \tag{135}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial \theta} \sim rd\theta \sim (r\sin\theta d\varphi) \wedge dr \tag{136}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial \varphi} \sim r\sin\theta d\varphi \sim dr \wedge (rd\theta) \tag{137}$$

Redefining the Natario vector nX as being the rate-of-strain tensor of fluid mechanics as shown below(pg 5 in [2]):

$$nX = X^r \mathbf{e}_r + X^\theta \mathbf{e}_\theta + X^\varphi \mathbf{e}_\varphi \tag{138}$$

$$nX = X^r dr + X^\theta r d\theta + X^\varphi r \sin\theta d\varphi \tag{139}$$

$$ds^2 = dt^2 - \sum_{i=1}^{3}(dx^i - X^i dt)^2 \tag{140}$$

$$X^i = r, \theta, \varphi \curvearrowright i = 1, 2, 3 \tag{141}$$

We are interested only in the coordinates r and θ according to pg 5 in [2])

$$ds^2 = dt^2 - (dr - X^r dt)^2 - (rd\theta - X^\theta dt)^2 \tag{142}$$

$$(dr - X^r dt)^2 = dr^2 - 2X^r dr dt + (X^r)^2 dt^2 \tag{143}$$

$$(rd\theta - X^{\theta}dt)^2 = r^2 d\theta^2 - 2X^{\theta} r d\theta dt + (X^{\theta})^2 dt^2 \tag{144}$$

$$ds^2 = dt^2 - (X^r)^2 dt^2 - (X^{\theta})^2 dt^2 + 2X^r dr dt + 2X^{\theta} r d\theta dt - dr^2 - r^2 d\theta^2 \tag{145}$$

$$ds^2 = [1 - (X^r)^2 - (X^{\theta})^2]dt^2 + 2[X^r dr + X^{\theta} r d\theta]dt - dr^2 - r^2 d\theta^2 \tag{146}$$

making $r = rs$ we have the Natario warp drive equation:

$$ds^2 = [1 - (X^{rs})^2 - (X^{\theta})^2]dt^2 + 2[X^{rs} drs + X^{\theta} rs d\theta]dt - drs^2 - rs^2 d\theta^2 \tag{147}$$

According with the Natario definition for the warp drive using the following statement(pg 4 in [2]):any Natario vector nX generates a warp drive spacetime if $nX = 0$ and $X = vs = 0$ for a small value of rs defined by Natario as the interior of the warp bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ with $X = vs$ for a large value of rs defined by Natario as the exterior of the warp bubble with $vs(t)$ being the speed of the warp bubble

The expressions for X^{rs} and X^{θ} are given by:(see pg 5 in [2])

$$nX \sim -2v_s n(rs)\cos\theta \mathbf{e}_{rs} + v_s(2n(rs) + (rs)n'(rs))\sin\theta \mathbf{e}_{\theta} \tag{148}$$

$$nX \sim 2v_s n(rs)\cos\theta \mathbf{e}_{rs} - v_s(2n(rs) + (rs)n'(rs))\sin\theta \mathbf{e}_{\theta} \tag{149}$$

$$nX \sim -2v_s n(rs)\cos\theta drs + v_s(2n(rs) + (rs)n'(rs))\sin\theta rs d\theta \tag{150}$$

$$nX \sim 2v_s n(rs)\cos\theta drs - v_s(2n(rs) + (rs)n'(rs))\sin\theta rs d\theta \tag{151}$$

But we already know that the Natario vector nX is defined by(pg 2 in [2]):

$$nX = X^{rs} drs + X^{\theta} rs d\theta \tag{152}$$

Hence we should expect for:

$$X^{rs} = -2v_s n(rs)\cos\theta \tag{153}$$

$$X^{rs} = 2v_s n(rs)\cos\theta \tag{154}$$

$$X^{\theta} = v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{155}$$

$$X^{\theta} = -v_s(2n(rs) + (rs)n'(rs))\sin\theta \tag{156}$$

We are interested in the two-dimensional $1+1$ version of the Natario warp drive in the dimensions rs and t (motion over the $x - axis$ only with $\theta = 0$ $cos\theta = 1$ and $\sin\theta = 0$) given by:

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs} drs dt - drs^2 \tag{157}$$

$$X^{rs} = 2v_s n(rs) \tag{158}$$

11 Appendix C:Differential Forms,Hodge Star and the Mathematical Demonstration of the Natario Vectors $nX = -vsdx$ and $nX = vsdx$ for a constant speed vs

This appendix is being written for novice or newcomer students on Warp Drive theory still not acquainted with the methods Natario used to arrive at the final expression of the Natario Vector nX

The Canonical Basis of the Hodge Star in spherical coordinates can be defined as follows(pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta) \wedge (r\sin\theta d\varphi) \sim r^2 \sin\theta (d\theta \wedge d\varphi) \tag{159}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi) \wedge dr \sim r\sin\theta (d\varphi \wedge dr) \tag{160}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr \wedge (rd\theta) \sim r(dr \wedge d\theta) \tag{161}$$

From above we get the following results

$$dr \sim r^2 \sin\theta (d\theta \wedge d\varphi) \tag{162}$$

$$rd\theta \sim r\sin\theta (d\varphi \wedge dr) \tag{163}$$

$$r\sin\theta d\varphi \sim r(dr \wedge d\theta) \tag{164}$$

Note that this expression matches the common definition of the Hodge Star operator * applied to the spherical coordinates as given by(pg 8 in [15]):

$$*dr = r^2 \sin\theta (d\theta \wedge d\varphi) \tag{165}$$

$$*rd\theta = r\sin\theta (d\varphi \wedge dr) \tag{166}$$

$$*r\sin\theta d\varphi = r(dr \wedge d\theta) \tag{167}$$

Back again to the Natario equivalence between spherical and cartezian coordinates(pg 5 in [2]):

$$\frac{\partial}{\partial x} \sim dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \sim r^2 \sin\theta\cos\theta d\theta \wedge d\varphi + r\sin^2\theta dr \wedge d\varphi = d\left(\frac{1}{2}r^2 \sin^2\theta d\varphi\right) \tag{168}$$

Look that

$$dx = d(r\cos\theta) = \cos\theta dr - r\sin\theta d\theta \tag{169}$$

Or

$$dx = d(r\cos\theta) = \cos\theta dr - \sin\theta r d\theta \tag{170}$$

Applying the Hodge Star operator * to the above expression:

$$*dx = *d(r\cos\theta) = \cos\theta(*dr) - \sin\theta(*rd\theta) \tag{171}$$

$$*dx = *d(r\cos\theta) = \cos\theta[r^2\sin\theta(d\theta\wedge d\varphi)] - \sin\theta[r\sin\theta(d\varphi\wedge dr)] \tag{172}$$

$$*dx = *d(r\cos\theta) = [r^2\sin\theta\cos\theta(d\theta\wedge d\varphi)] - [r\sin^2\theta(d\varphi\wedge dr)] \tag{173}$$

We know that the following expression holds true(see pg 9 in [14]):

$$d\varphi\wedge dr = -dr\wedge d\varphi \tag{174}$$

Then we have

$$*dx = *d(r\cos\theta) = [r^2\sin\theta\cos\theta(d\theta\wedge d\varphi)] + [r\sin^2\theta(dr\wedge d\varphi)] \tag{175}$$

And the above expression matches exactly the term obtained by Natario using the Hodge Star operator applied to the equivalence between cartezian and spherical coordinates(pg 5 in [2]).

Now examining the expression:

$$d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{176}$$

We must also apply the Hodge Star operator to the expression above

And then we have:

$$*d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \tag{177}$$

$$*d\left(\frac{1}{2}r^2\sin^2\theta d\varphi\right) \sim \frac{1}{2}r^2 * d[(\sin^2\theta)d\varphi] + \frac{1}{2}\sin^2\theta * [d(r^2)d\varphi] + \frac{1}{2}r^2\sin^2\theta * d[(d\varphi)] \tag{178}$$

According to pg 10 in [14] the term $\frac{1}{2}r^2\sin^2\theta * d[(d\varphi)] = 0$

This leaves us with:

$$\frac{1}{2}r^2 * d[(\sin^2\theta)d\varphi] + \frac{1}{2}\sin^2\theta * [d(r^2)d\varphi] \sim \frac{1}{2}r^2 2\sin\theta\cos\theta(d\theta\wedge d\varphi) + \frac{1}{2}\sin^2\theta 2r(dr\wedge d\varphi) \tag{179}$$

Because and according to pg 10 in [14]:

$$d(\alpha+\beta) = d\alpha + d\beta \tag{180}$$

$$d(f\alpha) = df\wedge\alpha + f\wedge d\alpha \tag{181}$$

$$d(dx) = d(dy) = d(dz) = 0 \tag{182}$$

From above we can see for example that

$$*d[(\sin^2\theta)d\varphi] = d(\sin^2\theta)\wedge d\varphi + \sin^2\theta\wedge dd\varphi = 2sin\theta\cos\theta(d\theta\wedge d\varphi) \tag{183}$$

$$*[d(r^2)d\varphi] = 2rdr\wedge d\varphi + r^2\wedge dd\varphi = 2r(dr\wedge d\varphi) \tag{184}$$

And then we derived again the Natario result of pg 5 in [2]

$$r^2\sin\theta\cos\theta(d\theta\wedge d\varphi) + r\sin^2\theta(dr\wedge d\varphi) \tag{185}$$

Now we will examine the following expression equivalent to the one of Natario pg 5 in [2] except that we replaced $\frac{1}{2}$ by the function $f(r)$:

$$*d[f(r)r^2\sin^2\theta d\varphi] \tag{186}$$

From above we can obtain the next expressions

$$f(r)r^2 * d[(\sin^2\theta)d\varphi] + f(r)\sin^2\theta * [d(r^2)d\varphi] + r^2\sin^2\theta * d[f(r)d\varphi] \tag{187}$$

$$f(r)r^2 2sin\theta\cos\theta(d\theta\wedge d\varphi) + f(r)\sin^2\theta 2r(dr\wedge d\varphi) + r^2\sin^2\theta f'(r)(dr\wedge d\varphi) \tag{188}$$

$$2f(r)r^2 sin\theta\cos\theta(d\theta\wedge d\varphi) + 2f(r)r\sin^2\theta(dr\wedge d\varphi) + r^2\sin^2\theta f'(r)(dr\wedge d\varphi) \tag{189}$$

Comparing the above expressions with the Natario definitions of pg 4 in [2]):

$$e_r \equiv \frac{\partial}{\partial r} \sim dr \sim (rd\theta)\wedge(r\sin\theta d\varphi) \sim r^2\sin\theta(d\theta\wedge d\varphi) \tag{190}$$

$$e_\theta \equiv \frac{1}{r}\frac{\partial}{\partial\theta} \sim rd\theta \sim (r\sin\theta d\varphi)\wedge dr \sim r\sin\theta(d\varphi\wedge dr) \sim -r\sin\theta(dr\wedge d\varphi) \tag{191}$$

$$e_\varphi \equiv \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi} \sim r\sin\theta d\varphi \sim dr\wedge(rd\theta) \sim r(dr\wedge d\theta) \tag{192}$$

We can obtain the following result:

$$2f(r)\ cos\theta[r^2 sin\theta(d\theta\wedge d\varphi)] + 2f(r)\ sin\theta[r\sin\theta(dr\wedge d\varphi)] + f'(r)r\sin\theta[r\sin\theta(dr\wedge d\varphi)] \tag{193}$$

$$2f(r)\ cos\theta e_r - 2f(r)\ sin\theta e_\theta - rf'(r)\sin\theta e_\theta \tag{194}$$

$$*d[f(r)r^2\sin^2\theta d\varphi] = 2f(r)\ cos\theta e_r - [2f(r) + rf'(r)]\sin\theta e_\theta \tag{195}$$

Defining the Natario Vector as in pg 5 in [2] with the Hodge Star operator * explicitly written :

$$nX = vs(t) * d\left(f(r)r^2\sin^2\theta d\varphi\right) \tag{196}$$

$$nX = -vs(t) * d\left(f(r)r^2\sin^2\theta d\varphi\right) \tag{197}$$

We can get finally the latest expressions for the Natario Vector nX also shown in pg 5 in [2]

$$nX = 2vs(t)f(r)\ cos\theta e_r - vs(t)[2f(r) + rf'(r)]\sin\theta e_\theta \tag{198}$$

$$nX = -2vs(t)f(r)\ cos\theta e_r + vs(t)[2f(r) + rf'(r)]\sin\theta e_\theta \tag{199}$$

With our pedagogical approaches

$$nX = 2vs(t)f(r)\ cos\theta dr - vs(t)[2f(r) + rf'(r)]r\sin\theta d\theta \tag{200}$$

$$nX = -2vs(t)f(r)\ cos\theta dr + vs(t)[2f(r) + rf'(r)]r\sin\theta d\theta \tag{201}$$

12 Appendix D:Solution of the Quadratic Form $ds^2 = 0$ for the Horizon problem in the Natario Warp Drive in a $1+1$ Spacetime

The equation of the Natario warp drive spacetime in $1+1$ dimensions is given by:[48]

$$ds^2 = [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 \tag{202}$$

Inserting the condition of the null-like geodesics ($ds^2 = 0$) in the equation of the Natario warp drive spacetime in $1+1$ dimensions we have:

$$ds^2 = 0 \dashrightarrow [1 - (X^{rs})^2]dt^2 + 2X^{rs}drsdt - drs^2 = 0 \tag{203}$$

$$[1 - (X^{rs})^2] + 2X^{rs}\frac{drs}{dt} - (\frac{drs}{dt})^2 = 0 \tag{204}$$

$$U = \frac{drs}{dt} \tag{205}$$

$$[1 - (X^{rs})^2] + 2X^{rs}U - U^2 = 0 \tag{206}$$

$$U^2 - 2X^{rs}U - [1 - (X^{rs})^2] = 0 \tag{207}$$

$$U = \frac{2X^{rs} \pm \sqrt{4(X^{rs})^2 + 4[1 - (X^{rs})^2]}}{2} \tag{208}$$

$$U = \frac{2X^{rs} \pm 2}{2} \tag{209}$$

$$U = \frac{drs}{dt} = X^{rs} \pm 1 \tag{210}$$

From above we can see that we have two solutions:one for the photon sent towards the front of the ship($-$ sign) and another for the photon sent to the rear of the ship($+$ sign).

$$U_{front} = \frac{drs_{front}}{dt} = X^{rs} - 1 \tag{211}$$

$$U_{rear} = \frac{drs_{rear}}{dt} = X^{rs} + 1 \tag{212}$$

$$X^{rs} = 2v_s n(rs) \tag{213}$$

[48]See Appendix B

13 Appendix E:Mathematical Demonstration of the Horizon Equation in the Natario Warp Drive for a $1+1$ Spacetime

An Horizon occurs every time we get:[49][50]

$$U_{front} = \frac{drs_{front}}{dt} = X^{rs} - 1 = 2v_s n(rs) - 1 = 0 \tag{214}$$

The value of the $n(rs)$ in the Horizon is given by:

$$2v_s n(rs) - 1 = 0 \dashrightarrow 2v_s n(rs) = 1 \tag{215}$$

$$n(rs) = \frac{1}{2vs} = [\frac{1}{2}][1 - f(rs)^{WF}]^{WF} \tag{216}$$

$$\frac{1}{vs} = [1 - f(rs)^{WF}]^{WF} \tag{217}$$

$$\sqrt[WF]{\frac{1}{vs}} = 1 - f(rs)^{WF} \tag{218}$$

$$f(rs)^{WF} = 1 - \sqrt[WF]{\frac{1}{vs}} \tag{219}$$

$$f(rs) = \sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}} = \frac{1}{2}[1 - tanh[@(rs - R)] \tag{220}$$

$$1 - tanh[@(rs - R)] = 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}} \tag{221}$$

$$tanh[@(rs - R)] = 1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}} \tag{222}$$

$$@(rs - R) = arctanh(1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) \tag{223}$$

$$rs - R = \frac{1}{@}arctanh(1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) \tag{224}$$

$$rs = R + \frac{1}{@}arctanh(1 - 2\sqrt[WF]{1 - \sqrt[WF]{\frac{1}{vs}}}) \tag{225}$$

[49]See Appendix D

[50]When reading this appendix remember that $WF >> 1$ and $vs >> 1$

14 Appendix F:Infinite Doppler Blueshift in the Horizon for the Natario Warp Drive in a $1+1$ Spacetime

Defining the following term X as being:

$$X = 2vsn(rs) \tag{226}$$

We know that in the Horizon the value of $n(rs)$ is given by:[51]

$$n(rs) = \frac{1}{2vs} \tag{227}$$

Then we have for X:

$$X = 2vsn(rs) = 1 \tag{228}$$

Also for the Horizon we have:

$$\frac{dx}{dt} = 0 \tag{229}$$

The photon stops!!![52]

- 1)- photon sent towards the front of the Warp Bubble $\frac{dx}{dt} = X - 1 = 2vsn(rs) - 1$
- 2)- photon sent towards the rear of the Warp Bubble $\frac{dx}{dt} = X + 1 = 2vsn(rs) + 1$

Or even better:

- 1)- photon sent towards the front of the Warp Bubble $\frac{dx}{dt} - X = \frac{dx}{dt} - 2vsn(rs) = -1$
- 2)- photon sent towards the rear of the Warp Bubble $\frac{dx}{dt} + X = \frac{dx}{dt} + 2vsn(rs) = +1$

Then we can easily see that(pg 6 in [2]).:

$$\parallel \frac{dx}{dt} - X \parallel = 1 \tag{230}$$

$$\parallel \frac{dx}{dt} - 2vsn(rs) \parallel = 1 \tag{231}$$

Defining now the term n in function of X we get the following values for n:

- 1)-photon sent to the front of the bubble

$$n = \frac{dx}{dt} - X = \frac{dx}{dt} - 2vsn(rs) = -1 \dashrightarrow 2vsn(rs) = 1 \dashrightarrow \frac{dx}{dt} = 0 \tag{232}$$

- 1)-photon sent to the rear of the bubble

$$n = \frac{dx}{dt} + X = \frac{dx}{dt} + 2vsn(rs) = +1 \dashrightarrow 2vsn(rs) = 1 \dashrightarrow \frac{dx}{dt} = 0 \tag{233}$$

We are interested in the results for the photon sent to the front of the bubble.

[51]See Appendix E

[52]See Appendix D

We are ready now to examine the topic of the Infinite Doppler Blueshifts in the Horizon suffered by photons sent towards the front of the Warp Bubble.See pg 6 and 8 in [2].

The unit vector n defined below represents the direction of the corresponding light ray from the point of view of the Eulerian observer outside the warp bubble.(See pg 6 in [2]).

$$n = \frac{dx}{dx} - X = \frac{dx}{dx} - 2vsn(rs) \tag{234}$$

In the Horizon ($\frac{dx}{dx} = 0$) and ($X = 2vsn(rs) = 1$)
Then we have:

$$n = \frac{dx}{dx} - X = \frac{dx}{dx} - 2vsn(rs) = 0 - 1 = -1 \tag{235}$$

The equation of the observed energy is given by(See pg 8 in [2]):

$$E_0 = E(1 + n.X) \tag{236}$$

So when a photon reaches the Horizon we have the following conditions: $n = -1$ and $X = 2vsn(rs) = 1$.Inserting these values in the equation of the energy(pg 8 in [2]) we have:

$$E = \frac{E_0}{(1 + n.X)} = \frac{E_0}{(1 + -1.1)} = \frac{E_0}{0} \tag{237}$$

And then we have the Infinite Doppler Blueshift in the Horizon as mentioned by Natario in pg 8 in [2].

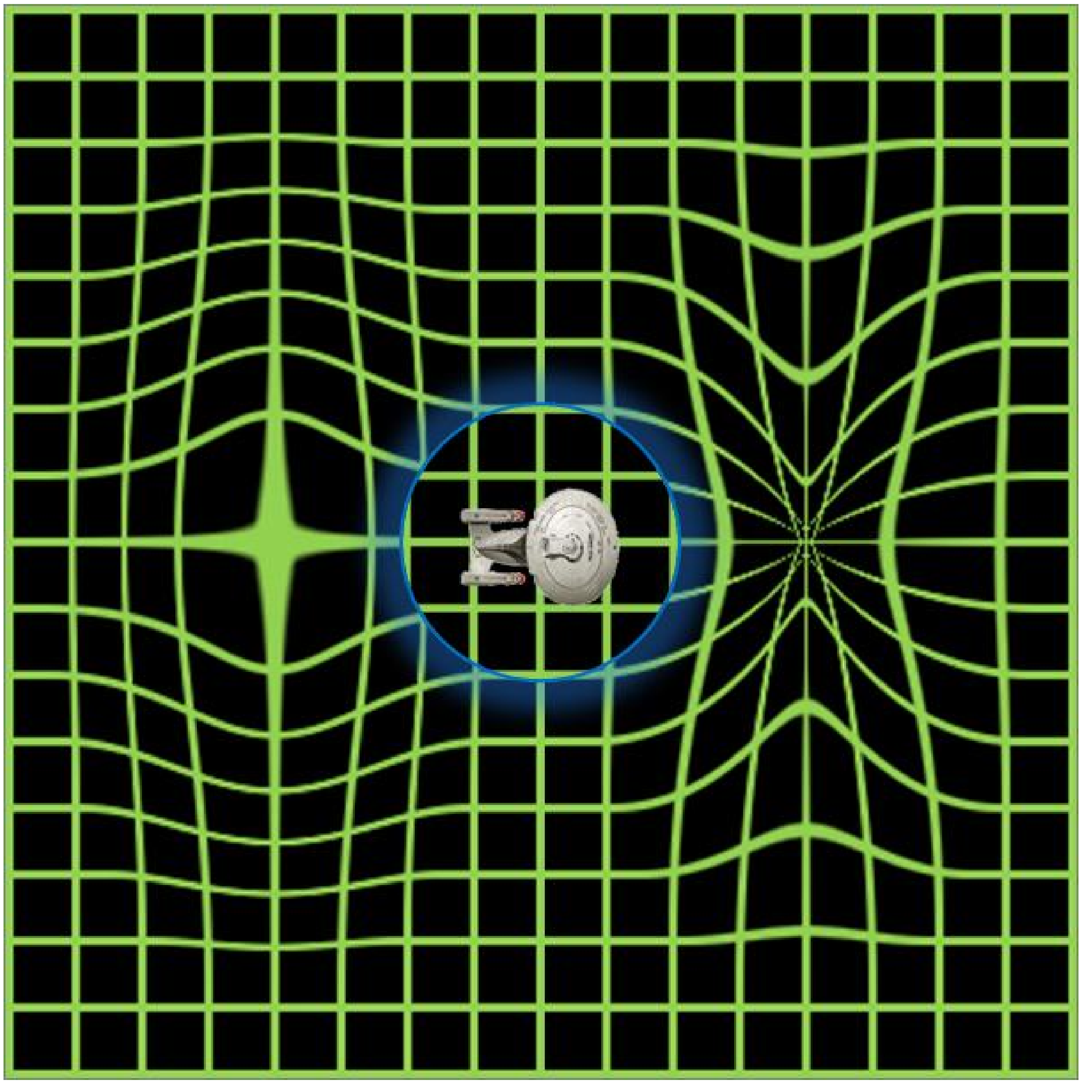

Figure 1: Artistic representation of the Natario Warp Bubble .(Source:Internet)

15 Appendix G:Artistic Presentation of the Natario Warp Bubble

According to the Natario definition for the warp drive using the following statement(pg 4 in [2]):

- 1)-Any Natario vector nX generates a warp drive spacetime if $nX = 0$ and $X = vs = 0$ for a small value of rs defined by Natario as the interior of the bubble and $nX = -vs(t)dx$ or $nX = vs(t)dx$ with $X = vs$ or $X = -vs$ for a large value of rs defined by Natario as the exterior of the bubble with $vs(t)$ being the speed of the bubble and X is the shift vector(pg 5 in [2]).

The blue region is the Natario warped region(bubble walls) where the negative energy is located.

Note that the negative energy surrounds entirely the "spaceship" inside the bubble deflecting incoming hazardous interstellar objects protecting the crew.[53]

A given Natario vector nX generates a Natario warp drive Spacetime if and only if satisfies these conditions stated below:

- 1)-A Natario vector nX being $nX = 0$ for a small value of rs(interior of the bubble)
- 2)-A Natario vector $nX = Xdx$ for a large value of rs(exterior of the bubble)
- 3)-A shift vector X depicting the speed of the bubble being $X = 0$(interior of the bubble) while $X = vs$ or $X = -vs$ being the shift vector seen by distant observers(exterior of the bubble).

The Natario vector nX is given by:

$$nX = v_s(t)d\left[n(rs)rs^2\sin^2\theta d\varphi\right] \sim 2v_s n(rs)\cos\theta drs - v_s(2n(rs) + rsn'(rs))rs\sin\theta d\theta \tag{238}$$

$$nX = -v_s(t)d\left[n(rs)rs^2\sin^2\theta d\varphi\right] \sim -2v_s n(rs)\cos\theta drs + v_s(2n(rs) + rsn'(rs))rs\sin\theta d\theta \tag{239}$$

This holds true if we set for the Natario vector nX a continuous Natario shape function being $n(rs) = \frac{1}{2}$ for large rs(outside the bubble) and $n(rs) = 0$ for small rs(inside the bubble) while being $0 < n(rs) < \frac{1}{2}$ in the walls of the bubble(pg 5 in [2])

The Natario vector $nX = 0$ vanishes inside the bubble because inside the bubble there are no motion at all because $dx = 0$ or $n(rs) = 0$ or $X = 0$ while being $nX = -vs(t)dx \neq 0$ or $nX = vs(t)dx \neq 0$ not vanishing outside the bubble because $n(rs)$ do not vanish.Then an external observer would see the bubble passing by him with a speed defined by the shift vector $X = -vs(t)$ or $X = vs(t)$.

The "spaceship" above lies in the interior of the bubble at the rest $X = vs = 0$ but the observer outside the bubble sees the "spaceship" passing by him with a speed $X = vs$.

See also pgs 14,15 and 16 in [4] and pgs 7,8 and 9 in [2] for more graphical presentations of the Natario warp bubble

[53]See Appendix M

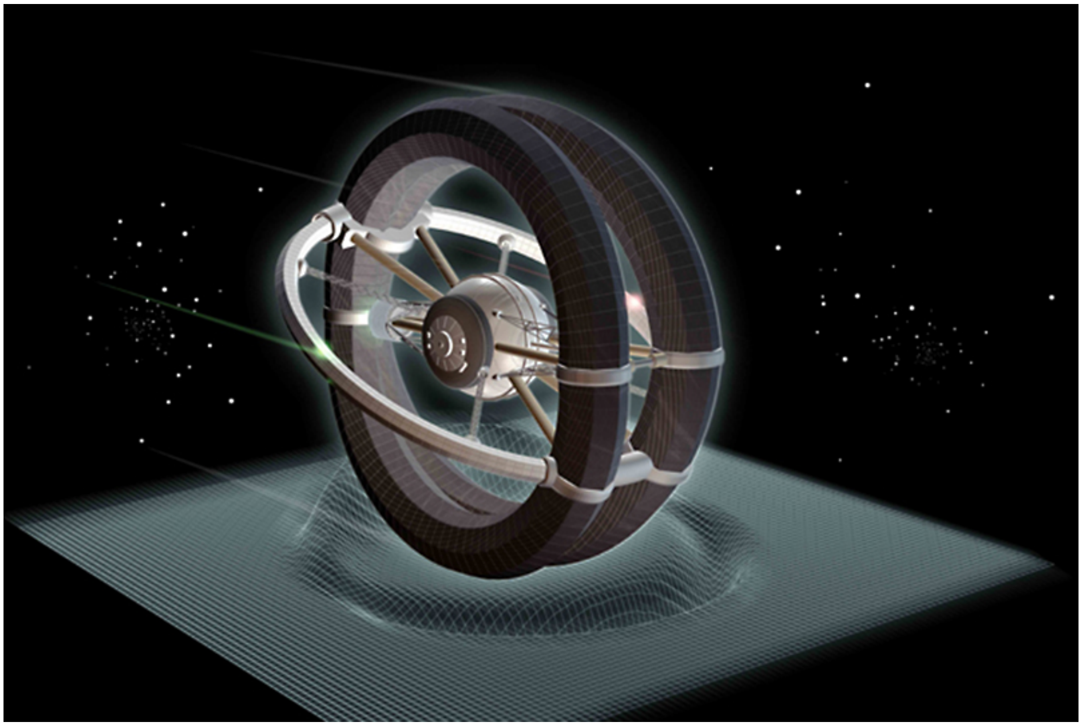

Figure 2: Artistic representation of the Natario Warp Drive .Note the Alcubierre Expansion of the Normal Volume Elements below.(Source:Internet)

16 Appendix H:Artistic Presentation of the Natario Warp Drive

Note that according to the geometry of the Natario warp drive the spacetime contraction in one direction(radial) is balanced by the spacetime expansion in the remaining direction(perpendicular).

Remember also that the expansion of the normal volume elements in the Natario warp drive is given by the following expressions(pg 5 in [2]). :

$$K_{rr} = \frac{\partial X^r}{\partial r} = -2v_s n'(r) \cos\theta \tag{240}$$

$$K_{\theta\theta} = \frac{1}{r}\frac{\partial X^\theta}{\partial \theta} + \frac{X^r}{r} = v_s n'(r) \cos\theta; \tag{241}$$

$$K_{\varphi\varphi} = \frac{1}{r\sin\theta}\frac{\partial X^\varphi}{\partial \varphi} + \frac{X^r}{r} + \frac{X^\theta \cot\theta}{r} = v_s n'(r) \cos\theta \tag{242}$$

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \tag{243}$$

If we expand the radial direction the perpendicular direction contracts to keep the expansion of the normal volume elements equal to zero.

This figure is a pedagogical example of the graphical presentarion of the Natario warp drive.

The "bars" in the figure were included to illustrate how the expansion in one direction can be counterbalanced by the contraction in the other directions.These "bars" keeps the expansion of the normal volume elements in the Natario warp drive equal to zero.

Note also that the graphical presentation of the Alcubierre warp drive expansion of the normal volume elements according to fig 1 pg 10 in [1] is also included

Note also that the energy density in the Natario Warp Drive being given by the following expressions(pg 5 in [2]):

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(n'(r))^2 \cos^2\theta + \left(n'(r) + \frac{r}{2} n''(r) \right)^2 \sin^2\theta \right]. \tag{244}$$

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta + \left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2n(r)}{dr^2} \right)^2 \sin^2\theta \right]. \tag{245}$$

Is being distributed around all the space involving the ship(above the ship $\sin\theta = 1$ and $\cos\theta = 0$ while in front of the ship $\sin\theta = 0$ and $\cos\theta = 1$).The negative energy in front of the ship "deflect" photons so these will not reach the Horizon and the Natario warp drive will not suffer from Doppler blueshifts.The illustrated "bars" are the obstacles that deflects photons or incoming particles from outside the bubble never allowing these to reach the interior of the bubble.[54]

-)-Energy directly above the ship($y - axis$)

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[\left(\frac{dn(r)}{dr} + \frac{r}{2} \frac{d^2n(r)}{dr^2} \right)^2 \sin^2\theta \right]. \tag{246}$$

-)-Energy directly in front of the ship($x - axis$)

$$\rho = -\frac{1}{16\pi} K_{ij} K^{ij} = -\frac{v_s^2}{8\pi} \left[3(\frac{dn(r)}{dr})^2 \cos^2\theta \right]. \tag{247}$$

Note that as fast as the ship goes by,then the negative energy density requirements grows proportionally to the square of the bubble speed vs.

For a bubble speed 200 times faster than light then ($vs = 6 \times 10^{10}$) and this affects the negative energy requirements by the factor vs^2 being ($vs^2 = 3,6 \times 10^{21}$) [55]

This raises the amount of negative energy to enormous levels making this a major concern when studying warp drive spacetimes.

The derivatives of the Natario shape function $n(rs)$ must be very low in order to obliterate the factor vs^2.

[54] See Appendix M

[55] the mass of the Earth according to Wikipedia is $M_{\oplus} = 5,9722 \times 10^{24}$ kilograms.In tons this reaches 10^{21} tons,exactly the factor vs^2 for 200 times light speed

Note also that even in a $1+1$ dimensional spacetime the Natario warp drive retains the zero expansion behavior:

$$K_{rr} = \frac{\partial X^r}{\partial r} = -2v_s n'(r)\cos\theta \tag{248}$$

$$K_{\theta\theta} = \frac{X^r}{r} = v_s n'(r)\cos\theta; \tag{249}$$

$$K_{\varphi\varphi} = \frac{X^r}{r} = v_s n'(r)\cos\theta \tag{250}$$

$$\theta = K_{rr} + K_{\theta\theta} + K_{\varphi\varphi} = 0 \tag{251}$$

Figure 3: Artistic representation of the Natario Warp Drive breaking the light speed barrier. (Source:Internet)

17 Appendix I:Artistic Presentation of the Natario Warp Drive entering in the luminal warp "shockwave" $X = 1$, Natario angle=$\alpha = \frac{\pi}{2}$

Although the figure above was "borrowed" from science fiction it depicts correctly what would happen for a Natario warp drive when it reaches luminal speeds($X = 1$). We already know that when $\|\mathbf{X}\| = 1$ and $vs = 1$ according to pg 6 in [2],(see also pg 15 in [4])

$$\sin\alpha = \frac{1}{\|\mathbf{X}\|} = 1 \tag{252}$$

$$\sin\alpha = 1 \dashrightarrow \alpha = \frac{\pi}{2} \dashrightarrow \|\mathbf{X}\| = 1 \tag{253}$$

From above we see that the white light "disk" appears in front of the ship perpendicular to the direction of motion because the Natario angle in this case is $\frac{\pi}{2}$.[56].

The white light "disk" represents the Natario "boom" that may harm the surface of a planet if the ship breaks the light speed barrier in the neighborhoods of it like the Mach "boom" harms glass of windows or human hears if a jet breaks the sound barrier at a low altitude and over populated areas .

[56]Compare this with the graphical presentation of the Mach cone angle Appendix L

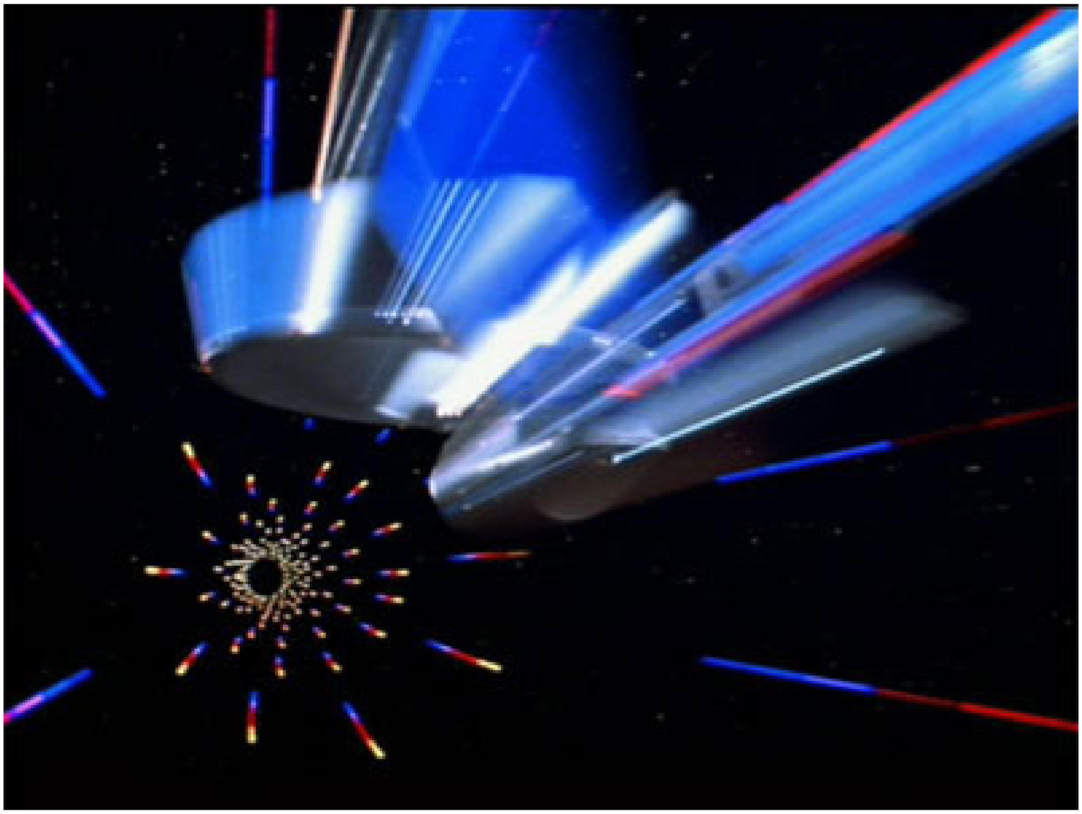

Figure 4: Artistic representation of the Natario Warp Drive at "low" superluminal(warp) speed. Note the Doppler lines . (Source:Internet)

18 Appendix J:Artistic Presentation of the Natario Warp Drive at "low" superluminal(warp) speed $X = vs$ and $vs > 1$. Natario angle $\alpha < \frac{\pi}{2}$ $\alpha \simeq \frac{\pi}{2}$

Like in the previous artistic presentation the figure above was also "borrowed" from science fiction and also depicts correctly what would happen for a Natario warp drive when it reaches "low" superluminal(warp) speeds($X > 1$). We already know that when $X = vs$ and $vs > 1$ (pg 6 in [2]).

$$\sin\alpha = \frac{1}{v_s} \tag{254}$$

$$\sin\alpha < 1 \dashrightarrow \alpha < \frac{\pi}{2} \dashrightarrow \|\mathbf{X}\| > 1 \dashrightarrow \sin\alpha \simeq 1 \dashrightarrow \alpha \simeq \frac{\pi}{2} \tag{255}$$

From above we can see that $\sin\alpha < 1$ and $\alpha < \frac{\pi}{2}$.This means that the Natario angle is no longer perpendicular to the direction of motion and now have a small inclination given by the Natario angle itself.[57].

[57]Compare this with the graphical presentation of the Mach cone angle Appendix L

Figure 5: Artistic representation of the Natario Warp Drive at high superluminal(warp) speed. Note the Doppler lines . (Source:Internet)

19 Appendix K:Artistic Presentation of the Natario Warp Drive at "high" superluminal(warp) speed $X = vs$ and $vs >> 1$. Natario angle $\alpha << \frac{\pi}{2}$ $\alpha \simeq 0$

Like in the two previous artistic presentation the figure above was also "borrowed" from science fiction and also depicts correctly what would happen for a Natario warp drive when it reaches "high" superluminal(warp) speeds($X >> 1$). We already know that when $X = vs$ and $vs >> 1$ (pg 6 in [2]).

$$\sin\alpha = \frac{1}{v_s} \tag{256}$$

$$\sin\alpha << 1 \dashrightarrow \alpha << \frac{\pi}{2} \dashrightarrow \|\mathbf{X}\| >> 1 \dashrightarrow \sin\alpha \simeq 0 \dashrightarrow \alpha \simeq 0 \tag{257}$$

In this case since $X >> 1$ $\sin\alpha << 1$ and $\alpha << \frac{\pi}{2}$.The Natario angle now appears almost parallel to the direction of motion and the inclination of the Natario angle is so hight that approaches zero.[58].

[58]Compare this with the graphical presentation of the Mach cone angle Appendix L

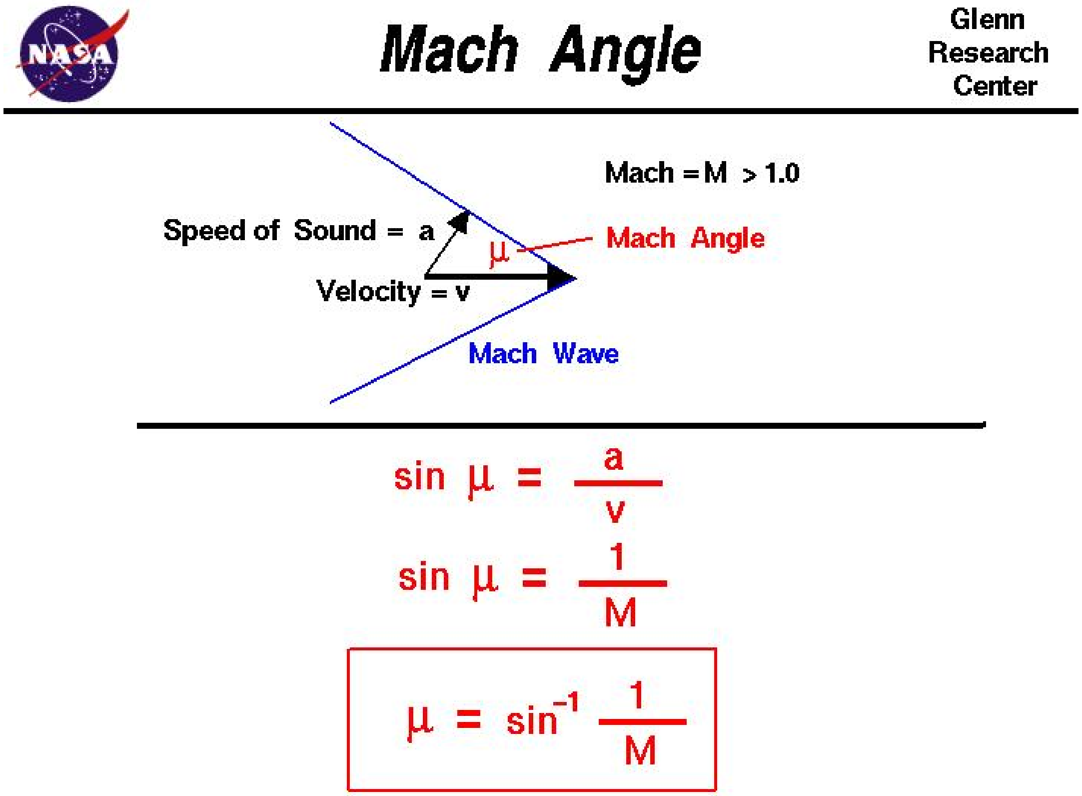

Figure 6: Graphical Presentation of the Mach cone angle . (Source:NASA)

20 Appendix L:Artistic Presentation of the Mach cone angle

Above is being presented the graphical presentation of the Mach cone angle.We included this presentation here[59] to demonstrate the similarities that exists between the supersonic(Mach) speeds and the Mach cone angle with the superluminal(warp) speeds and the Natario cone angle.

Note that when a plane achieves the speed of the sound its velocity v equals the speed of the sound a and then

$$\sin\mu = \frac{a}{v} = 1 \tag{258}$$

In the above case the angle $\mu = \frac{\pi}{2}$ and the Mach "shockwave" is perpendicular to the direction of motion of the plane.

But when the plane accelerates to supersonic(Mach) speeds then $v >> a$ and $\sin\mu = \frac{a}{v} << 1$ with the angle $\mu \cong 0$.As fast the plane accelerates the Mach angle approaches zero and the Mach shockwave inclination tend to be parallel to the speed of the plane itself.

[59] See the Remarks Section on Appendix L

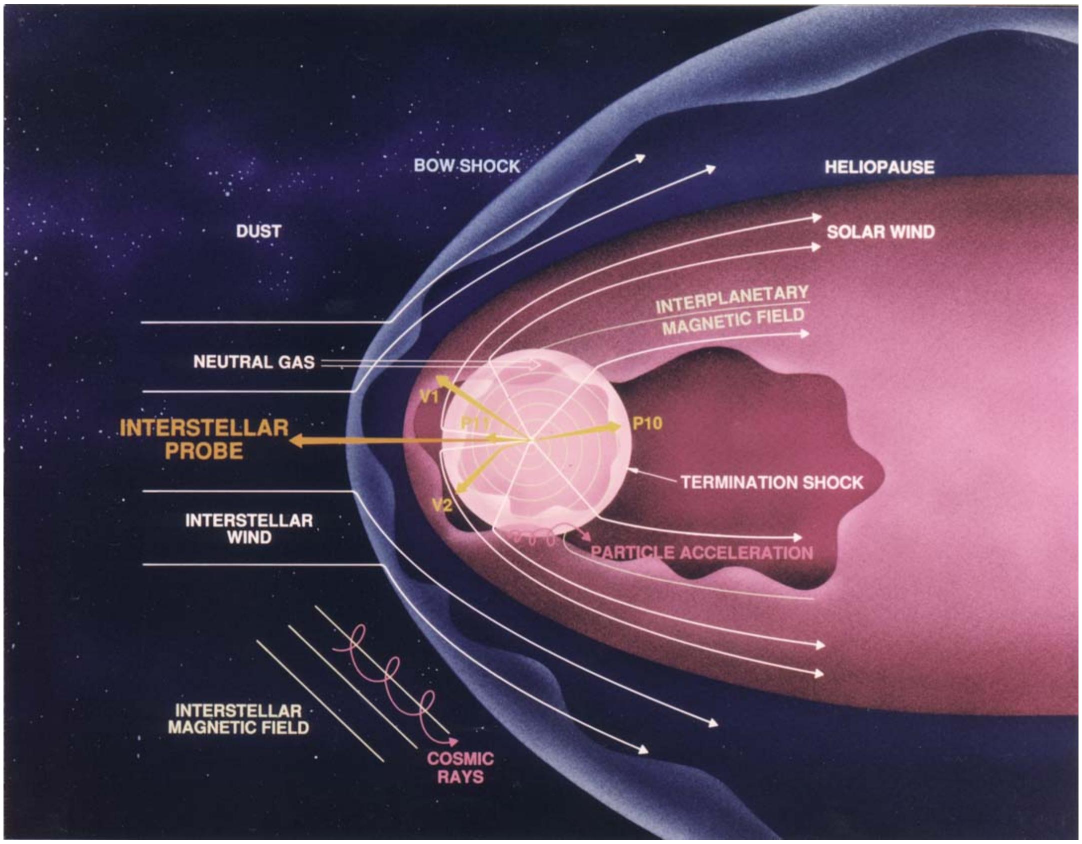

Figure 7: Artistic representation of a Natario Warp Drive in a real superluminal space travel .Note the negative energy in front of the ship deflecting incoming hazardous interstellar matter(brown arrows).(Source:Internet)

21 Appendix M:Artistic Presentation of a Natario Warp Drive In A Real Faster Than Light Interstellar Spaceflight

Above is being presented the artistic presentation of a Natario warp drive in a real interstellar superluminal travel.The "ball" or the spherical shape is the Natario warp bubble with the negative energy surrounding the ship in all directions and mainly protecting the front of the bubble.[60]

The brown arrows in the front of the Natario bubble are a graphical presentation of the negative energy in front of the ship deflecting dust,neutral gases,interstellar wind and Doppler Blueshifted photons.

The angle between the blue bow shock and the direction of motion is the Natario angle defined by pg 6 in [2].[61]

The spaceship is at the rest and in complete safety inside the Natario bubble.

[60]See the Appendices G and H
[61]See the Appendices I,J,K and L

Lets illustrate the dangers of the collisions with interstellar matter in a real superluminal travel.

Consider the Natario "ball" of this presentation.The ship is at the rest inside the "ball" but the "ball" (warp bubble) is moving with a speed of 200 times faster than light. Imagine that our "ball" in interstellar space is moving towards two clouds of interstellar dust initially at the rest but in front of the ship :cloud A and cloud B.Cloud A lies in front of cloud B. So the ship will reach cloud A first. When the ship arrives at cloud A the repulsive gravitational field will deflect the particles but particles directly over the axis of the "ball" motion will be pushed away by the repulsive gravitational field from the negative energy of the warp bubble walls like a jet "stream" or a "dragging". Now the "ball" and "dragged" particles from cloud A are moving 200 times faster than light in the direction of cloud B.The "dragged" particles from the cloud A will reach the particles from cloud B initially at the rest and collisions will happens.These collisions will be highly energetic releasing enormous bursts of energy. [62] Then in front of the bubble a region of high energy will be formed.The ship at the rest inside the bubble is protected by the negative energy of the warp bubble walls but superluminal spaceships generating these huge amounts of energy in front of their bubbles must avoid to use faster than light speeds in the neighborhoods of planets because these have no protections against these energetic bursts.

A ship must leave a planet(Earth) at subluminal speeds and travel to the limits of the planetary system still at subluminal speeds and only in real interstellar space far away from any planet the ship can then trigger the faster than light motion.Once arriving at its destiny(Gliese $667c$) but still in interstellar space the ship must slow down from superluminal speeds entering in the planetary system to be visited at subluminal speeds in order to do not harm any planet.

According with Natario in pg 7 before section 5.2 in [3] negative energy density means a negative mass density and a negative mass density generates a repulsive gravitational field. This repulsive gravitational field in front of the ship in the Natario warp drive spacetime protects the ship from impacts with the hazardous interstellar matter and also protects the ship from photons highly Doppler Blueshifted approaching the ship from the front.So in the Natario warp drive the problem of the collisions with the interstellar matter never happens.

Adapted from the negative mass in Wikipedia:The free Encyclopedia:

"if we have a small object with equal inertial and passive gravitational masses falling in the gravitational field of an object with negative active gravitational mass (a small mass dropped above a negative-mass planet, say), then the acceleration of the small object is proportional to the negative active gravitational mass creating a negative gravitational field and the small object would actually accelerate away from the negative-mass object rather than towards it."

[62] Think on how many particles of space dust exists in each cubic centimeter of interstellar space and think on how many cubic centimeters of space exists in 20 light-years.

22 Appendix N:Integration of the Negative Energy Density in the Natario Warp Drive using the original Natario shape function

The total energy integral for the Natario warp drive is given by:

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@^2}{16}\int([\frac{1}{cosh^4[@(rs-R)]}])drs \tag{259}$$

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@^2}{16}\int([\frac{drs}{cosh^4[@(rs-R)]}]) \tag{260}$$

Fortunately integrals of these forms are available in tables of integrals of hyperbolic functions and the one needed to compute the integration above is given below($n = 4$)[63]:

$$\int\frac{dx}{cosh^n(ax)} = \frac{sinh(ax)}{a(n-1)cosh^{n-1}(ax)} + \frac{n-2}{n-1}\int\frac{dx}{cosh^{n-2}(ax)} \tag{261}$$

$$\int\frac{dx}{cosh^4(ax)} = \frac{sinh(ax)}{(3a)cosh^3(ax)} + \frac{2}{3}\int\frac{dx}{cosh^2(ax)} \tag{262}$$

But:

$$\int\frac{dx}{cosh^2(ax)} = \frac{1}{a}tanh(ax) \tag{263}$$

Hence we have:

$$\int\frac{dx}{cosh^4(ax)} = \frac{sinh(ax)}{(3a)cosh^3(ax)} + \frac{2}{3a}tanh(ax) \tag{264}$$

$$\int\frac{dx}{cosh^4(ax)} = \frac{1}{3a}[\frac{sinh(ax)}{cosh^3(ax)} + 2tanh(ax)] \tag{265}$$

$$\int\frac{dx}{cosh^4(ax)} = \frac{tanh(ax)}{3a}[\frac{1}{cosh^2(ax)} + 2] \tag{266}$$

$$\int[\frac{drs}{cosh^4[@(rs-R)]}] = \frac{tanh(@(rs-R))}{3@}[\frac{1}{cosh^2(@(rs-R))} + 2] \tag{267}$$

The integral will then be given by:

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@^2}{16}\int([\frac{drs}{cosh^4[@(rs-R)]}]) \tag{268}$$

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@^2}{16}\frac{tanh(@(rs-R))}{3@}[\frac{1}{cosh^2(@(rs-R))} + 2] \tag{269}$$

$$E = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\int((n'(rs))^2)drs = -3\frac{c^2}{G}\frac{v_s^2}{8\pi}\frac{@}{16}\frac{tanh(@(rs-R))}{3}[\frac{1}{cosh^2(@(rs-R))} + 2] \tag{270}$$

[63]Wikipedia:The free Encyclopedia

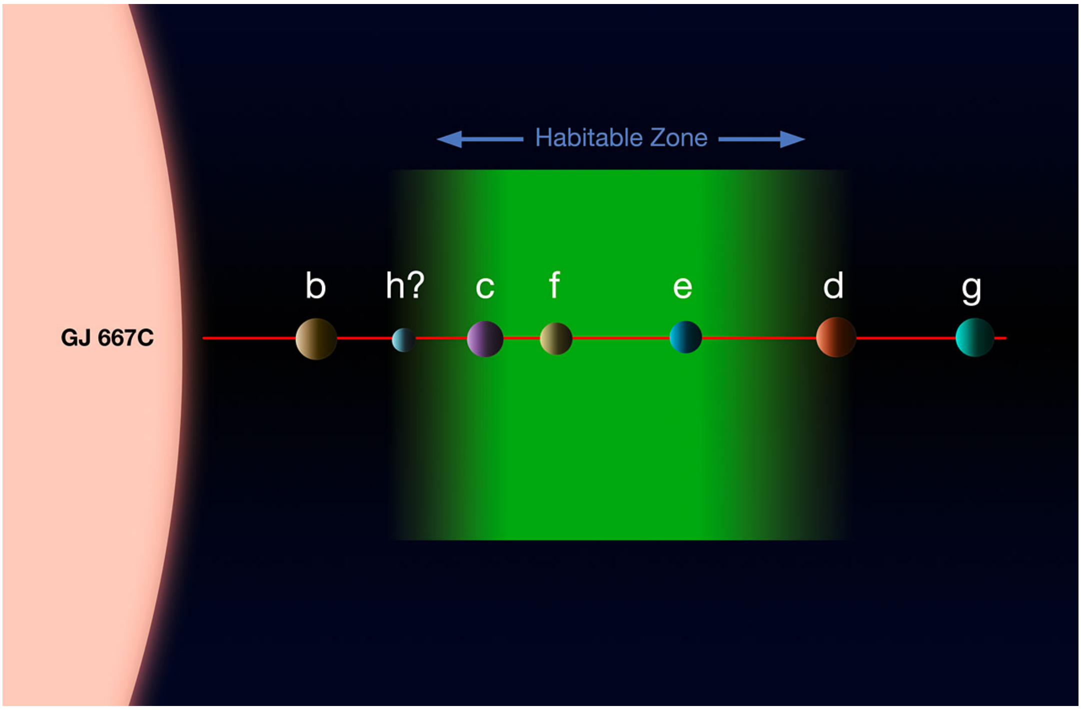

Figure 8: Artistic Presentation of the Habitable Zone(HZ) of the Star System $Gliese667C$. (Source:ESO)

23 Appendix O:Artistic Presentation of the Habitable Zone(HZ) of the Star System Gliese 667C

Above is being presented the artistic presentation of the HZ of $Gliese667C$.[64] Compare this with fig 15 pg 19 in [12],fig 15 pg 18 in [13].

Planets C,F and E are confirmed orbiting the star well within the limits of the HZ.According to the ESO team the planets D and H are in the outer and inner edges of the HZ borders respectively but planet H still needs confirmation.

In the inner edge of the HZ border (too hot) if planet H possesses an atmosphere with large amounts of waver vapor the major part of the star light rays will be reflected to space lowering the temperature and H might well be also habitable.

In the outer edge of the HZ border (too cold) if planet D possesses an atmosphere with large amounts of carbon dioxide the major part of the star light rays will be trapped in a greenhouse effect raising the temperature and D might well be also habitable.

So perhaps $Gliese667C$ might have 5 planets in the HZ after all.And there are 260 billions of stars class M in our galaxy like $Gliese667C$.

[64] See the Remarks Section on Appendix O

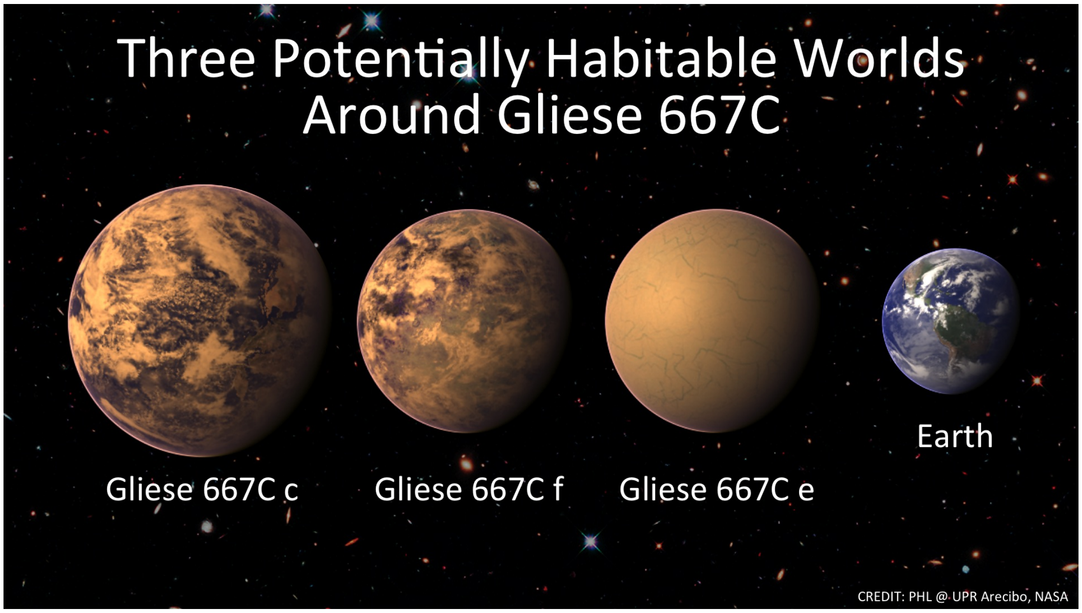

Figure 9: Artistic Presentation of the Planets of $Gliese667C$ Compared to Earth (Source:PHL)

24 Appendix P:Artistic Presentation of the Planets of Gliese 667C Compared to Earth

The artistic presentation above[65] depicts the 3 planets in the HZ of $Gliese667C$ when compared to Earth.The sizes are at scale.From the picture above we can see that each one of them is larger and heavier than Earth with gravity fields strong enough to sustain large atmospheres of carbon dioxide and oxygen and perhaps able to sustain advanced life forms and maybe who knows advanced intelligent life forms.

The exploration of these planets will be a goal accessible to the human race if and only if faster than light space travel could ever be developed.

[65] See Remarks Section on Appendix P

Figure 10: Artistic Presentation of the the $Gliese667C$ Landscape if habited by an advanced civilization (Source:$Internet$)

25 Appendix Q:Artistic Presentation of the Gliese 667C Landscape if habited by an advanced civilization

Almost immediately after the publication of [12] and [13] the enthusiasts of intelligent life in the Universe started to theorize about the possibility of advanced civilizations on $Gliese667C$. The picture above depicts how the landscape of the planets on $Gliese667C$ would look if habited by an advanced civilization.

The exploration of these planets will be a goal accessible to the human race if and only if starships equipped with the warp drive could ever be developed. [66]

This is the reason why our civilization needs to develop the technology of the warp drive.

[66] The reverse situation is also true:an advanced civilization on $Gliese667C$ can visit our planetary system if and only if they possesses the warp drive technology

26 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[67]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[68][69]

27 Remarks

Reference [7] was online at the time we picked it up for our records.It ceased to be online but we can provide a copy in PDF Acrobat reader of this reference for those interested.

Reference [8] we only have access to the abstract.

Reference [4] can be obtained from the web page of Professor Jose Natario at Instituto Superior Tecnico Lisboa Portugal. [70]

Appendix L is available to the public from the NASA Glenn Research Center Ohio United States under the program destined for the education of high school students ($K - 12$).[71]

Reference [13] can be obtained from the web page of the the ESO (European Southern Observatory)La Silla Chile. [72]

Appendix O can be obtained from the web page of the the ESO (European Southern Observatory)La Silla Chile. [73]

Appendix P can be obtained from the web page of the the PHL (Planetary Habitability Laboratory)University of Arecibo Puerto Rico [74]

[67]special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[68]"Ideas And Opinions" Einstein compilation, ISBN $0 - 517 - 88440 - 2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[69]appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978 - 0 - 9557068 - 0 - 6$

[70]http://www.math.ist.utl.pt/ jnatar/seminario.pdf

[71]http://www.grc.nasa.gov/WWW/K-12/airplane/machang.html

[72]http://www.eso.org/public/archives/releases/sciencepapers/eso1328/eso1328a.pdf

[73]http://www.eso.org/public/images/eso1328b/

[74]http://phl.upr.edu/press-releases/anearbystarwiththreepotentiallyhabitableworlds

28 Legacy

This work is dedicated to the team of exo-planetary astronomers of ESO (European Southern Observatory) La Silla Chile that discovered the star system $Gliese667C$.After this discovery the astronomy of exo-planets will never be the same.

If our galaxy have a numerous population of M stars with several potential habitable words in each star according to the ESO team then and joining our desires to the wishes of the ESO team we all hope that $GJ667C$ is the first of many star systems still waiting to be discovered.

The astronomers of ESO discoverers of $Gliese667C$ are:

Guillem Anglada-Escude,Mikko Tuomi,Enrico Gerlach,Rory Barnes,Rene Heller,James S.Jenkins,Sebastian Wende,Steven S.Vogt,R.Paul Butler,Ansgar Reiners,Hugh R.A.Jones

References

[1] *Alcubierre M., (*1994*). Class.Quant.Grav. 11 L73-L77,gr-qc/0009013*

[2] *Natario J.,(*2002*). Class.Quant.Grav. 19 1157-1166,gr-qc/0110086*

[3] *Natario J.,(*2006*).Gen.Rel.Grav.* 38 *(*2006*)* $475-484$*,gr-qc/0408085*

[4] *Natario J..(*2003*),Warp Drive Seminar Lectures.Instituto Superior Tecnico,Lisboa Portugal*

[5] *Lobo F.,(*2007*).,arXiv:0710.4474*

[6] *Everett A.E.,Roman T.A,(*1997*) ,Phys.Rev. D56 024023,gr-qc/9702049*

[7] *Gauthier C.,Gravel P.,Melanson J.,(*2003*). Gravitation and Cosmology. Vol* 9 *No.*4*(*36*)* 301-306

[8] *Gauthier C.,Gravel P.,Melanson J.,(*2002*). Int.J.Mod.Phys.A(IntJMPA). Vol* 17 *No.*20 2761

[9] *Loup F.,(*2013*).,HAL-00850438*

[10] *Loup F.,(*2013*).,HAL-00852077*

[11] *Loup F.,(*2013*).,HAL-00879381*

[12] *Guillem A.E.(ET.AL.),(*2013*).Astronomy and Astrophysics, Vol* 556*, A*126*,arXiv:1306.6074*

[13] *Guillem A.E.(ET.AL.),(*2013*).ESO-PRL*1328*a*

[14] *Introduction to Differential Forms,Arapura D.,*2010

[15] *Teaching Electromagnetic Field Theory Using Differential Forms,Warnick K.F. Selfridge R. H.,Arnold D. V.,IEEE Transactions On Education Vol* 40 *Num* 1 *Feb* 1997

.