APPROACH TO SOLVE P VS PSPACE WITH COLLAPSE OF LOGARITHMIC AND POLYNOMIAL SPACE

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1. Overview

This article describes about that P is not PSPACE. If P is PSPACE, we can derive P is L from relation of logarithm and polynomial reduction. But this result contracit Space Hierarchy Theorem. Therefore P is not PSPACE.

2. Preparation

In this article, we use description as follows;

Definition 1. We will use the term "$pDTM$" as Turing Machine set that compute P, "$LDTM$" as Turing Machine set that compute L. "$RpDTM$" as Reversible $pDTM$.

And we will use words and theorems of References [1, 2, 3] in this paper.

3. P is not PSPACE

To prove $P \subsetneq PSPACE$ to think in case $L = P$ or $L \subsetneq P$. If $L = P$ then $P \subsetneq PSPACE$ from Space Hierarchy Theorem. If $L \subsetneq P$ also $P \subsetneq PSPACE$ from relation of logarithm and polynomial reduction. Therefore $P \subsetneq PSPACE$.

Theorem 2. $L = P \rightarrow P \subsetneq PSPACE$

Proof. It is trivial. Because of Space Hierarchy Theorem, $NL \subsetneq PSPACE$. Therefore $L = P \rightarrow P \subsetneq PSPACE$. □

Theorem 3. $L \subsetneq P \rightarrow P \subsetneq NP$

Proof. We prove it using reduction to absurdity. We assume that $P = NP$, therefore all $A, B \in NP - Complete$ have $f \in LDTM$ that reduce A to B.

$\forall A, B \in NP - Complete \exists f \in LDTM\left(f\left(A\right) = B\right)$

If $A \in NP - Complete$ and $g \in RpDTM$ then

$A \leq_p g\left(A\right)$

and

$g\left(A\right) \leq_p g^{-1}\left(g\left(A\right)\right) = A \in NP \Longrightarrow g\left(A\right) \in NP$

Therefore

$g\left(A\right) \in NP - Complete$

That is,

$\forall A \in NP - Complete \forall g \in RpDTM \exists f \in LDTM\left(f\left(A\right) = g\left(A\right)\right)$

But all DTM can reduce RpDTM. Therefore this means $L = P$ and contradict $L \subsetneq P$.

Therefore, this theorem was shown than reduction to absurdity. □

Theorem 4. $P \subsetneq PSPACE$

Proof. Mentioned above 23, $P \subsetneq PSPACE$ either $L = P$ or $L \subsetneq P$. Therefore this theorem was shown. □

References

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[2] OGIHARA Mitsunori, Hierarchies in Complexity Theory, 2006

[3] MORITA Kenichi, Reversible Computing, 2012