Three conjectures about an infinity of subsets of integers, each with possible infinite terms primes or squares of primes

Marius Coman
Bucuresti, Romania
email: mariuscoman13@gmail.com

Abstract. In my previous paper «Twenty-four conjectures about "the eight essential subsets of primes"» are made three conjectures about each one from the following eight subsets: the primes of the form 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 respectively 30*k + 29. The conjectures from that paper state that each from these eight sets of primes has an infinity of terms and also that each one of them can be entirely defined with a recurrent formula starting from just three given terms. In this paper are generalized the three conjectures for an infinity of subsets, each having possibly an infinity of terms which are primes or squares of primes, subsets of integers of the form 2*p(1)*p(2)*...*p(m)*k + d, where p(1), p(2), ..., p(m) are the first m odd primes, k is a non-null positive integer and d an odd positive integer satisfying certain conditions.

Conjecture 1:

The sequence a(n), as it will be defined below, has an infinity of terms that are primes or squares of primes.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = q_1, a(2) = q_2, a(3) = q_3, where q_1, q_2 and q_3 are the first three integers which are primes or squares of primes of the form $2*p_1*p_2*...*p_m*k + d$, where p_1, p_2, ..., p_m are the first m odd primes, k is a non-null positive integer and d is equal to 1 or is equal to any odd positive integer which satisfies the following two conditions: d is co-prime to any of the primes p_1, p_2, ..., p_m and d < $2*p_1*p_2*...*p_m$.

: a(n) is the smallest prime or square of prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – d, where 1 ≤ i ≤ j < n (if such prime or square of prime exists for any n, as the conjecture states).

Conjecture 2:

Any prime or square of prime of the form $2*p_1*p_2*...*p_m*k + d$ is a term of the sequence a(n) as it is defined by Conjecture 1 [in other words, the sequence a(n) is the same with the sequence of the integers which are primes or squares of primes of the form $2*p_1*p_2*...*p_m*k + d$, where k > 0].

Conjecture 3:

If the Conjecture 2 doesn’t hold, than is true at least that any prime or square of prime of the form $2*p_1*p_2*...*p_m*k + d$ is a term of a sequence a(n) that can be defined as follows:

: a(1), a(2), a(3) are three distinct integers which are primes or squares of primes of the form $2*p_1*p_2*...*p_m*k + d$, where k > 0;

: a(n) is the smallest integer which is prime or square of prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – d, where 1 ≤ i ≤ j < n.

Verifying the Conjecture 2 for m = 1:

(for first few terms)

For m = 1 we have $p_1 = p_m = 3$ and only two possible values for d (because d must be odd, smaller than 2*3 = 6 and also co-prime with 3), *i.e.* d = 1 and d = 5.

I.

For d = 1 the first three integers which are primes or squares of primes of the form 6*k + 1, where k > 0, are 7, 13 and 19:

: Conjecture 1 states that there exist an infinity of integers of the form 6*k + 1 which are primes or squares of primes;

: Conjecture 2 states that any prime or square of prime of the form 6*k + 1 can be defined starting from the primes 7, 13 and 19 with the formula from Conjecture 1. The next six integers of this form which are primes or squares of primes are 5^2 = 1 + 19 – 1 = 13 + 13 – 1; 31 = 13 + 19 – 1; 37 = 7 + 31 – 1; 43 = 7 + 37 – 1, 7^2 = 7 + 43 – 1 = 13 + 37 – 1 = 19 + 31 - 1; 61 = 19 + 43 – 1 = 31 + 31 – 1.

II.

For d = 5 the first three integers which are primes or squares of primes of the form 6*k + 5, where k > 0, are 11, 17 and 23:

: Conjecture 1 states that there exist an infinity of integers of the form 6*k + 5 which are primes or squares of primes;

: Conjecture 2 states that any prime or square of prime of the form 6*k + 5 can be defined starting from the primes 11, 17 and 23 with the formula from Conjecture 1. The next six integers of this form which are primes or squares of primes are 29 = 17 + 17 – 5; 41 = 23 + 23 – 5; 47 = 11 + 41 – 5 = 23 + 29 – 5; 53 = 11 + 47 – 5 = 17 + 41 – 5 = 29 + 29 – 5; 59 = 11 + 53 – 5 = 17 + 47 – 5 = 23 + 41 – 5; 71 = 17 + 59 – 5 = 23 + 53 – 5 = 29 + 47 – 5.

Verifying the Conjecture 2 for m = 2:

For m = 2 we have p_1 = 3 and p_2 = p_m = 5 and the subsets of integers of the form 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23; for these eight subsets of integers we treated a more strict form of the three conjectures from above in the previous paper «Twenty-four conjectures about "the eight essential subsets of primes"».

Verifying the Conjecture 2 for m = 3:

(for d = 1, d = 11 and the first few terms)

For m = 3 we have p_1 = 3, p_2 = 5 and p_3 = p_m = 7 and the subsets of primes of the form 210*k + 1, 210*k + 11, 210*k + 13, 210*k + 17, 210*k + 19, 210*k + 23, 210*k + 29, 210*k + 31, 210*k + 37, 210*k + 41, 210*k + 43, 210*k + 47, 210*k + 53, 210*k + 59, 210*k + 61, 210*k + 67, 210*k + 71, 210*k + 73, 210*k + 79, 210*k + 83, 210*k + 89, 210*k + 97, 210*k + 101, 210*k + 103, 210*k + 107, 210*k + 109, 210*k + 13, 210*k + 121, 210*k + 127, 210*k + 131, 210*k + 137, 210*k + 139, 210*k + 143, 210*k + 149, 210*k + 151, 210*k + 157, 210*k + 163, 210*k + 167, 210*k + 169, 210*k + 173, 210*k + 179, 210*k + 181, 210*k + 187, 210*k + 191, 210*k + 193, 210*k + 197, 210*k + 199, 210*k + 209.

: Conjecture 1 states that there exist an infinity of integers of each of these forms which are primes or squares of primes.

: Conjecture 2 states that any prime or square of prime of this form can be defined with the formula from Conjecture 1 starting from the three integers which are primes or squares of primes of the respective form, considering k > 0.

: The first three integers which are primes or squares of primes of the form 210*k + 1, where k > 0, are 211, 421 and 631; the following next six integers which are primes or squares of primes of this form are 29^2 = 21 + 631 – 1 = 421 + 421 – 1; 1051 = 421 + 631 – 1 = 211 + 29^2 – 1; 1471 = 421 + 1051 – 1; 41^2 = 211 + 1471 – 1 = 631 + 1051 – 1; 2311 = 631 + 41^2 – 1; 2521 = 211 + 2311 – 1 = 1051 + 1471 – 1.

: The first three integers which are primes or squares of primes of the form 210*k + 11, where k > 0, are 431, 641 and 1061; the following next six integers which are primes or squares of primes of this form are 1481 = 431 + 1061 – 11; 1901 = 431 + 1481 – 11; 2111 = 641 + 1481 – 11; 2531 = 431 + 2111 – 11; 2741 = 641 + 2111 – 11; 3371 = 641 + 2741 – 11.