Natural Length Scale from Cosmological Holographic Principle Matches Proton

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Abstract

A natural unit of length, determined by applying the Holographic Principle to the Hubble sphere, matches the diameter of the proton, the smallest relevant size in nature. Unlike traditional Large Numbers coincidences, which are accurate only to order of magnitude, this agreement is precise.

Introduction

This brief Note discusses the natural length scale implied by the restriction on the information density of a region under the Holographic Principle.

The Planck length, $l_P = (hG/2\pi c^3)^{1/2} = 1.616\times10^{-35}$ m, is 20 orders of magnitude smaller than the atomic nucleus. Mathematically, this is trivial, following from the weakness of gravity. But from an informational perspective, it seems deeply disturbing. If we naïvely consider the world to be about as rich in information as a discrete 3-dimensional lattice of spacing l_P, reality appears to be severely over-specified.

Since John Archibald Wheeler's promotion of the epigrammatic "It from Bit" hypothesis [Wheeler, 1989], the physics community has become increasingly committed to the concept of information being the deep foundation underlying quantum mechanics [Zeilinger, 1999]. This visionary idea was recently given a rigorous mathematical formulation by Chiribella and colleagues [2011].

The relationship of information to geometry was first made explicit in black hole thermodynamics, where each bit requires an area of 4 l_P^2 on the event horizon [Bekenstein, 1973]. The universality of this relationship, implying dimensional reduction in quantum gravity, was first proposed by 't Hooft [1993], and through work of Thorn and Susskind, refined into a Holographic Principle [Susskind, 1994]. This has been found to be remarkably general and robust [Bousso, 2002]. The information content of a region is limited, not by its volume, but by the area of its boundary surface.

This powerfully counter-intuitive rule restricts the information density of a region and thus the informational over-specification implied in a naïve lattice model with 1 bit per l_P^3.

Thus, a sphere matching the r.m.s. charge radius of the proton, 0.8775 fm [CODATA, 2010], while containing 6.7×10^{59} Planck volumes, l_P^3, has a surface area of 3.7×10^{40} Planck areas, l_P^2, and could thus accommodate "only" 9.3×10^{39} bits, an amelioration of the informational overkill by 20 orders of magnitude. This amelioration improves as larger dimensions are considered, owing to the area scaling as l^2, while volume scales as l^3. When cosmological distances are considered, the informational over-specification vanishes.

Model

Consider a spherical region of radius, R. The surface area, $4\pi R^2$, encodes $\pi R^2/l_P^2$ bits. In a simple case, consider the corresponding information in the 3-dimensional space to be distributed uniformly throughout the volume of the sphere. Let us propose a natural elementary unit of length, l_H, such that each bit occupies a unit cube with sides $2\ l_H$, the 3-dimensional arrangement being harmonious with the layout of bits on the 2-dimensional boundary.

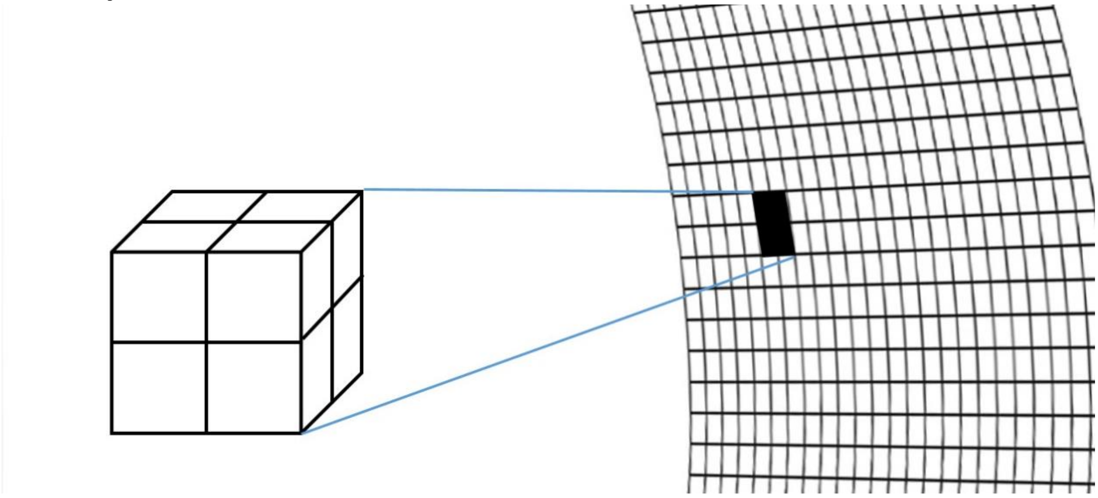

Fig. 1. Encoding 1 bit of information, an area $4\ l_P^2$ on the 2-dimensional boundary surface maps to a 3-dimensional cube of volume $8\ l_H^3$ within the enclosed space. When the radius of the enclosed space is large, $l_H >> l_P$

With $\pi R^2/l_P^2$ bits, each occupying $8\ l_H^3$, distributed in a volume of $^4/_3\ \pi R^3$, we find

$$l_H = (R\ l_P^2/6)^{1/3} \qquad (1)$$

We ask: what is the scale of R such that l_H matches relevant atomic dimensions? The relevant dimension seems to be the diameter of the proton, which is the majority constituent of the baryonic matter in the universe and is the smallest common entity with a non-zero size found in nature (quarks and electrons being point-like).

Setting l_H equal to the diameter of the proton, twice the r.m.s. charge radius of 0.8775 fm, we obtain

$$R = 6\ l_H^3\ /\ l_P^2\ = 1.242 \times 10^{26}\ \text{m} \qquad (2)$$

This value is remarkably close to c_0/H_0, the radius of the Hubble sphere. Using the current estimate from analysis of the Cosmic Microwave Background by the ESA Planck Collaboration [2013], the Hubble constant, H_0, is 67.8 Km/s/Mpc in traditional astrophysical units, giving the value of $c_0/H_0 = 1.364\times10^{26}$ m. Using conventional telescopic observations, a somewhat higher value of $H_0 = 74.3$ Km/s/Mpc is found [Freedman *et al*, 2012], giving the value of $c_0/H_0 = 1.245\times10^{26}$ m.

If we accept this striking coincidence as indicative of a meaningful relationship, we may reverse the calculation to determine the natural length scale, l_H, given the cosmological radius $R = c_0/H_0$, and see how well ½ l_H matches the r.m.s. radius of the proton. Using the Planck Collaboration value for H_0, we obtain 0.9054 fm. With the value from conventional telescopic methods, we have 0.8782 fm. The former value is 3% higher than the proton radius; the latter matches the CODATA experimental value of 0.8775(51) fm within its error band.

If this agreement is not fundamentally meaningful, it is a spectacular coincidence.

Discussion

The Holographic Principle, applied on a cosmological scale, abolishes the informational over-specification implied by the Planck length, l_P, being many orders of magnitude smaller than atomic dimensions. Applying the principle on the cosmological scale, however, raises non-trivial problems [Bousso, 2002]. The question is: which radius should be used in the calculation?

The “size of the observable universe” is not a simple concept in cosmologies subject to a Hubble expansion, where a time-dependent scaling factor for space appears in the metric [Davis & Lineweaver, 2003]. At least three radii merit consideration.

1. The *Hubble sphere*, whose radius of approximately 14 Glyr is the distance that light could travel in an inertial frame in the time since the Big Bang.
2. The *event horizon*, which lies at the slightly greater distance from which light emitted now will eventually be able to reach us, allowing for the universal expansion. Its relationship to the Hubble sphere is model-dependent. In the current *Λ-CDM* model, the two approach equality in the future, becoming almost indistinguishable by twice the current age of the universe [see Fig 1, upper panels, in Davis & Lineweaver, 2003].
3. The *particle horizon*, which is the location of the most remote objects we can now see by light they emitted long ago, and is about 46 Glyr distant, roughly three times the radius of the Hubble sphere.

Detailed analysis of the mathematical foundations of the Holographic Principle identifies the boundary which constrains the entropy and information content of our region as that within whose *future light cone* we presently lie: i.e. the event horizon [Bousso, 2002]. For simplicity, however, we choose the Hubble radius, c_0/H_0, as an approximation independent of the cosmological model.

Using this value for the cosmological radius, we find a spectacular agreement between an inferred natural length scale, l_H, and the size of the proton.

For almost a century now, various concordances have been noticed between atomic and cosmological measures, such as the ratio of electromagnetic to gravitational forces in the atom, on the order of 10^{40}, matching the ratio of cosmological to atomic time-scales. These concordances generate *Large Number Hypotheses* (LNH), which propose that physically significant mechanisms, rather than chance coincidences, are revealed by the numerical agreements [Ray, Mukhopadhyay & Ghosh, 2007]. Dirac was an early proponent of this idea, inferring that the gravitational constant, G, would vary inversely as cosmic time, t.

The numerical agreement reported in the present paper is clearly cognate with LNH, but is different in two important respects.

Firstly, the agreement is remarkably precise. LNH coincidences are usually only to orders of magnitude.

Secondly, though the model is intuitive rather than rigorous, and involves a very unfashionable fixed-matrix Newtonian view of space, the derivation is based on *physical* *principles*. LNH coincidences are typically discovered purely by numerical exploration and some explanation is sought, *post hoc*.

If we are to take coincidences of the LNH type seriously, we must challenge the consensus picture of cosmology, with its enormously successful *Standard Model, Λ-CDM*. For such coincidences to be maintained, the constants of physics would need to change over cosmological timescales. In the absence of any observational evidence for such changes, a conspiracy of compensating effects would be required to maintain a universe which always looks the same. Such notions remain a source of amusing speculation, but in the mainstream are treated with the healthy skepticism they deserve.

The coincidence reported in this paper, surprisingly however, does not challenge the observational basis of the Standard Model.

The proposed natural length scale, l_H, is a function of the radius of the cosmological event horizon, which is remarkably constant from the present era into the future, and changes substantially only in the distant past [Davis & Lineweaver, 2003, Fig 1, top panel]. To change l_H by 10% requires a 33% change in the radius, because of the cube root relationship in equation (1). Such a change will *never* occur in the future, and to find it in the past, we must venture back 11 Gyr, a mere 3 Gyr from the beginning of time, into a highly model-dependent epoch which challenges the limits of observational astronomy.

Unlike most LNH conjectures, the holographic natural length scale is not easily dismissed on observational grounds.

Conclusion

The Holographic Principle, applied on the cosmological distance scale, cures the informational over-specification implied by the smallness of the Planck length, l_P. The natural length scale thus derived is some 20 orders of magnitude larger than l_P and is concordant with atomic dimensions, precisely matching the r.m.s. electromagnetic diameter of the proton.

As others have remarked, the elegance of the universe in which we find ourselves manifests itself in many ways, some of them surprising.

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