# Special features of the mathematical apparatus for the classical electrodynamics

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## Abstract

The special features of the use of vector analysis in the electrodynamics are examined.

Vector analysis is the basic mathematical apparatus for electrodynamics. Such vector quantities, as force, speed, acceleration, electric field and current demonstrate well the physical nature of these values. However, with the use of a vector apparatus for describing the physical processes are introduced such of vector, which do not reflect the physical essence of those processes, which they describe. We will call such vectors vector- phantoms. Let us give several examples.

If is located the disk, which revolves with the angular velocity $\omega$, then they depict this process as the vector, which coincides with the rotational axis of disk and rests in its center. It does ask itself, is there this vector in reality and that it does represent? There is no doubt about the fact that this vector can be introduced by arrangement, but any physical sense as, for example, velocity vector, it it does not have. Thus the vector of momentum is accurately introduced. This vector also coincides with the rotational axis, it rests in the center of the plane of rotation and it is equal to the work of radial velocity to a radius. Similarly is introduced the vector of the magnetic dipole moment, which for the ring current is equal to the work of the current strength to the area of the circle streamlined with current. This vector coincides with the rotational axis of circle and rests on its plane. But any physical sense these a vector do not have.

Let us recall what is the vector is, which presents rotor. This vector is introduced as follows

$$rot\ \vec{a} = \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ a_x & a_y & a_z \end{pmatrix}$$

In order to explain the geometric sense of rotor let us examine solid body, which revolves with the angular velocity $\omega$ around the axis $z$. Then the linear speed of the body $v$ at point $(x, y, z)$ is numerically equal

$$v = \omega r = \omega\sqrt{x^2 + y^2}\ ,$$

and component it along the axes, for the right-handed coordinate system, will be equal

$$v_x = -\frac{vy}{\sqrt{x^2 + y^2}} = -\omega y\,,$$

$$v_y = -\frac{vx}{\sqrt{x^2 + y^2}} = -\omega x\,,$$

$$v_z = 0\,.$$

The vector components $rot\,v$ in this case to be determined by the relationships:

$$rot_x v = rot_y v = 0$$

$$rot_z v = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = 2\omega$$

Is again obtained the vector, directed in parallel to rotational axis and normal toward the plane of rotation. This vector also is introduced by arrangement and of any physical sense it does not have.

Thus, with the use of vector analysis for describing the physical phenomena are introduced two types of vectors. The first of them represents the real physical of vector, which characterize physical quantity itself taking into account of its value and direction (for example, the vector of force, speed, acceleration, tension of electric field and current). Another category of vectors - this those of vector, which can be presented with the aid of the operation of rotor or vector product. These vector do not represent physical quantities and they are introduced by arrangement, being vector- phantoms. Specifically, the vector of such type includes magnetic field.

Magnetic field is introduced or with the aid of the rotor of the electric field

$$\frac{\partial \vec{H}}{\partial t} = -\frac{1}{\mu} rot\vec{E} ,$$

or as the rotor of the vector potential

$$\vec{H} = rot\vec{A}_H .$$

This means that the magnetic field is not physical field, but represents the certain vector symbol, which is introduced by arrangement and of physical sense it does not have.

However, that does occur further? During writing of the Maxwell equations rotor from the magnetic field they make level to the full current

$$rot\vec{H} = rot\ rot\vec{A}_H = \vec{j}_\Sigma \ .$$

Is obtained so that rotor from the vector $\vec{H}$, which is introduced by arrangement, gives the real physical vector of current density. Thus, the vector of magnetic field represents typical vector-phantom.

It is possible to give another example. The Lorentz force, which acts on the moving charge, is determined by the vector product of the real velocity vector and of magnetic field:

$$\vec{F} = \mu\left[\vec{v} \times \vec{H}\right] .$$

Is again obtained so that the operation of vector product, which itself physical sense does not have, with the participation of real vector and vector of phantom it gives real physical force taking into account of its value and direction. Of this consists the sense of introduction in vector analysis of such operations as rotor and vector product. If we look to the mathematical apparatus for physics in connection with to vector analysis, then it appears that this apparatus represents the mixture of real physical vectors and vectors of the phantoms, the relation between which it is regulated with the aid of the, including and operations indicated.

Above it was convincingly shown that entire electrodynamics can be built without the use of this concept as magnetic field.