The tetrad in the curved time-space

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ABSTRACT

In the general relativity theory, defines the tetrad that moves in r-axis in the curved time-space.

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I.Introduction

This theory's object is that defines the tetrad that moves in r-axis in the curved time-space.

The Schwarzschild solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}[\frac{dr^2}{1-\frac{2GM}{rc^2}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2] \quad (1)$$

In this time, a moving matter's acceleration is the constant acceleration a_0 in the Schwarzschild time-space.

$$a_0 = \frac{d}{dt}(\frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}}) \quad (2)$$

$$a_0 t = \frac{u}{\sqrt{1-\frac{2GM}{rc^2}-\frac{u^2}{c^2}}} , \quad u = \sqrt{1-\frac{2GM}{rc^2}}\frac{a_0 t}{\sqrt{1+\frac{a_0^2 t^2}{c^2}}} \quad (3)$$

If $\frac{d\theta}{dt} = \frac{d\phi}{dt} = 0$, the solution is

$$d\tau^2 = (1-\frac{2GM}{rc^2})dt^2 - \frac{1}{c^2}\frac{dr^2}{1-\frac{2GM}{rc^2}} \quad (4)$$

In this time, if uses ψ,

$$1 = (1-\frac{2GM}{rc^2})(\frac{dt}{d\tau})^2 - \frac{1}{c^2}\frac{1}{1-\frac{2GM}{rc^2}}(\frac{dr}{d\tau})^2$$

$$\cosh\psi = \sqrt{1-\frac{2GM}{rc^2}}\frac{dt}{d\tau} , \quad \sinh\psi = \frac{1}{c}\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{d\tau} \quad (5)$$

Therefore, r-axis's velocity V_r is

$$V_r = \frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{dt} = u = \sqrt{1-\frac{2GM}{rc^2}}\frac{a_0 t}{\sqrt{1+\frac{a_0^2 t^2}{c^2}}} \quad (6)$$

According to Eq(5),Eq(6),

$$\frac{1}{c}\frac{dr}{\sqrt{1-\frac{2GM}{rc^2}}}=\frac{1}{c}\frac{a_0 t}{\sqrt{1+\frac{a_0{}^2t^2}{c^2}}}\sqrt{1-\frac{2GM}{rc^2}}dt \text{ , } \cosh\psi=\sqrt{1-\frac{2GM}{rc^2}}\frac{dt}{d\tau}$$

$$=\frac{1}{c}\frac{a_0 t}{\sqrt{1+\frac{a_0{}^2t^2}{c^2}}}\cosh\psi d\tau=\sinh\psi d\tau$$

$$\frac{1}{\cosh^2\psi}=1-(\frac{\sinh\psi}{\cosh\psi})^2=1-(\frac{a_0 t/c}{\sqrt{1+\frac{a_0{}^2t^2}{c^2}}})^2=\frac{1}{1+\frac{a_0{}^2t^2}{c^2}} \quad (7)$$

Hence,

$$\cosh\psi=\sqrt{1+\frac{a_0{}^2t^2}{c^2}} \quad , \ \sinh\psi=\frac{a_0 t}{c} \quad (8)$$

$$\cosh\psi=\sqrt{1-\frac{2GM}{rc^2}}\frac{dt}{d\tau}=\sqrt{1+\frac{a_0{}^2t^2}{c^2}} \text{ , } \sinh\psi=\frac{1}{c}\frac{1}{\sqrt{1-\frac{2GM}{rc^2}}}\frac{dr}{d\tau}=\frac{a_0 t}{c}$$

(9)

Therefore,

$$\frac{dt}{d\tau}=\frac{\sqrt{1+\frac{a_0{}^2t^2}{c^2}}}{\sqrt{1-\frac{2GM}{rc^2}}} \quad , \ \frac{1}{c}\frac{dr}{d\tau}=\frac{a_0 t}{c}\sqrt{1-\frac{2GM}{rc^2}} \quad (10)$$

II. The tetrad in the curved time-space

The tetrad $e_a{}^\mu$ is the unit vector that is each other orthographic and it use the following formula.

$$e_a{}^\mu e_b{}^\upsilon g_{\mu\upsilon}=\eta_{ab} \quad (11)$$

Therefore, Eq(11) is

$$g_{\mu\upsilon}e_0{}^\mu(r,t)e_0{}^\upsilon(r,t)=\eta_{00}=-1$$

$$d\tau^2=-\frac{1}{c^2}g_{\mu\upsilon}dx^\mu dx^\upsilon$$

$$\rightarrow \ -1 = g_{\mu\upsilon}(\frac{1}{c}\frac{dx^{\mu}}{d\tau})(\frac{1}{c}\frac{dx^{\upsilon}}{d\tau}) = g_{\mu\upsilon}e_0^{\ \mu}(r,t)e_0^{\ \upsilon}(r,t) \qquad (12)$$

According to Eq(10),Eq(12)

$$e_0^{\ \alpha}(r,t) = \frac{1}{c}\frac{dx^{\alpha}}{d\tau}$$

$$= (\frac{\sqrt{1+\frac{a_0^{\ 2}t^2}{c^2}}}{\sqrt{1-\frac{2GM}{rc^2}}}, \frac{a_0 t}{c}\sqrt{1-\frac{2GM}{rc^2}}, 0, 0) \qquad (13)$$

About θ -axis's and ϕ -axis's orientation

$$g_{22}e_2^{\ 2}(r,t)e_2^{\ 2}(r,t) = \eta_{22} = 1, \quad e_2^{\ \alpha}(r,t) = (0,0,\frac{1}{r},0) \qquad (14)$$

$$g_{33}e_3^{\ 3}(r,t)e_3^{\ 3}(r,t) = \eta_{33} = 1, \ e_3^{\ \alpha}(r,t) = (0,0,1/r\sin\theta,0) \qquad (15)$$

And the other vector $e_1^{\ \alpha}(r,t)$ has to satisfy the tetrad condition, Eq (11)

$$g_{00}e_0^{\ 0}(r,t)e_1^{\ 0}(r,t) + g_{11}e_0^{\ 1}(r,t)e_1^{\ 1}(r,t) = \eta_{01} = 0$$

$$e_1^{\ \alpha}(r,t) = (\frac{a_0 t/c}{\sqrt{1-\frac{2GM}{rc^2}}}, \sqrt{1+\frac{a_0^{\ 2}t^2}{c^2}}\sqrt{1-\frac{2GM}{rc^2}}, 0, 0) \qquad (16)$$

III. Conclusion

In the general relativity theory, defines the tetrad that moves in r-axis in the curved time-space.

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