Twenty-four conjectures about “the eight essential subsets of primes”

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Abstract. In this paper are made twenty-four conjectures about eight subsets of prime numbers, i.e. the primes of the form 30*k + 1, 30*k + 7, 30*k + 11, 30*k + 13, 30*k + 17, 30*k + 19, 30*k + 23 respectively 30*k + 29. Because we strongly believe that this classification of primes can have many applications, we refered in the title of this paper to these subsets of primes as to “the eight essential subsets of primes”. The conjectures state that each from these eight sets of primes has an infinity of terms and also that each one of them can be entirely defined with a recurrent formula starting from just three given terms.

I.

Conjecture 1:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 31, a(2) = 61, a(3) = 151 [the first three terms of the sequence are the smallest three primes of the form 30*k + 1];
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 1, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 2:
Any prime of the form 30*k + 1 is a term of the sequence a(n) as it is defined by Conjecture 1 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 1].

Verifying the conjecture 2:
(for the primes of the form 30*k + 1 up to 421)

: a(4) = a(1) + a(3) – 1 = 181;
: a(5) = a(2) + a(3) – 1 = 211;
: a(6) = a(1) + a(5) – 1 = a(2) + a(4) – 1 = 241;
: a(7) = a(1) + a(6) – 1 = a(2) + a(5) – 1 = 271;
: a(8) = a(2) + a(7) – 1 = a(3) + a(4) – 1 = 331;
: a(9) = a(3) + a(7) – 1 = a(5) + a(5) – 1 = 421.

Conjecture 3:

If the Conjecture 2 doesn’t hold, than is true at least that any prime of the form 30*k + 1 is a term of a sequence a(n) that can be defined as follows:

: a(1), a(2), a(3) are three distinct primes of the form 30*k + 1;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 1, where 1 ≤ i ≤ j < n.

II.

Conjecture 4:

The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 37, a(2) = 67, a(3) = 97;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 7, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 5:

Any prime of the form 30*k + 7, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 4 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 7, for k > 0].

Verifying the conjecture 5:

(for the primes of the form 30*k + 7 up to 367)

: a(4) = a(1) + a(3) – 7 = a(2) + a(2) – 7 = 127;
: a(5) = a(1) + a(4) – 7 = a(2) + a(3) – 7 = 157;
: a(6) = a(4) + a(5) – 7 = 277;
: a(7) = a(1) + a(6) – 7 = a(5) + a(5) – 7 = 307;
: a(8) = a(1) + a(7) – 7 = a(2) + a(6) – 7 = 337;
: a(9) = a(1) + a(8) – 7 = a(2) + a(7) – 7 = 367.

Conjecture 6:

If the Conjecture 5 doesn’t hold, than is true at least that any prime of the form 30*k + 7, where k > 0, is a term of a sequence a(n) that can be defined as follows:

: a(1), a(2), a(3) are three distinct primes of the form 30*k + 7;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 7, where 1 ≤ i ≤ j < n.

III.

Conjecture 7:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 41, a(2) = 71, a(3) = 101;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 11, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 8:
Any prime of the form 30*k + 11, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 7 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 11, for k > 0].

Verifying the conjecture 8:
(for the primes of the form 30*k + 11 up to 367)

: a(4) = a(1) + a(3) – 11 = a(2) + a(2) – 11 = 131;
: a(5) = a(2) + a(4) – 11 = a(3) + a(3) – 11 = 191;
: a(6) = a(2) + a(5) – 11 = a(4) + a(4)- 11 = 251;
: a(7) = a(1) + a(6) – 11 = a(3) + a(5) – 11 = 281;
: a(8) = a(1) + a(7) – 11 = a(2) + a(6) – 11 = 311;
: a(9) = a(3) + a(8) – 1 = a(4) + a(7) – 1 = 401.

Conjecture 9:
If the Conjecture 8 doesn't hold, than is true at least that any prime of the form 30*k + 11, where k > 0, is a term of a sequence a(n) that can be defined as follows:

: a(1), a(2), a(3) are three distinct primes of the form 30*k + 11;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 11, where 1 ≤ i ≤ j < n.

IV.

Conjecture 10:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 43, a(2) = 73, a(3) = 103;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 13, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 11:

Any prime of the form 30*k + 13, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 10 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 13, for k > 0].

Conjecture 12:

If the Conjecture 11 doesn’t hold, than is true at least that any prime of the form 30*k + 13, where k > 0, is a term of a sequence a(n) that can be defined as follows:

: a(1), a(2), a(3) are three distinct primes of the form 30*k + 13;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 13, where 1 ≤ i ≤ j < n.

V.

Conjecture 13:

The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 47, a(2) = 107, a(3) = 137;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 17, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 14:

Any prime of the form 30*k + 17, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 13 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 17, for k > 0].

Conjecture 15:

If the Conjecture 14 doesn’t hold, than is true at least that any prime of the form 30*k + 17, where k > 0, is a term of a sequence a(n) that can be defined as follows:

: a(1), a(2), a(3) are three distinct primes of the form 30*k + 17;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 17, where 1 ≤ i ≤ j < n.

VI.

Conjecture 16:

The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 79, a(2) = 109, a(3) = 139;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 19, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 17:
Any prime of the form 30*k + 19, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 16 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 19, for k > 0].

Conjecture 18:
If the Conjecture 17 doesn’t hold, than is true at least that any prime of the form 30*k + 19, where k > 0, is a term of a sequence a(n) that can be defined as follows:
: a(1), a(2), a(3) are three distinct primes of the form 30*k + 19;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 19, where 1 ≤ i ≤ j < n.

VII.

Conjecture 19:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:
: a(1) = 53, a(2) = 83, a(3) = 113;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 23, where 1 ≤ i ≤ j < n (if such prime exists for any n, as the conjecture states).

Conjecture 20:
Any prime of the form 30*k + 23, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 19 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 23, for k > 0].

Conjecture 21:
If the Conjecture 20 doesn’t hold, than is true at least that any prime of the form 30*k + 23, where k > 0, is a term of a sequence a(n) that can be defined as follows:
: a(1), a(2), a(3) are three distinct primes of the form 30*k + 23;
: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 23, where 1 ≤ i ≤ j < n.

VIII.

Conjecture 22:
The sequence a(n), as it will be defined below, has in infinity of terms that are prime numbers.

The sequence a(n), where n non-null positive integer, is defined in the following way:

: a(1) = 59, a(2) = 89, a(3) = 149;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 29, where $1 \leq i \leq j < n$ (if such prime exists for any n, as the conjecture states).

Conjecture 23:

Any prime of the form 30*k + 29, where k > 0, is a term of the sequence a(n) as it is defined by Conjecture 22 [in other words, the sequence a(n) is the same with the sequence of the primes of the form 30*k + 29, for k > 0].

Conjecture 24:

If the Conjecture 23 doesn't hold, than is true at least that any prime of the form 30*k + 29, where k > 0, is a term of a sequence a(n) that can be defined as follows:

: a(1), a(2), a(3) are three distinct primes of the form 30*k + 29;

: a(n) is the smallest prime greater than a(n – 1) that can be written as a(n) = a(i) + a(j) – 29, where $1 \leq i \leq j < n$.