# The proof of the Bill conjecture

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**The math problem is Bill conjecture:**

**Guess one:** if $A^x + B^y = C^z$, and A, B, C, x, y, z are all positive integers, and x, y, z are greater than 2, then the A, B, C must have a common prime factor.

**Guess two:** if A, B, C are positive integers and overall coprime. Then the equation $A^x + B^y = C^z$, no x, y, z all positive integer solutions of greater than 2.

The essence of these two kinds of conjecture is the same.

**Prove:**

Let x, y, z is a positive integer, and greater than 2.

Let $x = y$, $z = x + 1$, have to

$$2^x + 2^y = 2^z \tag{1}$$

Obviously 2 and 2 are not coprime.

Let $n = y = z$, D, E, F is a positive integer, and $D > E$. Then $F = D^n - E^n$, according to the Catalan (Eugne Charles Catalan) theorem, F will not equal to 1.

Have to

$$F + E^n = D^n \tag{2}$$

Both sides also multiplied by $F^n$ term, have to

$$F^{(n+1)} + (EF)^n = (DF)^n$$

.

Let $F = GH$, G is a positive integer, H is a prime.

Have to

$$(GH)^{(n+1)} + (EGH)^n = (DGH)^n \tag{3}$$

Let $A = GH$, $B = EGH$, $C = DGH$, and $x = n + 1$, $y = z = n$. Because E, D can be any value, n also can be any value, for the type (2) both sides only multiplied by $F^n$ term can turn into the indefinite equation form $A^x + B^y = C^z$. So in addition to the type (1), the type (3) is the indefinite equation $A^x + B^y = C^z$ solution form only. In this kind of indefinite equation A, B, C must have a common a prime factor H.

So for $y \neq z$ case? In addition to $y|z$ or $z|y$, for $F + E^y = D^z$ equation on both sides, no matter what multiplied by about y, z, D, E, F term can't turn into the indefinite equation form $A^x + B^y = C^z$. When $y|z$ or $z|y$, $F = D^z - E^y$ can be written as $F = (D^p)^y - E^y$ or $F = D^z - (E^q)^z$ (p, q is an integer greater than 1, and $yp = z$ or $zq = y$), then the type (3) is the same indefinite equation $A^x + B^y = C^z$ solution form, just the D turn into $D^p$ or the E turn into $E^q$.

**By the discussion above all, the Bill conjecture is proved.**