An Elementary Proof of the Kepler's Conjecture

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Abstract

Heap together equivalent spheres into a cube up to most possible, then variant general volumes of equivalent spheres inside the cube depend on variant arrangements of equivalent spheres fundamentally. This π/√18 which the Kepler's conjecture mentions is the ratio of the general volume of equivalent spheres under the maximum to the volume of the cube. We will do a closer arrangement of equivalent spheres inside a cube. Further let a general volume of equivalent spheres to getting greater and greater, up to tend upwards the super-limit, in pace with which each of equivalent spheres is getting smaller and smaller, and their amount is getting more and more. We will prove the Kepler's conjecture by such a way in this article.

Keywords

Equivalent spheres, rearrangements, cube, cuboid, square, rectangle, spherical center, volume, differential cubes, point-like spheres, super-limit, infinity, Ratio, π/√18.

Basic Concepts

First let us review following a few of basic concepts once more, for they relating to the proof.

The Kepler's conjecture states that heap together equivalent spheres into a cube, then, a ratio of any general volume of equivalent spheres inside the cube to the volume of the cube is not greater than π/√18 always.

Suppose that the length of each edge of a cube is α, then the volume of the cube is $\alpha^{3,}$ and the volume of internal tangent sphere of the cube is equal to $\pi\alpha^3/6$.

The ratio of the volume of internal tangent sphere of a cube to the volume of the cube is equal to $\pi/6$.

Suppose that the length of edges of a cuboid is α, β and γ respectively, then the volume of the cuboid is $\alpha\beta\gamma$.

Suppose that the length of a rectangle is α , and its breadth is β, then its area is $\alpha\beta$.

Suppose that the length of each side of a square is α , then its area is α^2.

Aforementioned α , β and γ are real numbers. Here we need to stress a bit that you can get a line segment whereby any real number measures from the number axis to act as an edge of rectangular parallelopiped or a diameter of sphere.

The Proof

Suppose that the length of each edge of cube K is L, then its volume is equal to L^3, and either diagonal of each square surface of cube K is equal to $\sqrt{2}L$, where L is a real number.

We will use cube K_2 which is equivalent to cube K to act as a container which holds equivalent spheres, in this proof.

Suppose that the length of each edge of cube N is $\sqrt[6]{2}L$, then its volume is equal to $\sqrt{2}L^3$, and its internal tangent sphere's volume is equal to $\sqrt{2}L^3\pi/6$.

Thus it can computed, the ratio of the volume of the internal tangent sphere of cube N to the volume of cube K is equal to $\sqrt{2}\pi/6$, i.e. $\pi/\sqrt{18}$. This ratio is just that ratio which the Kepler's conjecture mentions.

Suppose that the length of each edge of cube M is $\sqrt{2}L$, then its volume is equal to $2\sqrt{2}L^3$.

Since there is $L<\sqrt[6]{2}L<\sqrt{2}L$, thus we let cube K lie amidst cube N, and let cube N lie amidst cube M, also enable each of them to have two horizontal surfaces, and any pair of opposite surfaces of each of them parallels a pair of opposite surfaces of either of the other two.

The volume of the annular solid between six square surfaces of cube K and six square surfaces of cube N is equal to $(\sqrt{2}-1)L^{3,}$ including the area of six square surface of cube N theoretically.

There is a rectangular parallelepiped between the upper horizontal square surface of cube K and the surface of amidst the upper horizontal square surface of cube M. Also there is a rectangular parallelepiped between the bottom horizontal square surface of cube K and the square of amidst the bottom horizontal square surface of cube M, and either rectangular parallelepiped has four edges whose each equals $1/2(\sqrt{2}L-L)$ and eight edges whose each equals L. So via the simple computation, the general volume of the two rectangular parallelepipeds is equal to $(\sqrt{2}-1)L^3$ including two areas $2L^2$ of two square of amidst the upper and bottom horizontal square surfaces of cube M theoretically.

Thus it can seen, the general volume of two such rectangular parallelepipeds is just equal to the general volume of aforementioned the annular solid, for either general volume is equal to $(\sqrt{2}-1)L^3$.

Two such rectangular parallelepipeds plus cube K constitute a larger rectangular parallelepiped, and we name the larger rectangular parallelepiped “cuboid R”.

The length, width and height of cuboid R be L, L and $\sqrt{2}L$ respectively, so its volume is equal to $\sqrt{2}L^{3.}$

It is known that the length of each edge of cube N is $^6\sqrt{2}L$, and its volume is equal to $\sqrt{2}L^3$ too.

So the volume of cuboid R is equal to the volume of cube N, clearly cube K is both central section of cube N and the middle of cuboid R.

Now let us divide M into y^3 smaller equivalent cubes or y^3 smaller equivalent cubes plus the remainders which cannot form any such smaller cube, where y is a natural number.

In pace with which y is getting greater and greater, cube M is divided into smaller and smaller equivalent cubes or such smaller cubes plus the remainders, of course not excepting cube N and cuboid R inside cube M.

After cube M is divided for each once, there is a difference between the volume of cube N and a general volume of all smaller equivalent cubes within cube N, and there is a difference between the volume of cuboid R and a general volume of all smaller equivalent cubes within cuboid R. And two such differences are getting less and less in pace with which y is getting greater and greater.

If y tends upwards infinity, then two such differences tend downwards the zero, or at most one is equal to the zero because the length or width of cuboid R is L, while the length of each edge of cube N is $\sqrt[6]{2}L$, obviously L and $\sqrt[6]{2}L$ have not any common factor within the limits of real numbers, yet can only whereby such a common factor acts as the length of every edge of smaller equivalent cubes, justly the two differences are equal-ability to the zero simultaneously.

That is to say, a general volume of all very tiny equivalent cubes within cube N tends upwards the volume of cube N, and a general volume of all

very tiny equivalent cubes within cuboid R tends upwards the volume of cuboid R, when y tends upwards infinity.
In addition, the volume of cuboid R is equal to the volume of cube N, therefore a general volume of all very tiny equivalent cubes within cuboid R tends towards a general volume of all very tiny equivalent cubes within cube N.

Thus it can otherwise seen, on the one hand, there are merely very tiny equivalent cubes or very tiny equivalent cubes plus the remainders within cube M when y tends upwards infinity; on the other, very tiny equivalent cubes and their internal tangent spheres within cube M can only come into being after tends upwards infinitely divide cube M. Hereinafter such very tiny equivalent cubes are termed “differential cubes”, and very tiny equivalent internal tangent spheres of differential cubes are termed “point-like spheres”.

Also differential cubes within cube M equal one another, so the sum total of all differential cubes within cuboid R tends towards the sum total of all differential cubes within cube N.
Moreover there is a point-like sphere within every differential cube only, thus the general volume of all point-like spheres inside cuboid R tends towards the general volume of all point-like spheres inside cube N.

Also the ratio of the volume of the internal tangent sphere of a cube to the volume of the cube is equal to $\pi/6$. Then, the ratio of the general volume of all point-like spheres inside cube N to the general volume of all differential cubes within cube N is equal to $\pi/6$.

In addition, the ratio of the volume of the internal tangent sphere of cube N to the volume of cube N is equal to $\pi/6$, and the general volume of all differential cubes within cube N tends upwards the volume of cube N, therefore the general volume of all point-like spheres inside cube N tends upwards the volume of the internal tangent sphere of cube N.

Moreover the general volume of all point-like spheres inside cuboid R tends towards the general volume of all point-like spheres inside cube N, therefore the general volume of all point-like spheres inside cuboid R tends upwards the volume of the internal tangent sphere of cube N.

In addition, the ratio of the volume of the internal tangent sphere of cube N to the volume of cube K is equal to $\pi/\sqrt{18}$, therefore the ratio of the general volume of all point-like spheres inside cuboid R to the volume of cube K tends upwards $\pi/\sqrt{18}$.

After cube M is divided to y^3 smaller equivalent cubes or y^3 smaller equivalent cubes plus the remainders, spherical centers of internal tangent spheres of smaller equivalent cubes within cuboid R including cube K lie both at some equivalent horizontal squares and at some equivalent vertical rectangles. Besides two adjacent such horizontal squares are a diameter of the internal tangent sphere apart, and two adjacent such vertical rectangles are the diameter apart too.

Every such sphere's center arranges both at a rank which has more such spheres' centers and at a file which has more such spheres' centers at each such horizontal square inside cuboid R including cube K. Besides two adjacent spheres' centers at a line tend downward a diameter of the internal tangent sphere apart, and here "line" denotes both "rank" and "file", the same below. The arrangement of the spheres' centers at every such horizontal square be just the same as compared with the other each.

Every sphere's center arranges both at a rank which has more such spheres' centers and at a file which has more such spheres' centers at each such vertical rectangle inside cuboid R. Besides, two adjacent spheres' centers at a line tend downward a diameter of the internal tangent sphere apart.

The arrangement of the spheres' centers at every such vertical rectangle be just the same as compared with the other each.

Two adjacent point-like spheres inside cuboid R tend downward externally tangent, namely the distance between two adjacent point-like spheres' centers tends downwards a diameter of point-like sphere, also point-like spheres on the marginality inside cuboid R and surfaces of cuboid R tend downward internally tangent.

Now we will make an attempt to put all point-like spheres inside cuboid R into cube K, of course this is merely an explanation at the theory.

Since cube K is a part of cuboid R, thus in order to differentiate between cube K and cuboid R, let us adopt a reproduction of cube K to replace cube K, and put the reproduction out of cuboid R, and enable it to have two horizontal surfaces. We otherwise term the reproduction "cube K_2". Justly cube K_2 is equivalent to cube K , but there are no spheres inside cube K_2 by now.

For the sake of aforementioned purpose, first we need to determine points as spheres' centers of all point-like spheres inside cube K_2 under the prerequisite which can inlay every point-like sphere inside cuboid R, and the way of doing is as the follows.

Let us name the common rectangular bisector of any pair of opposite right dihedral angles which have two horizontal surfaces and two vertical surfaces inside cube K_2 "rectangle B_k".

Obviously rectangle B_k and the either horizontal surface form a dihedral angle of $\pi/4$, and rectangle B_k and the either vertical surface form a dihedral angle of $\pi/4$ too. Thereupon we reckon rectangle B_k as a slantwise rectangle.

Also we name any vertical rectangle which has more spheres' centers of point-like spheres inside cuboid R "rectangle H_R".

Since the breadth of rectangle B_k is equal to the breadth of rectangle $H_{R,}$ and the length of rectangle B_k is equal to the length of rectangle $H_{R,}$ so rectangle B_k is equivalent to rectangle $H_{R.}$

Up to now, we may make a breakthrough exactly to the beginning. First determine points as point-like spheres' centers at rectangle B_k according to the arrangement of all point-like spheres' centers at rectangle $H_{R.}$

Then the sum total of ranks of point-like spheres' centers at rectangle B_k is equal to the sum total of ranks of point-like spheres' centers at rectangle H_R, and the sum total of point-like spheres' centers at the each rank at rectangle B_k is equal to the sum total of point-like spheres' centers at the each rank at rectangle H_R.

Secondly, determine a horizontal square inside cube K_2 by means of each rank of point-like spheres' centers at rectangle $B_{k\,,}$ of course each such horizontal square inside cube K_2 is equivalent to each horizontal square inside cuboid R, and every such horizontal square inside cube K_2 continues to have a rank of point-like spheres' centers at rectangle $B_{k.}$.

Moreover take such a rank as the datum rank wherewith to determine other ranks of point-like spheres' centers at each such horizontal square.

After that, leave from such a datum rank to determine orderly other ranks of point-like spheres' centers at each and every such horizontal square inside cube K_2 according to the interval between two adjacent ranks of point-like spheres' centers at any such horizontal square inside cuboid R, viz. two adjacent spheres' centers tend downward a diameter of point-like sphere apart, until cannot continue to determine such sphere's center.

Since the sum total of ranks of point-like spheres' centers at rectangle B_k is equal to the sum total of ranks of point-like spheres' centers at rectangle H_R, from this determined that the sum total of horizontal squares which contain more point-like spheres' centers inside cube K_2 is equal to the sum total of horizontal squares which contain more point-like spheres' centers inside cuboid R.

Moreover every horizontal square inside cube K_2 is equivalent to every horizontal square inside cuboid R, and the arrangement of point-like spheres' centers at each horizontal square inside cube K_2 and the arrangement of point-like spheres' centers at each horizontal square inside cuboid R are just the same, to wit the identical number of point-like spheres' centers to the both, consequently the sum total of determined point-like spheres' centers inside cube K_2 is equal to the sum total of point-like spheres' centers inside cuboid R.

After points as point-like spheres' centers inside cube K_2 are determined, we are not difficult to seen, all point-like spheres' centers inside cube K_2 are both at some equivalent horizontal squares and at slantwise rectangle B_k plus some slantwise rectangles which parallel slantwise rectangle B_k, and the farther slantwise rectangle leaves from slantwise rectangle B_k, the smaller is it.

Hereunder we need to test and verify whether can inlay a point-like sphere for every determined point-like sphere's center inside cube K_2.

First, considering the horizontal direction, two determined adjacent point-like spheres' centers tend downward a diameter of point-like sphere apart. If whereby both of them act as two point-like spheres' centers, then inlaid two point-like spheres tend toward the externally tangent on the horizontal direction.

Secondly, considering the slantwise direction whose inclination is $\pi/4$, two determined adjacent point-like spheres' centers tend downward a diameter of point-like sphere apart. If whereby both of them act as two point-like spheres' centers, then inlaid two point-like spheres tend toward the externally tangent on the slantwise direction whose inclination is $\pi/4$.

Thirdly, since the distance between two adjacent horizontal squares which contain point-like spheres' centers tend downward $\sqrt{2}/2$ diameter of point-like sphere apart, and the distance between two adjacent slantwise rectangles which contain point-like spheres' centers tend downward $\sqrt{2}/2$ diameter of point-like sphere apart too, thus it can seen, the distance between the two adjacent planes on the either direction is greater than a radius of point-like sphere. If whereby every determined point-like sphere's center's point acts as a point-like sphere's center, then can inlay all point-like spheres enough on the vertical direction and slantwise direction whose inclination is $3\pi/4$.

Altogether, there is possible on eight directions which divide averagely the 2π-space round every determined point-like sphere's center inside cube K_2, consequently it is able to inlay all point-like spheres inside cuboid R into cube K_2 barely enough.

After put all point-like spheres inside cuboid R into cube K_2, the ratio of the general volume of all point-like spheres inside cube K_2 to the volume of cube K_2 tends upwards $\pi / \sqrt{18}$.

In other words, the ratio of the general volume of all point-like spheres after the rearrangement inside cube K to the volume of cube K tends upwards $\pi / \sqrt{18}$.

For above-mentioned the way of doing, we can only from the theory to understand, because after tend upwards infinitely divide cube M in pace with which y tends upwards infinity, we utterly impossibly probe to being any point-like sphere.

Nevertheless we may let y to equal a non-large natural number first, then, put internal tangent spheres of all equivalent cubes inside cuboid R into cube K_2 really according to aforementioned the way of doing.

After that, once by once put internal tangent spheres of smaller and smaller equivalent cubes inside cuboid R into cube K_2 according to the way of doing, in pace with which y be getting greater and greater, and so on and so forth, up to y tends upwards infinity, then the way of doing is from feasibility on the practice up to feasibility at the theory.

If y tends upwards infinity, then it reaches to the super-limit as to put all point-like spheres inside cuboid R into cube K_2 barely enough. Under this case, two adjacent point-like spheres tend toward the externally tangent, and each and every point-like sphere on the marginality and a surface of cube K_2 tend toward the internally tangent.

Since there are infinitely many natural numbers, namely there are infinitely many values of y, thus y has not a maximum value, therefore the ratio of the general volume of all point-like spheres inside cube K_2 to the volume of cube K_2 can only tend infinitesimally upwards $\pi/\sqrt{18}$, however it cannot equal $\pi/\sqrt{18}$ always. In other words, the super-limit of the ratio is $\pi/\sqrt{18}$.

We may also so imagine that the volume of a smallest sphere is less than a volume of any point-like sphere, if the ratio is equal to $\pi/\sqrt{18}$, then the general volume of all smallest spheres within cube K_2 is equal to the volume of the internal tangent sphere of cube N. But, every contact among smallest spheres, among smallest spheres on the marginality and surfaces of cube K_2 is a common tangent point of two geometrical solids. So all smallest spheres within cube K_2 and cube K_2 link into an impartible object, yet the object is neither a cube nor many independent smallest spheres, even it is no a geometrical solid. Thus the ratio of the general volume of all point-like spheres inside cube K_2 to the volume of cube K_2 always cannot come up to $\pi/\sqrt{18}$.

Thus far, proving for the Kepler's conjecture has ended. If there is no any nonlogical inference, then the conjecture is proven as the true.