A Complete Proof of the Polignac's Conjecture

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Abstract

Let us consider odd numbers which share a prime factor>1 as a kind, then the number axis's positive half line which begins with odd point 3 consists of infinite many equivalent line segments on same permutation of χ kinds' odd points plus odd points amongst the χ kinds' odd points, where χ≥1. In this article, we shall prove the unproved half of the Polignac's conjecture by mathematical induction with the aid of such equivalent line segments and kinds of odd points thereon.

Keywords

Number axis's positive half line, Kinds of odd points, Mathematical induction, The Coexisting theorem,

Basic Concepts

In 1849, Polignac conjectured that for every even number 2n are there infinitely many pairs of consecutive primes which differ by 2n, where n≥1. Yet this is an unproved conjecture up to now.

Happily, I had proven a half of the conjecture by the end of last year, and the half states that there are infinitely many pairs of consecutive odd prime numbers. The paper with relation to the half is published at pp. 17-26, Number 1 (2013) of learned journal "Advances in Theoretical and Applied Mathematics" of Research India Publications.

What we need is to successively prove another half of the conjecture by now, namely prove that every positive even number 2n is a difference of two consecutive odd prime numbers. Nevertheless apply more or less alike method as compared with the proven half, therefore following basic concepts expounded have more repeats. Of course, this is inevitable and indispensable for the new proof.

Everyone knows, each and every odd point at positive half line of the number axis expresses a positive odd number. Also infinite many a distance between two consecutive odd points at the positive half line equal one another. Afterwards, the number axis's positive half line which begins with odd point 3 is called the half line for short.

Let us use symbol "•" to denote an odd point at the beginning's half line and in formulations. Moreover the half line is marked merely with symbols of odd points here. Please, see following first illustration.

First Illustration

We use also symbol "•s" to denote at least two undefined odd points in formulations. We consider smallest positive odd prime number 3 as №1 odd prime number, and consider positive odd prime number J_χ as №χ odd prime number, where $\chi \geq 1$, then odd prime number 3 is written as J_1 as well. And then, we consider positive odd numbers which share prime factor J_χ as №χ kind of odd numbers. If an odd number contains α different prime factors, then, the odd number concurrently belongs in α kinds of odd numbers, where $\alpha \geq 1$.

There is an only odd prime number J_χ within №χ kind's odd numbers. Existing J_χ, we term others as №χ kind of odd composite numbers.

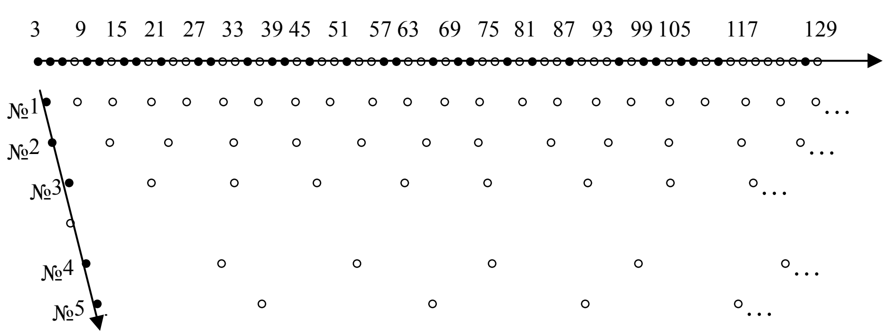
If one • is defined as an odd composite point, then we change symbol "◦" for symbol "•". And use symbol "◦s" to denote the plural in formulations.

If one • is affirmed as an odd prime point, then this • is rewritten as one ♠. And use ♠s to denote the plural in formulations.

In the course of the proof, we shall change ◦s for •s at places of ∑№χ [χ≥1] kind's odd composite points orderly according as χ is from small to large.

Since №χ kind's odd numbers are infinitely many a product which multiplies every odd number by J_χ, so there is a №χ kind's odd point within consecutive J_χ odd points at the half line.

We analyze seriatim №χ kind of odd points at the half line according to χ =1, 2, 3 … in one by one, and range them as second illustration.



Second Illustration

Thus it can seen, one another's permutation of χ kinds of odd points plus odd points amongst the χ kinds of odd points assumes always infinite many recurrences on same pattern at the half line, irrespective of their prime/composite attribute. We consider one another's-equivalent shortest line segments of the half line according to same permutation of χ kinds' odd points plus odd points amongst the χ kinds' odd points as recurring segments of the χ kinds' odd points. And use character "$RLS_{№1\sim№\chi}$" to express a recurring segment of $\sum№\chi$ [$\chi\geq1$] kind of odd points, also use $RLSS_{№1\sim№\chi}$ to express the plural.

Number the ordinals of odd points at seriate each $RLS_{№1\sim№\chi}$ by consecutive natural numbers ≥1, namely from left to right each odd point at seriate each $RLS_{№1\sim№\chi}$ is marked with from small to great a natural number which begins with 1 in the proper order.

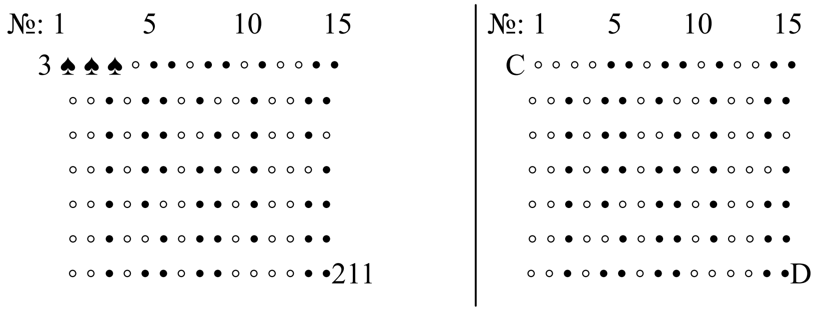
Then, there is one №(χ+1) kind's odd point within $J_{\chi+1}$ odd points which share an ordinal at $J_{\chi+1}$ $RLSS_{№1\sim№\chi}$ of seriate each $RLS_{№1\sim№(\chi+1)}$.

Excepting most left one at №1 $RLS_{№1\sim№\chi}$ is an odd prime point, others are all odd composite points, in №χ kind's odd points. Thus №1 $RLS_{№1\sim№\chi}$ is a particular $RLS_{№1\sim№\chi}$ in contradistinction to each of others.

There are $\prod J_\chi$ odd points at each $RLS_{№1\sim№\chi}$, where $\prod J_\chi=J_1*J_2*\ldots*J_\chi$, and $\chi\geq1$. Justly №1 $RLS_{№1\sim№\chi}$ begins with odd point 3. Yet №1 $RLS_{№1\sim№1}$ ends with odd point 7; №1 $RLS_{№1\sim№2}$ ends with odd point 31; №1 $RLS_{№1\sim№3}$ ends with odd point 211; №1 $RLS_{№1\sim№4}$ ends with odd point 2311, etc. Undoubtedly one $RLS_{№1\sim№(\chi+1)}$ consists of $J_{\chi+1}$ consecutive $RLSS_{№1\sim№\chi}$, and they link, one by one.

$J_{\chi+1}$ $RLSS_{№1\sim№\chi}$ of any $RLS_{№1\sim№(\chi+1)}$ may be folded at an illustration, so as to view conveniently. For instance, after change ◦s for •s at places of $\sum№\chi$ [$\chi\leq3$] kind's odd composite points, for odd points at №1 $RLS_{№1\sim№3}$

i.e. 3(211) and at another $RLS_{№1\sim№3}$ CD, please, see third illustration.



Third Illustration

Thus it can seen, after change ◦s for •s at places of $\sum№\chi$ [$\chi\geq1$] plus №(χ+1) kind's odd composite points, there is one №(χ+1) kind's odd composite point within $J_{\chi+1}$ odd points on an ordinal of every odd point of a $RLS_{№1\sim№\chi}$ at seriate each $RLS_{№1\sim№(\chi+1)}$ on the right of №1 $RLS_{№1\sim№(\chi+1)}$.

After change ◦s for •s at places of $\sum№\chi$ [$\chi\geq1$] kind's odd composite points, if an odd point P_1 is separated from another odd point P_2 by λ_χ ◦s, then express such a combinative form as a pair of P_1 λ_χ(◦s) P_2, where $\lambda_\chi \geq 0$.

After change ◦s for •s at places of $\sum№\chi$ [$\chi\geq1$] kind's odd composite points, there are pairs of ♠ λ_{χ}(◦s) ♠ on the right of J_χ at №1 $RLS_{№1\sim№\chi}$ and pairs of • λ_χ(◦s) • on ordinals of ♠ λ_{χ}(◦s) ♠ at seriate each $RLS_{№1\sim№\chi}$ on the right of №1 $RLS_{№1\sim№\chi}$, where $\lambda_\chi \geq 0$.

From the definition for recurring segments of χ kinds' odd points, we can conclude that after change ◦s for •s at places of $\sum№\chi$ [$\chi\geq1$] kind's odd composite points, provided there is a pair of ♠ λ_{χ}(◦s) ♠ on the right of J_χ at №1 $RLS_{№1\sim№\chi}$, then there is surely a pair of • λ_{χ}(◦s) • on ordinals of the pair of ♠ $\lambda_{\chi(\circ s)}$ ♠ at seriate each $RLS_{№1\sim№\chi}$ on the right of №1 $RLS_{№1\sim№\chi}$. Undoubtedly, the converse proposition is tenable too. Namely after change ◦s for •s at places of ∑№χ [$\chi\geq1$] kind's odd composite points, provided there is a pair of • $\lambda_{\chi(\circ s)}$ • at seriate each $RLS_{№1\sim№\chi}$ on the right of №1 $RLS_{№1\sim№\chi}$, and all such pairs of • $\lambda_{\chi(\circ s)}$ • share a set of ordinals, then there is surely a pair of ♠ $\lambda_{\chi(\circ s)}$ ♠ on ordinals of any such pair of •$\lambda_{\chi(\circ s)}$• at №1 $RLS_{№1\sim№\chi}$. Of course, either ♠ of the pair of ♠ $\lambda_{\chi(\circ s)}$ ♠ and every prime factor of an odd number which each • of all such pairs of •$\lambda_{\chi(\circ s)}$ • expresses are greater than J_χ .

To be brief, after change ◦s for •s at places of ∑№χ [$\chi\geq1$] kind's odd composite points, a pair of ♠ $\lambda_{\chi(\circ s)}$ ♠ on the right of J_χ at №1 $RLS_{№1\sim№\chi}$ and infinite many pairs of • $\lambda_{\chi(\circ s)}$ • on ordinals of the pair of ♠ $\lambda_{\chi(\circ s)}$ ♠ at seriate $RLSS_{№1\sim№\chi}$ on the right of №1 $RLS_{№1\sim№\chi}$ coexist at the half line.

We term the aforesaid conclusion as the coexisting theorem for a pair of ♠$\lambda_{\chi(\circ s)}$ ♠ and infinite many pairs of •$\lambda_{\chi(\circ s)}$ • at the half line, or term it as the coexisting theorem for short.

The Proof

We shall prove indirectly the unproved half of the Polignac's conjecture by mathematical induction with the aid of the coexisting theorem for a pair of ♠ $\lambda_{\chi(\circ s)}$ ♠ and infinite many pairs of • $\lambda_{\chi(\circ s)}$ • at the half line, below.

(1). When $\chi=1$, there is a pair of ♠ $0_{(\circ s)}$ ♠ on the right of J_1 at №1 $RLS_{№1}$, and the pair of ♠ $0_{(\circ s)}$ ♠ is the very odd prime points 5 and 7.

When $\chi=2$, there are pairs of ♠ $\Omega_{2(\circ s)}$ ♠ on the right of J_2 at №1 $RLS_{№1\sim№2,}$ where $\Omega_2 = 0, 1, 2$.

When $\chi=3$, there are pairs of ♠ $\Omega_{3(\circ s)}$ ♠ on the right of J_3 at №1 $RLS_{№1\sim№3,}$ where $\Omega_2 \leq \Omega_3 \leq 6$.

When $\chi=4$, there are pairs of ♠ $\lambda_{4(\circ s)}$ ♠ on the right of J_4 at №1 $RLS_{№1\sim№4}$, where $\lambda_4 = \Omega_4$ plus κ_4, $\Omega_3 \leq \Omega_4 \leq 11$, and $\kappa_4 = 16$.

(2). When $\chi = \beta \geq 4$, suppose that there are pairs of ♠ $\lambda_{\beta(\circ s)}$ ♠ on the right of J_β at №1 $RLS_{№1\sim№\beta,}$ where $\lambda_\beta = \Omega_\beta$ plus κ_β, and Ω_β expresses any of consecutive natural numbers ≥ 1 plus 0, and $\Omega_\beta \geq \Omega_4$. In addition, let greatest value of Ω_β is η_β, then $\eta_\beta \geq 11$, and $\kappa_\beta > \eta_\beta + 1$.

(3). When $\chi = \beta+1$, we must prove that there are pairs of ♠ $\lambda_{\beta+1(\circ s)}$ ♠ on the right of $J_{\beta+1}$ at №1 $RLS_{№1\sim№(\beta+1),}$ where $\lambda_{\beta+1} = \Omega_{\beta+1}$ plus $\kappa_{\beta+1}$, and $\Omega_{\beta+1}$ expresses any of consecutive natural numbers ≥ 1 plus 0, and $\Omega_{\beta+1} \geq \Omega_\beta$. That is to say, at least one of $\Omega_{\beta+1}$ must be equal to $\eta_\beta + 1$ on a minimum, and let greatest value of $\Omega_{\beta+1}$ is $\eta_{\beta+1,}$ and $\kappa_{\beta+1} > \eta_{\beta+1} + 1$.

Proof. For the number axis's positive half line which is marked merely with symbols of undefined odd points , after change ∘s for •s at places of №1 kind's odd composite points, there is a pair of • $0_{(\circ s)}$ • on the right of J_1 at №1 $RLS_{№1}$. Besides there are pairs of • $\Omega_{1(\circ s)}$ • on the right of J_1 at seriate each $RLS_{№1\sim№2}$, where $\Omega_1 = 0, 1$. And every pair of •$\Omega_{1(\circ s)}$ • with a pair of •$\Omega_{1(\circ s)}$• on either side except for the left side of №1 pair of •$0_{(\circ s)}$•

forms two concurrent pairs which share an odd point.

Provided successively change ◦s for •s at places of №2 kind's odd composite points, since there is one №2 kind's odd composite point within J_2 odd points on an ordinal of every odd point of a $RLS_{№1}$ at seriate each $RLS_{№1\sim№2}$ on the right of №1 $RLS_{№1\sim№2}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $\Omega_1+1+\Omega_1 = 2$.

After successively change ◦s for •s at places of №2 kind's odd composite points, there are both pairs of • $\Omega_{1(◦s)}$ • and pairs of • $\Omega_{2(◦s)}$ • on the right of J_2 at seriate each $RLS_{№1\sim№2}$, where $\Omega_1 \leq \Omega_2 \leq 2$. Excepting a part of pairs of •$\Omega_{2(◦s)}$• belong to pairs of •$\Omega_{1(◦s)}$•, each of others exists at the place of two concurrent pairs of original •$\Omega_{1(◦s)}$•, hence every pair of •$\Omega_{2(◦s)}$ • with a pair of •$\Omega_{1(◦s)}$• on either side of the pair of •$\Omega_{2(◦s)}$ • is still two concurrent pairs, where Ω_1=0, 1.

Provided successively change ◦s for •s at places of №3 kind's odd composite points, since there is one №3 kind's odd composite point within J_3 odd points on an ordinal of every odd point of a $RLS_{№1\sim№2}$ at seriate each $RLS_{№1\sim№3}$ on the right of №1 $RLS_{№1\sim№3}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $2<\Omega_2+1+\Omega_1 \leq 6$.

After successively change ◦s for •s at places of №3 kind's odd composite points, there are both pairs of • $\Omega_{2(}$◦s$_{)}$ • and pairs of • $\Omega_{3(}$◦s$_{)}$ • on the right of J_3 at seriate each $RLS_{№1\sim№3}$, where $\Omega_2 \leq \Omega_3 \leq 6$.

Since every pair of • $\Omega_{3(}$◦s$_{)}$ • is either a pair of • $\Omega_{2(}$◦s$_{)}$ •, or at the place of two concurrent pairs of original • $\Omega_{2(}$◦s$_{)}$ •, hence every pair of •$\Omega_{3(}$◦s$_{)}$• with a pair of •$\Omega_{2(}$◦s$_{)}$• on either side of the pair of •$\Omega_{3(}$◦s$_{)}$• is still two concurrent pairs, where Ω_2=0, 1 and 2.

Provided successively change ◦s for •s at places of №4 kind's odd composite points, since there is one №4 kind's odd composite point within J_4 odd points on an ordinal of every odd point of a $RLS_{№1\sim№3}$ at seriate each $RLS_{№1\sim№4}$ on the right of №1 $RLS_{№1\sim№4}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $6<\Omega_3+1+\Omega_2\leq 11$.

After successively change ◦s for •s at places of №4 kind's odd composite points, there are both pairs of • $\Omega_{3(}$◦s$_{)}$ • and pairs of • $\lambda_{4(}$◦s$_{)}$ • on the right of J_4 at seriate each $RLS_{№1\sim№4}$, where $\lambda_4=\Omega_4$ plus κ_4, $\Omega_3\leq \Omega_4 \leq 11$, and $\kappa_4=16$.

Since every pair of • $\lambda_{4(}$◦s$_{)}$ • is either a pair of •$\Omega_{3(}$◦s$_{)}$ •, or at the place of two concurrent pairs of original • $\Omega_{3(}$◦s$_{)}$ •, hence every pair of •$\lambda_{4(}$◦s$_{)}$• with a pair of •$\Omega_{3(}$◦s$_{)}$• on either side of the pair of •$\lambda_{4(}$◦s$_{)}$• is still two concurrent pairs, where Ω_3=0, 1, 2, 3, 4, 5 and 6.

Let υ_4 expresses any of consecutive natural numbers $\geq$12. Provided successively change ◦s for •s at places of №5 kind's odd composite points, since there is one №5 kind's odd composite point within J_5 odd points on an ordinal of every odd point of a $RLS_{№1\sim№4}$ at seriate each $RLS_{№1\sim№5}$ on the right of №1 $RLS_{№1\sim№5}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $\Omega_4+1+\Omega_3=\upsilon_4$.

And so on and so forth…

Up to after successively change ◦s for •s at places of №β kind's odd composite points, there are both pairs of • $\lambda_{\beta-1(}$◦s$_{)}$ • and pairs of •$\lambda_{\beta(}$◦s$_{)}$ • on the right of J_β at seriate each $RLS_{№1\sim№\beta}$, where $\lambda_\beta=\Omega_\beta$ plus κ_β, Ω_β expresses any of consecutive natural numbers ≥1 plus 0, $\Omega_\beta \geq \Omega_{\beta-1} \geq \Omega_{4,}$ and $\kappa_\beta \geq \kappa_{\beta-1} \geq \kappa_4$.

Since every pair of • $\lambda_{\beta(}$◦s$_{)}$ • is either a pair of • $\lambda_{\beta-1(}$◦s$_{)}$ •, or at the place of two concurrent pairs of original • $\lambda_{\beta-1(}$◦s$_{)}$ •, hence every pair of •$\lambda_{\beta(}$◦s$_{)}$• with a pair of •$\lambda_{\beta-1(}$◦s$_{)}$• on either side of the pair of •$\lambda_{\beta(}$◦s$_{)}$• is still two concurrent pairs, where $\lambda_{\beta-1} \geq \lambda_4$.

Let greatest value of Ω_β is η_β, and υ_β expresses any of consecutive natural numbers $\geq\eta_\beta+1$. Provided successively change ◦s for •s at places of №(β+1) kind's odd composite points, since there is one №(β+1) kind's odd composite point within $J_{\beta+1}$ odd points on an ordinal of every odd point of a $RLS_{№1\sim№\beta}$ at seriate each $RLS_{№1\sim№(\beta+1)}$ on the right of №1 $RLS_{№1\sim№(\beta+1)}$, so this has made preparations for an increase of the number of consecutive odd composite points, where Ω_β+1+$\Omega_{\beta-1}$=υ_β. Evidently when Ω_β+1+$\Omega_{\beta-1}$= η_β+1, it will exceed first the super-limit of Ω_β, for example, a pair of • $\eta_{\beta(}$°s$_{)}$ • with a pair of • 0 $_{(}$°s$_{)}$ •, a pair of • (η_β-1)$_{(}$°s$_{)}$ • with a pair of •1 $_{(}$°s$_{)}$ •, a pair of •(η_β-2)$_{(}$°s$_{)}$ • with a pair of • 2 $_{(}$°s$_{)}$ •, etc.

After successively change ◦s for •s at places of №(β+1) kind's odd composite points, there are both pairs of •$\lambda_{\beta(}$°s$_{)}$ • and pairs of •$\lambda_{\beta+1(}$°s$_{)}$ • on the right of $J_{\beta+1}$ at seriate each RLS$_{№1\sim№(\beta+1)}$, where $\lambda_{\beta+1}$=$\Omega_{\beta+1}$ plus $\kappa_{\beta+1,}$ $\Omega_{\beta+1}$ expresses any of consecutive natural numbers ≥1 plus 0, $\Omega_\beta \leq \Omega_{\beta+1}$ $\leq \upsilon_\beta$, and $\kappa_{\beta+1} \geq \kappa_\beta$. Thus, where $\Omega_{\beta+1}$ > greatest value η_β of Ω_β, $\Omega_{\beta+1}$ exactly oversteps the super-limits of Ω_β.

Since the half line has infinitely many RLSS$_{№1\sim№(\beta+1)}$, thus there are infinitely many pairs of •$\lambda_{\beta+1(}$°s$_{)}$ • which share a set of ordinals, then there is a pair of ♠$\lambda_{\beta+1(}$°s$_{)}$♠ on the set of ordinals at №1 RLS$_{№1\sim№(\beta+1)}$ according to the aforesaid coexisting theorem, besides $\Omega_{\beta+1}$ of $\lambda_{\beta+1}$ contains natural number υ_β.

In order to attain the final goal, we need yet to further explain hereinafter, though we have proven the conclusion when χ= β+1 hereinbefore.

Since every pair of •$\lambda_{\beta+1(}$°s$_{)}$ • is either a pair of •$\lambda_{\beta(}$°s$_{)}$ •, or at the place of two concurrent pairs of original • $\lambda_{\beta(}$°s$_{)}$ •, hence every pair of •$\lambda_{\beta+1(}$°s$_{)}$• with a pair of •$\lambda_{\beta(}$°s$_{)}$• on either side of the pair of •$\lambda_{\beta+1(}$°s$_{)}$• is still two concurrent pairs, where $\lambda_\beta \geq \lambda_{\beta-1} \geq \lambda_4$.

Let greatest value of $\Omega_{\beta+1}$ is $\eta_{\beta+1}$, and $\upsilon_{\beta+1}$ expresses any of consecutive natural numbers $\geq\eta_{\beta+1}+1$. Provided successively change ◦s for •s at places of №(β+2) kind's odd composite points, since there is one №(β+2) kind's odd composite point within $J_{\beta+2}$ odd points on an ordinal of every odd point of a $RLS_{№1\sim№(\beta+1)}$ at seriate each $RLS_{№1\sim№(\beta+2)}$ on the right of №1 $RLS_{№1\sim№(\beta+2)}$, so this has made preparations for an increase of the number of consecutive odd composite points, where $\Omega_{\beta+1}+1+\Omega_{\beta}= \upsilon_{\beta+1}$.

If let such steps proceed infinitely according to the aforesaid way of doing, and let χ to express any natural number, then after change ◦s for •s at places of ∑№χ [χ≥1] kind's odd composite points, there are pairs of •$\lambda_{\chi(◦s)}$ • on the right of J_χ at seriate each $RLS_{№1\sim№\chi}$, where $\lambda_\chi=\Omega_\chi$ plus κ_χ, Ω_χ expresses any of consecutive natural numbers ≥1 plus 0, and $\kappa_\chi-1>$ greatest value of Ω_χ. Since the half line has infinitely many $RLSS_{№1\sim№\chi}$, thus there are infinitely many pairs of • $\lambda_{\chi\,(◦s)}$ • on ordinals of a set of odd points of a $RLS_{№1\sim№\chi}$, consequently there is a pair of ♠$\lambda_{\chi\,(◦s)}$♠ on ordinals of the set of odd points at №1 $RLS_{№1\sim№\chi}$ according to the aforesaid coexisting theorem, where $\lambda_\chi=\Omega_\chi$ plus κ_χ. Obviously, if χ tends to infinitely great ∞, then Ω_χ tends to equal every natural number plus 0, and $2\Omega_\chi+2$ tends to equal every positive even number, then it can replace $2\Omega_\chi+2$ by 2n, where n ≥1.

Since a pair of ♠ $\Omega_{\chi(◦s)}$ ♠ expresses a pair of consecutive odd prime numbers which differ by $2\Omega_\chi+2$, so there are pairs of consecutive odd prime numbers which differ by 2n always. Namely every positive even number 2n is a difference of two consecutive odd prime numbers. Thus far, we have proven the remainder half of the Polignac's conjecture.

Pro tanto, I firmly believe that the Polignac's conjecture is proven quite into the true according to the former and now proven propositions.