Prove Beal's Conjecture by Fermat's Last Theorem

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Abstract

In this article, we shall prove the Beal's conjecture by certain usual mathematical fundamentals with the aid of proven Fermat's last theorem, and finally reach a conclusion that the Beal's conjecture is tenable.

Keywords

Beal's conjecture, Inequality, Indefinite equation, Fermat's last theorem, Mathematical fundamentals, Attribute of A, B and C.

The proof

The Beal's Conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are all positive integers, and X, Y and Z are greater than 2, then A, B and C must have a common prime factor.

We regard limits of values of above-mentioned A, B, C, X, Y and Z as known requirements.

First, we must remove following two kinds from $A^X+B^Y=C^Z$ under the known requirements.

1. If A, B and C are all positive odd numbers, then A^X+B^Y is a positive even number, yet C^Z is a positive odd number, evidently there is only $A^X+B^Y\neq C^Z$ under the known requirements according to a positive odd number ≠ a positive even number.

2. If any two within A, B and C are positive even numbers, yet another is a positive odd number, then when A^X+B^Y is a positive even number, C^Z is a positive odd number, yet when A^X+B^Y is a positive odd number, C^Z is a positive even number, so there is only $A^X+B^Y \neq C^Z$ under the known requirements according to a positive odd number≠a positive even number.

Thus we reserve merely indefinite equation $A^X+B^Y=C^Z$ under following either qualification.

1. A, B and C are all positive even numbers.

2. A, B and C are two positive odd numbers and a positive even number.

For indefinite equation $A^X+B^Y=C^Z$ under the known requirements plus aforementioned either qualification, in fact, it has certain solutions of positive integers. Let us use following four concrete examples to explain such a viewpoint.

When A, B and C are all positive even numbers, if let A=B=C=2, X=Y=3, and Z=4, then, it is exactly equality $2^3+2^3=2^4$. Evidently $A^X+B^Y=C^Z$ at here has a set of solutions of positive integers (2, 2, 2), and A, B and C have common even prime factor 2.

In addition, if let A=B=162, C=54, X=Y=3, and Z=4, then, it is exactly equality $162^3+162^3=54^4$. Evidently $A^X+B^Y=C^Z$ at here has a set of solutions of positive integers (162, 162, 54), and A, B and C have two common prime factors, i.e. even 2 and odd 3.

When A, B and C are two positive odd numbers and a positive even number, if let A=C=3, B=6, X=Y=3, and Z=5, then, it is exactly equality $3^3+6^3=3^5$. Evidently $A^X+B^Y=C^Z$ at here has a set of solutions of positive integers (3, 6, 3), and A, B and C have common prime factor 3.

In addition, if let A=B=7, C=98, X=6, Y=7, and Z=3, then, it is exactly equality $7^6+7^7=98^3$. Evidently $A^X+B^Y=C^Z$ at here has a set of solutions of positive integers (7, 7, 98), and A, B and C have common prime factor 7.

Thus it can seen, above-mentioned four concrete examples have proved that indefinite equation $A^X+B^Y=C^Z$ under the known requirements plus aforementioned either qualification can exist, but also A, B and C have at least one common prime factor.

If we can prove that there is only $A^X+B^Y\neq C^Z$ under the known requirements plus the qualification that A, B and C have not any common prime factor, then $A^X+B^Y=C^Z$ under the known requirements, A, B and C must have a common prime factor.

Since when A, B and C are all positive even numbers, A, B and C have common prime factor 2, therefore, when A, B and C are two positive odd numbers and a positive even number, A, B and C are able to have not any common prime factor.

If A, B and C have not any common prime factor, then any two of them have not any common prime factor either. Since on the supposition that any two have a common prime factor, namely A^X+B^Y or C^Z-A^X or C^Z-B^Y have the prime factor, yet another has not it, then there is only to

$A^X+B^Y \neq C^Z$ or $C^Z-A^X \neq B^Y$ or $C^Z-B^Y \neq A^X$ according to the unique factorization theorem for a natural number.

Such being the case, provided we can prove that there is only inequality $A^X+B^Y \neq C^Z$ under the known requirements plus the qualification that A, B and C have not any common prime factor, then the Beal's conjecture is surely tenable, otherwise it will be negated.

Unquestionably, following two inequalities together can replace $A^X+B^Y \neq C^Z$ under the known requirements plus the aforesaid qualification.

1. $A^X+B^Y \neq 2^Z G^Z$ under the known requirements plus the qualification that A, B and G are all positive odd numbers without any common prime factor, where 2G=C.

2. $A^X+2^Y D^Y \neq C^Z$ under the known requirements plus the qualification that A, D and C are all positive odd numbers without any common prime factor, where 2D=B.

We again divide $A^X+B^Y \neq 2^Z G^Z$ into two kinds, i.e. (1) $A^X+B^Y \neq 2^Z$, when G=1, and (2) $A^X+B^Y \neq 2^Z G^Z$, where G>1.

Likewise, we again divide $A^X+2^Y D^Y \neq C^Z$ into two kinds, i.e. (3) $A^X+2^Y \neq C^Z$, when D=1, and (4) $A^X+2^Y D^Y \neq C^Z$, where D>1.

We will prove that aforesaid four inequalities hold water under under the known requirements plus respective qualification.

On purpose of the citation for convenience, let us first Prove $E^P+F^V \neq 2^M$, where E and F are two positive odd numbers without any common prime divisor, and P, V and M are positive integers >2. Since E and F have not any common prime factor, so it has $E^P \neq F^V$ according to the unique factorization theorem for a natural number, and let $F^V > E^P$.

In other words, Prove that indefinite equation $E^P+F^V=2^M$ has not a set of solutions of positive integers, where P, V and M are positive integers >2.

When P is a positive integer >2, indefinite equation $E^P+1^P=2^P$ has not a set of solutions of positive integers according to proven Fermat's last theorem [REFERENCES at the finale], then E is not a positive integer.

In the light of the same reason, when V is a positive integer >2, indefinite equation $F^V-1^V=2^V$ has not a set of solutions of positive integers, then F is not a positive integer.

Next, two sides of equal-sign of $E^P+1^P=2^P$ added respectively to two sides of equal-sign of $F^V-1^V=2^V$ makes $E^P+F^V=2^P+2^V$.

For indefinite equation $E^P+F^V=2^P+2^V$, when P=V, $2^P+2^V=2^{P+1}$, and $E^P+F^V=2^{P+1}$, let P+1=M, we get $E^P+F^V=2^M$, but E and F are not two positive integers according to preceding two conclusions. If enable E and F into two positive odd numbers, then, there is to $E^P+F^V\neq 2^M$ only.

However, when $P\neq V$, $2^P+2^V\neq 2^M$, then $E^P+F^V=2^P+2^V\neq 2^M$, i.e. $E^P+F^V\neq 2^M$, where E and F are not positive integers. If let E and F into two positive odd numbers, then either multiply E^P+F^V by a corresponding no positive integer such as ζ, or E^P added to a corresponding no positive integer such as μ, and F^V added to a corresponding no positive integer such as ξ , so either multiply 2^P+2^V by ζ, or 2^P+2^V added to $\mu+\xi$ at another side of the equality. But it has only $\zeta(2^P+2^V)\neq 2^M$ and $2^P+2^V+\mu+\xi \neq 2^M$, thus when E and F are two positive odd numbers, there is $E^P+F^V\neq 2^M$ only.

In a word, we have proven $E^P+F^V\neq 2^M$, where E and F are two positive odd numbers, and P, V and M are all positive integers >2.

On the basis of proven $E^P+F^V\neq 2^M$, we just proceed to determine and prove aforementioned four inequalities in one by one, thereinafter.

Firstly, let $A^X=E^P$, $B^Y=F^V$, and $2^Z=2^M$ for proven $E^P+F^V\neq 2^M$, we get $A^X+B^Y\neq 2^Z$, where X, Y and Z are all positive integers >2, and A and B are two positive odd numbers without any common prime factor.

Secondly, let us successively prove $A^X+B^Y\neq 2^Z G^Z$ under the known requirements plus the qualification that A, B and G are all positive odd numbers without any common prime factor, where G >1.

To begin with, multiply each term of proven $E^P+F^V\neq 2^M$ by G^M, then we get $E^P G^M+F^V G^M\neq 2^M G^M$.

For any positive even number, either it is able to be written as A^X+B^Y, or it is unable. Justly $E^P G^M+F^V G^M$ is a positive even number.

If $E^P G^M+F^V G^M$ is able to be written as A^X+B^Y, then it has $A^X+B^Y\neq 2^M G^M$.

If $E^P G^M+F^V G^M$ is unable to be written as A^X+B^Y, then $E^P G^M+F^V G^M$ at here have nothing to do with proving $A^X+B^Y\neq 2^M G^M$.

Under this case, there are still $E^PG^M+F^VG^M \neq A^X+B^Y$ and $E^PG^M+F^VG^M \neq 2^MG^M$, so let $E^PG^M+F^VG^M$ be equal to A^X+B^Y+2b or A^X+B^Y-2b, where b is a positive integer. And use sign "±" to denote sign "+" and sign "-" hereinafter, then we get $A^X+B^Y \pm 2b \neq 2^MG^M$, i.e. $A^X+B^Y \neq 2^MG^M \pm 2b$.

Since 2b can express every positive even number, then $2^MG^M \pm 2b$ can express all positive even numbers except for 2^MG^M.

For a positive even number, either it is able to be written as 2^KN^K, or it is unable, where K is a positive integer >2, and N is a positive odd number.

So where $2^MG^M \pm 2b = 2^KN^K$, there is $A^X+B^Y \neq 2^KN^K$. Yet where $2^MG^M \pm 2b \neq 2^KN^K$, $2^MG^M \pm 2b$ have nothing to do with proving $A^X+B^Y \neq 2^KN^K$.

That is to say, for inequality $E^PG^M+F^VG^M \neq 2^MG^M$, if $E^PG^M+F^VG^M$ is unable to be written as A^X+B^Y, we are also able to deduce $A^X+B^Y \neq 2^KN^K$ elsewhere.

Hereto, we have proven this kind of $A^X+B^Y \neq C^Z$, whether it is $A^X+B^Y \neq 2^MG^M$ or it is $A^X+B^Y \neq 2^KN^K$, so long as let C=2G and Z=M, or C=2N and Z=K, as far as OK's.

Thirdly, we carry on with proving $A^X+2^Y \neq C^Z$ under the known requirements plus the qualification that A and C are two positive odd numbers without any common prime factor.

In the former passage, we have proven $E^P+F^V \neq 2^M$, and $F^V > E^P$, so let $C^Z = F^V$, then we get $E^P+C^Z \neq 2^M$.

Moreover, let $2^M > 2^3$, then it has $2^M = 2^{M-1}+2^{M-1}$. So either there is $E^P+C^Z >$

$2^{M-1}+2^{M-1}$, or there is $E^P+C^Z < 2^{M-1}+2^{M-1}$. Namely either there is $C^Z-2^{M-1}>2^{M-1}-E^P$, or there is $C^Z-2^{M-1}<2^{M-1}-E^P$.

In addition, there is $A^X+E^P\neq 2^{M-1}$ according to proven $E^P+F^V\neq 2^M$.

Then, from $A^X+E^P\neq 2^{M-1}$, either get $2^{M-1}-E^P>A^X$, or get $2^{M-1}-E^P<A^X$.

Therefore, either there is $C^Z-2^{M-1}>2^{M-1}-E^P>A^X$, or there is $C^Z-2^{M-1}<2^{M-1}-E^P<A^X$.

Consequently, either there is $C^Z-2^{M-1}>A^X$, or there is $C^Z-2^{M-1} < A^X$.

In a word, there is $C^Z-2^{M-1}\neq A^X$, i.e. $A^X+2^{M-1}\neq C^Z$.

For inequality $A^X+2^{M-1}\neq C^Z$, let $2^{M-1}=2^Y$, we get inequality $A^X+2^Y\neq C^Z$.

Fourthly, let us last prove $A^X+2^YD^Y\neq C^Z$ under the known requirements plus the qualification that A, D and C are all positive odd numbers without any common prime factor, where D>1.

We have the aid of proven $A^X+2^Y\neq C^Z$ to complete the proof of $A^X+2^YD^Y\neq C^Z$ successively, that is achievable according to the preceding way of doing.

We need to use an inequality $H^U+2^Y\neq K^T$ according to proven $A^X+2^Y\neq C^Z$, where H and K are two positive odd numbers without any common prime factor, and U, Y and T are all positive integers>2, so we get $K^T-H^U\neq 2^Y$.

Like that, multiply each term of $K^T-H^U\neq 2^Y$ by D^Y, then we get $K^TD^Y-H^UD^Y\neq 2^YD^Y$.

For any positive even number, either it is able to be written as C^Z-A^X, or it is unable. Undoubtedly, $K^TD^Y-H^UD^Y$ is a positive even number.

If $K^T D^Y - H^U D^Y$ is able to be written as $C^Z - A^X$, then we get $C^Z - A^X \neq 2^Y D^Y$, i.e. $A^X + 2^Y D^Y \neq C^Z$.

If $K^T D^Y - H^U D^Y$ is unable to be written as $C^Z - A^X$, then $K^T D^Y - H^U D^Y$ at here have nothing to do with proving $A^X + 2^Y D^Y \neq C^Z$. Under this case, there are $K^T D^Y - H^U D^Y \neq C^Z - A^X$ and $K^T D^Y - H^U D^Y \neq 2^Y D^Y$ still.

Let $K^T D^Y - H^U D^Y$ be equal to $C^Z - A^X \pm 2d$, where d is a positive integer, then there is $C^Z - A^X \pm 2d \neq 2^Y D^Y$, i.e. $C^Z - A^X \neq 2^Y D^Y \pm 2d$.

Since 2d can express every positive even number, then $2^Y D^Y \pm 2d$ can express all positive even numbers except for $2^Y D^Y$.

For a positive even number, either it is able to be written as $2^S R^S$, or it is unable, where S is a positive integer>2, and R is a positive odd number.

So where $2^Y D^Y \pm 2d = 2^S R^S$, we get $C^Z - A^X \neq 2^S R^S$, i.e. $A^X + 2^S R^S \neq C^Z$, where R>1. Yet where $2^Y D^Y \pm 2d \neq 2^S R^S$, evidently $2^Y D^Y \pm 2d$ at here have nothing to do with proving $A^X + 2^S R^S \neq C^Z$.

That is to say, where $K^T D^Y - H^U D^Y \neq C^Z - A^X$, there is $A^X + 2^S R^S \neq C^Z$ still, elsewhere.

At aforesaid events, we have proven another kind of $A^X + B^Y \neq C^Z$, whether it is $A^X + 2^Y D^Y \neq C^Z$ or it is $A^X + 2^S R^S \neq C^Z$, so long as let B=2D, or B=2R and Y=S, as far as OK's.

To sun up, we have proven every kind of $A^X + B^Y \neq C^Z$ under the known requirements plus the qualification that A, B and C have not any common prime factor.

Then again, we review previous four concrete examples, themselves have proven that indefinite equation $A^X+B^Y=C^Z$ under the known requirements has certain solutions of positive integers, when A, B and C contain at least one common prime factor.

Overall, after the compare between $A^X+B^Y=C^Z$ and $A^X+B^Y\neq C^Z$ under the known requirements, we reach inevitably such a conclusion, namely an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the known requirements is that A, B and C have a common prime factor.

The proof was thus brought to a close, as a consequence, the Beal conjecture is tenable.

REFERENCES: Modular Elliptic Curves and Fermat's Last Theorem, By Andrew Wiles, Annals of Mathematics, Second Series, Vol. 141, №.3, (May, 1995), pp. 443-551.