Self-similar Doppler shift: An example of correct derivation that Einstein Relativity was preventing us to break through

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Abstract: In this short paper I present a simple but correct derivation of the complete Doppler shift effect. I will prove that Doppler effect of electromagnetic waves is a self-similar process, and therefore Special Relativity, that pretends to be complete for every inertial system, is excluded from that self-similarity of the Doppler effect.

I. THE DERIVATION: SUCCESSIVE STAGES TOWARDS SELF-SIMILARITY

Let a source of electromagnetic waves emit at a frequency f_0, and let a detector move away from it at a speed v, as messured in the reference frame of the source. If that speed v is close to c, then Special Relativity claims the meassured frequency in that detector is relativistic, so:

$$f = f_0\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}} \tag{1}$$

But, under classical mechanics assumptions, that Doppler shift is computed as follows:

$$f = f_0\left(1-\frac{v}{c}\right) \tag{2}$$

Let us split that speed into two, $v = u+u$, and place an antenna in the intermediate reference frame that moves at u with respect to the source. The detector is placed in a final third collinear reference frame, which is moving away at a speed u with respect to the second one. Then, this detector meassures a **double Doppler effect**, which in classical mechanics is:

$$f_1 = f_0\left(1-\frac{u}{c}\right)\left(1-\frac{u}{c}\right) = f_0\left(1-\frac{v}{2c}\right)^2 \tag{3}$$

therefore, we observe that f_1 is not equal to f in [2], and that means the Doppler effect under classical mechanics assumption is not self-similar. so it is incomplete. But, let's see now how the prediction under Special Relativity assumptions is for that double Doppler:

$$f_2 = f_0\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} = f_0\frac{1-\frac{v}{2c}}{1+\frac{v}{2c}} \tag{4}$$

We can see that f_2 is not equal to f in [1] either, so Special Relativity predicts a non self-similar Doppler, an incorrect one, since that theory pretends to be complete for every inertial system. A complete Doppler must be strictly self-similar. So, how can we design a model that can predict a complete Doppler? Let's do the following:

Above, we have divided the speed v into two halves, u. Let's now divide v into n parts, $v = u+u+...+u$. Then, we compound those n parts, under classical mechanics, assuming there is an antenna in each intermediate reference frame that relays the signal to the next collinear one. Then, we have:

$$f_n = f_0\left(1-\frac{v}{nc}\right)^n \tag{5}$$

If we now compute the limit of f_n when n tends to infinity, we get:

$$f = \lim_{n\to\infty} f_n = \lim_{n\to\infty} f_0\left(1-\frac{v}{nc}\right)^n = f_0\exp\left(-\frac{v}{c}\right) \tag{6}$$

and we can happily see that [6] does express a strictly self-similar Doppler effect, and therefore it is the correct formula for the complete Doppler. If someone still can't see the point about the meaning of equation [6] being an expression of a strictly self-similiar Doppler, it means the speed v of the detector with respect to the source can be split into so many parts as you like, and each part will correspond to an intermediate antenna that relays the signal to the next collinear antenna, so the compound Doppler will match the simple one, that is directly observed between source and final detector.

II. A TEST OF SELF-SIMILARITY FOR DETECTING FALSE THEORIES OF RELATIVITY

In the above derivation, someone could argue that I did not take into account the Einstein addition of velocities when I claimed that Special Relativity failed the test of self-similarity for the Doppler effect. Actually, Special Relativity equation [1] is not self-similar if we apply a **canonical sum of velocities**, $v = v_1 + v_2$, but if we apply the convention of Einstein addition of velocities we can achieve that equation [1] becomes self-similar.

Why that amazing feat?. Actually, it is not any amazing feat at all. Any theory of relativity that owns a formula for Doppler effect of electromagnetic waves can be proclaimed as self-similar if we define a method of how the composition of velocities must be. Any theory of relativity exhibits the following generic Doppler formula:

$$f = f_0 \exp\left(\mathrm{S}(\beta)\right) \tag{7}$$

where obviously, $\beta = \frac{v}{c}$ and $\mathrm{S}(\beta)$ is a function of β. Since Special Relativity has the following equation for Doppler effect:

$$f = f_0 \sqrt{\frac{1+\beta}{1-\beta}} \tag{8}$$

that means the function $\mathrm{S}(\beta)$ must be:

$$\exp\left(\mathrm{S}(\beta)\right) = \sqrt{\frac{1+\beta}{1-\beta}} \tag{9}$$

$$\mathrm{S}(\beta) = \ln \sqrt{\frac{1+\beta}{1-\beta}} \tag{10}$$

$$\mathrm{S}(\beta) = \frac{1}{2} \ln \frac{1+\beta}{1-\beta} \tag{11}$$

$$\mathrm{S}(\beta) = \mathrm{artanh}\ (\beta) \tag{12}$$

So, we can clearly see that when we apply the generic Doppler to the case of Special Relativity, we automatically attain a law of composition of velocities that makes it self-similar. In other words, let's suppose we want to compound two distinct β, β_1 and β_2, then we have:

$$\exp\left(\mathrm{S}(\beta_1)\right) \exp\left(\mathrm{S}(\beta_2)\right) = \exp\left(\mathrm{S}(\beta)\right) \tag{13}$$

$$\exp\left(\mathrm{S}(\beta_1) + \mathrm{S}(\beta_2)\right) = \exp\left(\mathrm{S}(\beta)\right) \tag{14}$$

$$\mathrm{artanh}\ (\beta_1) + \mathrm{artanh}\ (\beta_2) = \mathrm{artanh}\ (\beta) \tag{15}$$

so, we clearly have:

$$\beta = \frac{\beta_1 + \beta_2}{1 + \beta_1 \beta_2} \tag{16}$$

as the well-known Einstein's addition of velocities.

Above, we have seen that Doppler equation under classical mechanics is

$$f = f_0 \left(1 + \beta\right) \tag{17}$$

and we proved it is not self-smilar under a canonical sum of velocities. But curiously, we can transform it into a self-similar equation, as we did for Special Relativity, if we can find a suitable law of non canonical composition of velocities for it. Let's see how we can do it:

$$\left(1 + \beta\right) = \exp\left(\mathrm{S}(\beta)\right) \tag{18}$$

$$\mathrm{S}(\beta) = \ln\left(1 + \beta\right) \tag{19}$$

So the sum of betas would be now:

$$\mathrm{S}(\beta_1) + \mathrm{S}(\beta_2) = \mathrm{S}(\beta) \tag{20}$$

$$\ln\left(1 + \beta_1\right) + \ln\left(1 + \beta_2\right) = \ln\left(1 + \beta\right) \tag{21}$$

$$\left(1 + \beta\right) = \left(1 + \beta_1\right)\left(1 + \beta_2\right) \tag{22}$$

$$\beta = \left(1 + \beta_1\right)\left(1 + \beta_2\right) - 1 \tag{23}$$

$$\beta = \beta_1 + \beta_2 + \beta_1 \beta_2 \tag{24}$$

This would be the law of composition of velocities under classical mechanics assumptions if we want a self-similar Doppler. Logically, if we have endowed classical mechanics with that law of composition, it becomes a different theory, so we would have to called it with a different name. Anyway, now it is easy to prove the only theory of relatvity that is self-similar with a canonical sum of velocities is **the Extended Galilean Theory of Relativity**, which we already know exhibits the complete Doppler formula:

$$f = f_0 \exp\left(\beta\right) \tag{25}$$

$$f = f_0 \exp\left(\frac{v}{c}\right) \tag{26}$$

Regards

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[1] "The Complete Doppler Formula: Return to the Origin".Albert Zotkin. viXra:1209.0003

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[3] "La Autosimilaridad Estricta Destroza la Consistencia Interna de la Relatividad Especial de Einstein".

[4] "Fractals and Self-Similarity". Hutchinson, J. Indiana Univ. J. Math. 30, 713-747, 1981.