The Smarandache-Coman congruence on primes and four conjectures on Poulet numbers based on this new notion

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Abstract. In two previous articles I defined the Smarandache-Coman divisors of order k of a composite integer n with m prime factors and I made few conjectures about few possible infinite sequences of Poulet numbers, characterized by a certain set of Smarandache-Coman divisors. In this paper I define a very related notion, the Smarandache-Coman congruence on primes, and I also make five conjectures regarding Poulet numbers based on this new notion.

Definition 1:

We define in the following way *the Smarandache-Coman congruence on primes*: we say that *two primes p and q are congruent sco n* and we note p ≡ q(sco n) if S(p – n) = S(q – n) = k, where n is a positive non-null integer and S is the Smarandache function (obviously k is also a non-null integer). We also may say that k is equal to p sco n respectively k is also equal to q sco n and note k = p sco n = q sco n.

Note:

The notion of *Smarandache-Coman congruence* is very related with the notion of *Smarandache-Coman divisors*, which we defined in previous papers in the following way (Definitions 2-4):

Definition 2:

We call *the set of Smarandache-Coman divisors of order 1 of a composite positive integer n with m prime factors*, n = $d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 2, the set of numbers defined in the following way: $SCD_1(n) = \{S(d_1 - 1), S(d_2 - 1), ..., S(d_m - 1)\}$, where S is the Smarandache function.

Definition 3:

We call *the set of Smarandache-Coman divisors of order 2 of a composite positive integer n with m prime factors*, n = $d_1*d_2*...*d_m$, where the least prime factor of n, d_1, is greater than or equal to 3, the set of numbers defined in the following way: $SCD_2(n) = \{S(d_1 - 2), S(d_2 - 2), ..., S(d_m - 2)\}$, where S is the Smarandache function.

Examples:

1. The set of SC divisors of order 1 of the number 6 is $SCD_1(6)$ = {S(2 – 1), S(3 – 1)} = {S(1), S(2)} = {1, 2};
2. The set of SC divisors of order 2 of the number 21 is $SCD_2(21)$ = {S(3 – 2), S(7 – 2)} = {S(1), S(5)} = {1, 5}.

Definition 4:

We call *the set of Smarandache-Coman divisors of order k of a composite positive integer n with m prime factors*, n = $d_1*d_2*…*d_m$, where the least prime factor of n, d_1, is greater than or equal to k + 1, the set of numbers defined in the following way: $SCD_k(n)$ = {S(d_1 – k), S(d_2 – k), …, S(d_m – k)}, where S is the Smarandache function.

Note:

As I said above, in two previous articles I applied the notion of *Smarandache-Coman divisors* in the study of Fermat pseudoprimes; now I will apply the notion of *Smarandache-Coman congruence* in the study of the same class of numbers.

Conjecture 1:

There is at least one non-null positive integer n such that the prime factors of a Poulet number P, where P is not divisible by 3 or 5 and also P is not a Carmichael number, are, all of them, congruent sco n.

Verifying the conjecture:

(for the first five Poulet numbers not divisible by 3 or 5; see the sequence A001567 in OEIS for a a list of these numbers; see also the sequence A002034 for the values of Smarandache function)

: For P = 341 = 11*31, we have S(11 – 1) = S(31 – 1) = 5, so the prime factors 11 and 31 are congruent sco 1, which is written 11 ≡ 31(sco 1), or, in other words, 11 sco 1 = 31 sco 1 = 5; we also have S(11 – 7) = S(31 – 7) = 4, so 11 ≡ 31(sco 7);

: For P = 1387 = 19*73, we have S(19 – 1) = S(73 – 1) = 6, so the prime factors 19 and 73 are congruent sco 1, or, in other words, 6 is equal to 19 sco 1 and also with 73 sco 1;

: For P = 2047 = 23*89, we have S(23 – 1) = S(89 – 1) = 11, so the prime factors 19 and 73 are congruent sco 1;

: For P = 2701 = 37*73, we have S(37 – 1) = S(73 – 1) = 6, so the prime factors 19 and 73 are congruent sco 1;

: For P = 3277 = 29*113, we have S(29 – 1) = S(113 – 1) = 7, so the prime factors 29 and 113 are congruent sco 1.

Note:

If the conjecture doesn't hold in this form might be considered only the 2-Poulet numbers not divisible by 3 or 5.

Conjecture 2:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, ..., d_{k-1}) beside 3 of a k-Poulet number P divisible by 3 and not divisible by 5 is true that there exist the primes q_1, q_2, ..., q_n (not necessarily distinct) such that $q_1 = d_1$ sco n, $q_2 = d_2$ sco n, ..., $q_{k-1} = d_{k-1}$ sco n.

Verifying the conjecture:

(for the first four Poulet numbers divisible by 3 and not divisible by 5)

: For P = 561 = 3*11*17, we have 7 = 11 sco 4 and 13 = 17 sco 4;

: For P = 4371 = 3*31*47, we have 31 = 7 sco 3 and 47 = 11 sco 3;

: For P = 8481 = 3*11*257, we have 11 = 7 sco 4 and 257 = 23 sco 4;

: For P = 12801 = 3*17*251, we have 17 = 5 sco 2 and 251 = 83 sco 2.

Conjecture 3:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, ..., d_{k-1}) beside 5 of a k-Poulet number P divisible by 5 and not divisible by 3 is true that there exist the primes q_1, q_2, ..., q_n (not necessarily distinct) such that $q_1 = d_1$ sco n, $q_2 = d_2$ sco n, ..., $q_{k-1} = d_{k-1}$ sco n.

Verifying the conjecture:

(for the first four Poulet numbers divisible by 5 and not divisible by 3)

: For P = 1105 = 5*13*17, we have 13 = 11 sco 2 and 17 = 5 sco 2;

: For P = 10585 = 5*29*73, we have 29 = 13 sco 3 and 73 = 7 sco 3;

: For P = 11305 = 5*7*17*19, we have 7 = 5 sco 2, 17 = 5 sco 2 and 19 = 17 sco 2;

: For P = 41665 = 5*13*641, we have 13 = 11 sco 2 and 641 = 71 sco 2.

Conjecture 4:

There is at least one non-null positive integer n such that, for all the prime factors (d_1, d_2, ..., d_k) of a k-Poulet number P not divisible by 3 or 5 is true that there exist the primes q_1, q_2, ..., q_n (not necessarily distinct) such that $q_1 = d_1$ sco n, $q_2 = d_2$ sco n, ..., $q_k = d_k$ sco n.

Note:

In other words, because we defined the Smarandache-Coman congruence only on primes, we can say that for any set of divisors d_1, d_2, ..., d_k of a k-Poulet number P not divisible by 3 or 5 there exist a non-null positive integer n such that for any d_i (where i from 1 to k) can be defined a Smarandache-Coman congruence $d_i \equiv q_i$(sco n).

References:

1. Coman, Marius, *The math encyclopedia of Smarandache type notions*, Educational publishing, 2013;
2. Coman, Marius, *Two hundred conjectures and one hundred and fifty open problems about Fermat pseudoprimes*, Educational publishing, 2013.