GRAVITY *AS* THE INTRINSIC VORTICITY *OF* POINTS .

The article shows ***quaternion way*** *of transforming the embedded* ***Linear Work of points*** *in [PNS] to* ***Rotational Work*** *(Spin) on points of Spaces and* ***on infinite Dipole*** *of their Sub-spaces* ***and motion of particles*** *from the two perpendicular Energy conservational force fields* **in** *k - Range Scales .*

Markos Georgallides : Tel-00357-99 634628
Civil Engineer(NATUA) : Fax-00357-24 653551
N .Mylona St 15 , 6010 , Larnaca Cyprus
Expelled from Famagusta town occupied by
The Barbaric Turks - Au.1974 .
Email < georgallides.marcos@cytanet.com.cy>

1.. Introduction .

Primary Space Anti-Space ***Points*** {*the ENTITY with the embodied LAW* [**A,B-P$_{A}^{-}$, P$_{B}^{-}$**]}is a stationary force field from forces dP = [P$_B^-$ -P$_A^-$] with work W= ∫A-B [P.ds]= 0 ,where their relation (*position*) ***creates infinite dipole*** **z̄₀ =[s ,v̄n.∇i] =[λ ,± Λ̄.∇i]** = [**λ ,±Λ̄]** (*quaternion*) from infinite Sub-Spaces with *the only two elements* λ , Λ. Euler[1]s rotation in 3D space is represented by an axis (vector) and an angle of rotation , which is a property of complex number and defined as **z = [s ± v̄.i]** where **s, |v̄|** are real numbers (scalar, vectors) and **i** imaginary part such that i^2 = -1.Extending imaginary part to three dimensions $\bar{v}_1$ i ,$\bar{v}_2$ j ,$\bar{v}_3$ k → **v̄.**∇i becomes quaternion. Because Dipole **dš =Ā B is a mould** that transforms scalar magnitudes to vectors and vice-versa by rotation and **ĀB** is the ENTITY with the embodied LAW [**A , B - P$_A^-$, P$_B^-$]** (*this is quaternion's property*) and because Primary Quaternion is constant Mould , ***then in motionless (T = 0) Primary Wave Mould , Physical Universe Spaces seem to be like a simple harmonic oscillator* . Quantization of Points** (***gauge is a measure of distance* [20])** ***, is done through Vector Unit*** **dš** ***as*** **wavelength (λ)** *, **and of Energy Unit dP as*** **Spin (±Λ** ***) , in bound States (loops***) which ***withhold diffusion (flow)*** **.**The two fundamental magnitudes (***quanta of Points, ds = wavelength) and quanta of Energy (dP.ds = spin), are connected on*** **Primary dipole** and on any dipole AiBi . The position of dipole in the equilibrium Space , Anti-Space creates the ***charge = momentum*** or the ***Angular momentum*** which is the Spin (***it is the intrinsic twist of Space , Anti-Space***) which inextricably unify geometry of Space and motion .Principles are holding on Points A . For two points A , B not coinciding exists Principle of Inequality which consists another quality. These Points consist the monad AB with all *Spaces Anti-Spaces ,Sub-Spaces* and has steady boundaries ,which exist in their Position under the Principle ,Equality and Stability in Virtual displacement which presupposes Zero Work in a Restrain System. ***i.e. A point , which is nothing and has not any Position ,may be anywhere in Space, therefore the Primary point A ,being nothing also in no Space, is the only point and nowhere which means that , Primary point is the only Space and from this all the others , so then this primary point A to exist at a second point B somewhere else , point A must move at point B , where then A ≡ B.*** Since Spaces Anti–Spaces and Sub-Spaces are created from the first Unit AB , and are Property of this Unit only ,therefore these are also a Restrained System (S) .Presupposition for unit **AB =ds̄** means an Impulse (P**)** ***removes point A to B .***Since in each Restrained System (S) the Work done (W) by Impulse (P) on a Virtual displacement (ds > 0) is zero , or W = ∫A-B [P.ds] = 0 →[ds.(P_A + P $_B$) = 0] where Point A is *in* Space [S] and B *in* Anti-Space [AS] , [ds.(P_A + P $_B$) = 0] , *Therefore* ***, Each Unit AB = ds > 0 exists , by this Inner Impulse (P)*** *and also* **P$_A$ + P$_B$ = 0** . *i.e.* ***The Position and Dimension of all Points which are connected across the Universe and that of Spaces exist , because of this Static Inner Impulse* , on the contrary *should be one point only* (*Primary Point A = Black Hole → ds = 0 and P = ∞*) *. Black Hole may happen anywhere in Universe when Points of [PNS] coincide , A=B , ds = 0 and P = ∞ . The fact that on every point in [PNS] exist infinite Impulse P = 0 → P→ ∞ , grows the idea that Matter was never concentrated at a point and Energy was never very high energy ,*** i.e. **Bing Bang has never been existed** but as Space conservation State → W = ∫A-B [P.ds] = 0 . *Impulse is 0 → ∞ and may be Vacuum , Momentum or Potential or Induced Potential and it is one Type of Effect of Push nature* **,** Impulse → Motion = Energy**.** Because points A ,B are embodied with Opposite Forces and couple P,- P creates moment M = [P.λ] = Work and since force P = d(mv/dt) = dp/dt = constant , therefore P = p = Λ = momentum **.** The two elements λ , Λ of [PNS] consist the ***Mould Quaternion*** **z**$_o$ = [λ , ±Λ.∇i] = **| z₀|.e**^θ.∇i with its Conjugate **z'**$_o$ = [λ²-|Λ|²] , and its Negation Truth **eo =** [∇λ , ∓ ∇xΛ] = 0,{ Because functions f(Λ),f(-Λ) are Stationary in inverted order of rotation and their zero sum creates their *conjugation operation* through ***mould* z₀** independently of time (*negation truth*) by following Boolean logic operations **z**$_o$ = **1**, **eo = 0** and since also quaternion of the differential time operator ∂/∂t and 3D angular speed vector ϖ is → **[(∂/∂t,ϖ)©(∇λ , ±∇xΛ) = 0,ϖ.Λ= 0 , p]** ***restrain* p *only*** } where,

λ = ***the length of geometry primary dipole which is a scalar magnitude*** *,*

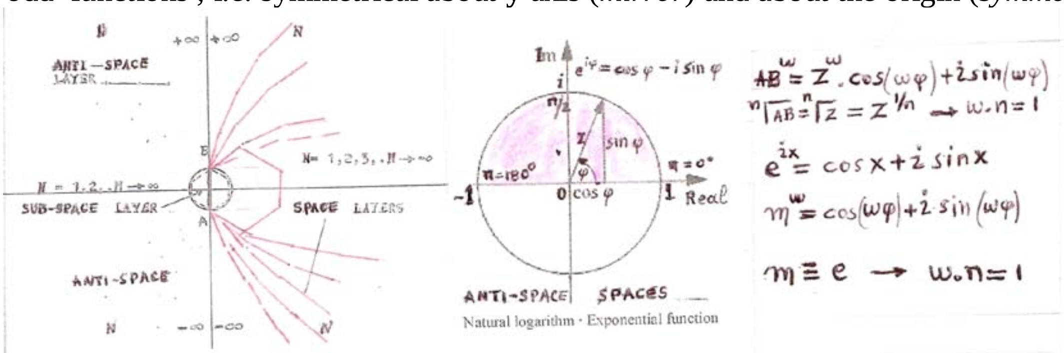
Λ = ***the spin of dipole , source term , equal to the angular momentum vector* = p ,** defining that this ***negation truth Unit*** **eo =** [∇λ,∓ ∇xΛ] = 0 instantly transfers Intrinsic Spin ***momentum*** **Λ= p = ϖ. λ = m.v** to all , *Inertial or not , Frame Layers* **K**$_{1,2,3}$ = **λ. Λ** *points* and over spaces , which is → ***Gravity of Spaces .***

2.. Complex Numbers :

De Moivre´s formula for complex numbers states that the multiplication of any two complex numbers say z1,z2, or [$z1=x1+i.y1$, $z2=x2+i.y2$] where $x=Re[z]$ the real part and $y=Im[z]$ the Imaginary part of z , is the multiplication of their moduli r1,r2 , where moduli ,r, is the magnitude [$r=|r|=\sqrt{x^2+y^2}$] and the addition of their angles ,φ1,φ2, where $\varphi=\arg z=\operatorname{atan2}(y,x)$ and so , $z1.z2 = (x1+i.y1).(x2+i.y2) = r1..r2[\cos(\varphi1+\varphi2)+i.\sin(\varphi1+\varphi2)]$ and when z1=z2=z and φ1=φ2=φ then z=x+iy and $z.z = z^2 = r^2(\cos2\varphi+i.\sin2\varphi)$ and for , w , complex numbers $z^w= r^w. [\cos(w\varphi)+i\sin(w\varphi)]$….(a) , and so for r=1 then → $z^w= 1^w. [\cos\varphi+i\sin\varphi]^w = [\cos.w\varphi+i.\sin.w\varphi]$ …......(a1)

The n.th root of any number **z** is a number b ($\sqrt[n]{z}=b$) such that $b^n =z$ and when z is a point on the unit circle, for r =1 , the first vertex of the polygon where φ=0 , is then $[b=(\cos\varphi+i\sin\varphi)]^n = b^n = z = \cos(n\varphi)+i.\sin(n\varphi) = [\cos(360/n)+i.\sin(360/n)]^n = \cos360^o + i.\sin360^o = 1+0.i = 1$ .. (b) , i.e.

the **w** spaces which are the repetition of any unit complex number **z** (multiplication by itself) is equivalent to the addition of their angle and the mapping of the regular polygons on circles with unit sides , while the **n** spaces which are the different roots of unit 1 and are represented by the unit circle and have the points z=1 as one of their vertices are mapped as these regular polygons inscribed the unit circle. Remarks : (F.1). Since $z^w= z^{-n}$ and $z^{-n} = z^{-1/w} = z^{+n}$ therefore complex numbers are even and odd functions , i.e. symmetrical about **y** axis (*mirror*) and about the origin (*symmetry*).



Spaces Anti-Spaces on Monad AB Natural logarithm – Exponential function ……..(F.1)

Complex number , z , the first dimentional unit AB = z , is such that either repeated by itself as monad (z^w = z.z.z.z. w-times) or repeated by itself in monad ($\sqrt[n]{z} = z^{1/n} = z^w$, $z^{1/n}.z^{1/n}.z^{1/n}$….w = 1/n-times , or the nth roots of **z** equal to w = 1/n) remains unaltered forming Spaces Anti-spaces {z^w, $-z^w$} and the inversing Sub-spaces {$\sqrt[n]{z}$ }, meaning that ,unit circle is mapped on itself simultaneously on the two bases , 1 and n=1/w , where w.n = 1. Let us see how this coexistence is happening by the operation of exponentiation . The logarithm of a number **x** with respect to base **b** is the exponent by which b must be raised to yield **x** .

In other words , the logarithm of **x** to base **b** is the solution **w** to the equation $b^w = x$,{eg.→ Log2(8) = 3 , since $2^3=2x2x2=8$} and in case of the reciprocal (1/x) then $b^w = (1/x)$,{eg. → Log3(1/3) = -1, since $3^{-1} = 1/3$ } . If w or n , is any natural and real number then we refer to natural logarithm else to logarithm .

This duality of coexistance on AB {***the w.th power and the n.th root of z where w.n =1***} presupposes a common base ,**m**, which creates this unit polynomial exponentiation on all these Spaces and Anti-Spaces for which happens $m^w= r^w.[\cos(w\varphi)+i.\sin(w\varphi)]= r^{1/n}.[\cos.(\varphi+2\lambda\pi)/n+i.\sin.(\varphi+2\lambda\pi)/n]$ where λ is an integer from 0 to n-1 ,and for r =1 , $m^w= [\cos\varphi+i.\sin\varphi]^w$ or $m^w= [\cos(\varphi+2\lambda\pi)/n+i.\sin(\varphi+2\lambda\pi)/n = m^{1/n} = [\cos.(\varphi+2\lambda\pi)/n + i.\sin.(\varphi+2\lambda\pi)/n]$ = ... (c) i.e.

Since Spaces are composed of monads (the entities ĀB) which are the harmonic repetition in them {{all the regular , for w > 2 polygons with monad ĀB as side limit to straight line AB for w = ±∞ , the (+) Space and the equilibrium (-) anti-Space of AB , to the complex plane for w = 2 , and to the circle with diameter AB for w=1 ,where on it exist all the roots of monad ĀB and are the circumscribed regular polygons in this Sub-Space}} and since monads are composed of purely real , |AB| , and purely Imaginary parts , ±d.∇i , therefore , all these to exist as regular polygons must be maped (sited) with natural and real numbers only and thus all Spaces , anti-Spaces and Subspaces of unit monad AB are represented , as the polygonal exponentiation , on this common base **m .**

Since Spaces , anti-Spaces {z^w, $-z^w$} and Subspaces {$\sqrt[n]{z}$}, which are the similar regular polygons , on and in unit monad AB = z , are both simultaneously created by the Summation of the exponentially unified in monad $\bar{A}B$ complex exponential dualities w, n where w. n= 1 ,so the repetition of monads AB exist as this constant Summation on this common base **m ,** which is according to one of the four basic properties of logs as →
log.w(**1**=w.n) = log.w(w)+log.w(n =1/w) = 1+1/w = 1+ n ………… …......(**c1**)
which is the base of natural logarithms **e** and since 1= w.n then →
$(1+n)^w = (1+1/w)^w$ = constant **= m = e ← ……(d)** which is independed of any Space and cordinate system that may be used , meaning also that Spaces , anti-Spaces (the conjugates) and Subspaces , all as regular polygons represent the mapping (to any natural real and complex number as power w = 1/n) of any unit $\bar{A}B$ which is a complex number **z** , on the constant base **m** , where then is ..
$m^\wedge \pm ({}^a+{}^d.i) = (x+i.y)^w = |z|^w.[\cos.w\varphi+i.\sin.w\varphi]$, a multi valued function where , $\sin\varphi = y/\sqrt{x^2+y^2}$, $\cos\varphi = x/\sqrt{x^2+y^2}$, $|z|=\sqrt{x^2+y^2}$, and for **|z|=1** $\mathbf{m^\wedge \pm ({}^a+{}^d.i) = (x+i.y)^w = [\cos.w\varphi + i.\sin.w\varphi]}$ **…….(d1)**
Equation (**d1**) ***represents the general interconnection of Spaces and Subspaces , on and in all monad units*** **$\bar{A}$B = [x+i.y]** where $({}^a+{}^d.\nabla i)$ is the new exponent complex number ,*vector*, corresponding to the ,w, power.
For y = 0 then $m^\wedge \pm ({}^a+{}^d.i) = (x+i.y)^w = x^w$ and it is the Normal element – x – of base m .
For x = 0 then $m^\wedge \pm ({}^a+{}^d.i) = (x+i.y)^w = (i.y)^w= i.(y)^w$ and it is the Normal element – i.y – of base m.
In Euclidean spaces the Dot product of two vectors is simply the cosine of the angle between them and the Cross product of two orthogonal vectors is another vector , orthogonal to both of them . The same also for unit vectors .
For w > = ± 1 then we have the Spaces and Anti-spaces. **Fo**r w = ± 1 then we have Spaces and Anti-spaces with unit circles r = ±1 and the Sub-Spaces on these circles .
For w = a ≠ 0 and d = 0 then $m^\wedge \pm({}^a) = (x+i.y)^w = |z|^w.[\cos.\varphi+ i.\sin.\varphi]^w = |z|^w. [\cos.(w\varphi) + i.\sin.(w\varphi)]$, and it is De Moivre´s formula for exponentiation .
For x=0 then $m^\wedge \pm({}^a+{}^d.i) = (i.y)^w= |z|^w.[\cos.w\varphi+i.\sin.w\varphi]$ and on unit circle , $|z|^w= 1$, the common base $m^\wedge \pm({}^a+{}^d.i) = [\cos.w\varphi+i.\sin.w\varphi]$ and for a=0 ***the non-regular Polygons*** **, d.i ≡ d.∇i** , then $m^\wedge\pm d.[\nabla i] = [\cos.w\varphi+ i.\sin.w\varphi]$ ……....(d2) i.e. (d2) is a quaternions exponentiation a system that extends Imaginary part of complex numbers which products of (**i**) is not commutative where the order of the variables follow the standard right hand rule , and for parallel vectors their quotient is scalar and tensor (Tz) of a unit vector **z** is one, then, base **m** becomes $m^\wedge \pm [\nabla i]^d = [\cos.d + i .\sin.d]$, which is for unit d .[∇i] = i , the Euler's formula which is [$e^\wedge (i.{}^d) = \cos.d + i .\sin.d$] and thus , this is the geometrical interpretation of **m and e .** For two complex numbers ,**a**, in polar coordinates (r,θ) and **w** , and by using the identity $[e^\wedge\ln(a)]^w = a^w$ then $a^w= [re^\wedge\theta i]^w = [e^\wedge(\ln.r+\theta i]^w = e^\wedge[(\ln.r+\theta i].w = r^w.[\cos\varphi +i.\sin\varphi]^w = r^w[\cos.w\varphi + i. \sin.w\varphi]$.
For φ =180° = π , cos.wφ = -1 , sin.wφ = 0 , (d2) becomes $m^\wedge\pm d.[\nabla i] = -1$, which is Euler's identity in general form and for Ellipsoid of axes y1,y2,y3 $m^\wedge \pm \pi.[y1+y2+y3]/\sqrt{3} = -1$, and for (a=0) the known Euler's identity for quaternions . Complex numbers are a subfield of quaternions .
De Moivre´s formula for nth roots of a quaternion where q =k.[cos.φ+ [∇i].sin.φ] is for w =1/n ,
$q^w = k^w.[\cos.w\varphi + \varepsilon.\sin.w\varphi]$ where q = z = ± (x+y.i) , decomposed into its scalar (x) and vector part (y.i) and this because all inscribed regular polygons in the unit circle have this first vertex at points 1 or at -1 (for real part φ = 0, φ = 2π) and all others at imaginary part where , k = Tz = Tensor (the length) of vector z in Euclidean coordinates which is $k = Tz = \sqrt{x^2+y1^2+y2^2+ yn^2}$, and for imaginary unit vector ã (a1,a2,a3,a.n..w) , the unit vector **ε** of imaginary part is →
$\varepsilon = (y.i/Ty) = [y.\nabla i]/[Ty] = \pm(y1.a1+y2.a2+) / (\sqrt{+y1^2+y2^2+yn^2})$ the rotation angle 0 < = φ < 2π , φ = $\pm \sin^{-1}Ty/Tz$, cosφ = x/Tz , which follow Pythagoras theorem for them and for all their reciprocal quaternions ã' (ã.ã' =1) . Since also the directional derivative of the scalar field y(y1 , y2 , yn ..) in the direction i is→ i (y1 , y2 , yn .) = i1.y1+ i2.y2 .+ i n .yn and defined as i.grad y = i1.(∂y/∂1) + i2.(∂y/∂2) +.. = [i.∇].y which gives the change of field y in the direction→ i, and [i.∇] is the single coherent unit , so coexistance between Spaces Antispaces and Sub-Spaces of any monad
$\bar{z}$ = x+y.i = $\bar{A}$B is happening through the general equation (d) and (d1) as follows →

$m^\wedge\pm({}^a+{}^d.\nabla i) = q^w = (Tq)^w.[\cos.w\varphi + \varepsilon.\sin.w\varphi]$ ..…(e) *where*
$m = \lim(1+1/w)^w = e$, for w = 1→ ∞ , q = z = ± (x+y.i)
$\sin\varphi = y/\sqrt{x^2+y^2}$, $\cos\varphi = x/\sqrt{x^2+y^2}$, $|z| = \sqrt{x^2+y^2}$,
$Tq = \sqrt{x^2+y1^2+y2^2+ \ldots.yn^2}$, $Ty = \sqrt{y1^2+y2^2+ \ldots.yn^2}$
$\varepsilon = (\bar{y}.i/Ty) = [\bar{y}.\nabla i] / [Ty]=(y1.a1+y2.a2+.)/(\sqrt{y1^2+y2^2+yn^2})$
$({}^a+{}^d.\nabla i)$ is the new exponent complex number ,*vector*, corresponding to the ,w, power .

Equation (e) is connecting Spaces , Anti-spaces , Subspaces of any unit ĀB

1… $q = \pm x \rightarrow$ then

$q^W = (Tq)^W.[\cos.w\varphi + \varepsilon.\sin.w\varphi] = \pm x^W.[\cos.w\varphi]$

since $Tq = \sqrt{x^2} = \pm x$ and $\varphi = 0$

$\varepsilon = (\bar{y}.i/Ty) = [\bar{y}.\nabla i] / [Ty] = 0$ and

$m\hat{}\pm({}^{a}+{}^{d}.\nabla i) = q^W = \pm x^W.\cos.x\varphi = \pm x^W$ ….(e1)

represents the mapping of monad AB = $\bar{z}$ in the real domain ,which is straight line -∞ , 0 , +∞ , and simultaneously all real roots of monad , $\bar{z}$, in unit circle $|\bar{z}|=1$ on the base of natural logarithms and when ,

2… $q = \pm y.i \rightarrow$ then

$\bar{z}$, $q^W = (Tq)^W.[\cos.w\varphi+\varepsilon.\sin.w\varphi] = (Tq)^W.[1+\boldsymbol{\varepsilon}.\sin.w\varphi]$

$Tq = \sqrt{1+ y1^2+y2^2+ \ldots.yn^2}$, $Ty = \sqrt{y1^2+y2^2+ \ldots.yn^2}$

$\varepsilon = (\bar{y}.i/Ty) = [\bar{y}.\nabla i] / [Ty]$ and

$m\hat{}\pm({}^{a}+{}^{d}.\nabla i) = q^W = \pm (Tq)^W.[1+ \boldsymbol{\varepsilon}.\sin.w\varphi] =$

$\pm (Tq)^W/(Ty) . [(Ty)+y.\nabla i]. \sin.w\varphi$ …...(e2)

represents the mapping of monad ĀB = $\bar{z}$ in Imaginary domain ,which are all regular polygons with side monad ĀB , with the first vertice on line AB and simultaneously all roots on monad ,z, as the circumscribed in unit circle regular polygons on the base of natural logarithms , i.e.

$m\hat{}\pm({}^{a}+{}^{d}.\nabla i) = m\hat{}(\pm {}^{d}.\nabla i) = [\cos.w\varphi + i.\sin.w\varphi]$, and as $q^W = (Tq)^W.[1+ [y.\nabla i].\sin.w\varphi / (Ty)]$ then this is De Moivre's formula for quaternion and for for $\bar{y}.[\nabla i] = i$ $m\hat{}\pm({}^{a}+{}^{d}.\nabla i) = m\hat{}(\pm{}^{d}.i) = [\cos.w\varphi + i.\sin.w\varphi]$ it is Euler´s number $[e\hat{}(i.{}^{d}) = \cos.d + i.\sin.d]$.

3… $\bar{q} = \bar{z} = \pm(x+y.i) \rightarrow m\hat{}\pm({}^{a}+{}^{d}.\nabla i) = q^W$

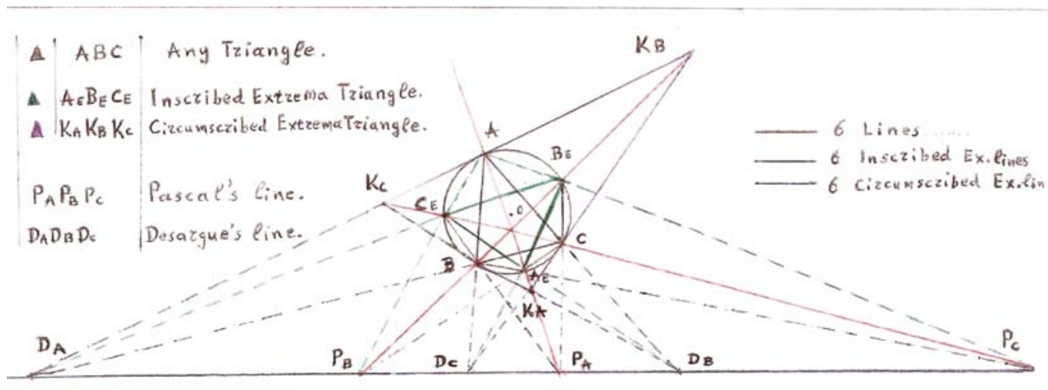
Since Spaces , anti-Spaces $\{\bar{z}^W, -\bar{z}^W\}$ and Subspaces $\{\sqrt[n]{z}\}$, have q = ĀB as common base , so similar Polygons have all their sides parallel between them , therefore this parallel transport (formation) of the infinit bases ***it gets rotated*** into a mixure of exponential vectors $\pm({}^{a}+{}^{d}.\nabla i)$.
With this way is possible for geometry of spaces , to be in the presence of the infinite units Ap , where p = 0 →∞ (the quantization of points Ap , Bp as monad ĀpBp , with the quantized Potential dPp = PĀp - PBp) , which are the vector bundle of matter , (the quanta of Points (ĀpBp) and the quanta of energy = ε → dPp simultaneously on points Āp ,Bp, and on unit vector ĀpBp).

It is shown in quaternion that if *any unit vector* $\bar{\mathbf{w}} = \mathbf{1}$ and $\bar{\mathbf{v}}$ *any vector* then $e\hat{}(w\varphi).(\bar{\mathbf{v}}).e\hat{}(-w\varphi)$, which gives the resultant rotating $\bar{\mathbf{v}}$ about the axis in the $\bar{\mathbf{w}}$ direction by $\mathbf{2\varphi}$ degrees .

3.. Definitions .

In general differential geometry terms , parallel transport of a base vector (∂i) along a base vector (∂j) is expressed through Christoffel symbols $\nabla i\, g\, j = \Gamma\, ij^{n}\, g\, n$ where ,n, is a covariant derivative and i < n < j of base vector g , and the metric (distance) $g = ds^2 = |\bar{A}B|^2 = g\,\mu\nu.dx\hat{}\mu.dx\hat{}\nu$.
Christoffel symbols are the connection coefficients induced by the metric which deriv is 0 (metric free) for parallel transports and for symmetric (torsion free) is generally holding , the Levi-Civita connection coefficients gμν , μ = ν → 0 το 3 induced by the metric **g** alone , without recourse to Cartesian coordinates .

It was proved as a theorem [16] that on any triangle ABC and on circumcircle exists one inscribed triangle AEBECE and another one circumscribed Extrema triangle KAKBKC such that the **Six** points of intersection of the six pairs of **triple lines** are collinear → (3+3) . 3 = 18 lines (F.2)
In Projective geometry , Space points are placed in Plane and in Perspective theory < Points at Infinity > and so thus are extrema points [17] . Extrema points follow Euclidean axioms and it is another way of mapping and not a new geometry contradicting the Euclidean .



The Six , Triple points , line [STPL] ……… (F.2)

It was also shown that Projective geometry is an Extrema in Euclidean geometry and [STPL] their boundaries . In case of Orthogonal system (angle A = 90°) then the inscribed triangle $A_E B_E C_E$ is in circle and the Extrema triangle $K_A K_B K_C$ has the two sides perpendicular to diameter BC and the third vertice in ∞ so any non orthogonal transformational system on an constant **vector $\bar{A}$ B** , which is the orthogonal system , is happening on the supplementary of θ angle i.e. (90-θ) where then on AB exists → cscθ = constant and equal to $\pm [1/\sqrt{1-\cos^2(90-\theta)}] = \pm [1/\sqrt{1-(BA/BD_B)^2}]$ which is identified for AB = v to Lorentz factor $\gamma = \pm[1/\sqrt{1-\beta^2}]$ where β = v/c , i.e

$$\csc\theta = \text{constant} = \pm [1/\sqrt{1-(BA/BD_B)^2}] ,$$ and it is the geometrical interpretation of Projective geometry as an Extrema in Euclidean geometry .

In Projective geometry ,Space points are placed in Plane and in Perspective theory < Points at Infinity > and so thus are extrema points[17].Extrema points follow Euclidean axioms either by translation geometry [s,0] or by rotation [0,v̄] or both [s,v̄] , where s = scalar and v̄ = vector .The Projective sphere comprehending great circles of the sphere as < lines > and pairs of antipodal points as <points> does not follow Euclidean axioms 1- 4 , because < points at Infinity > must follow 1-4 , which do not accept lines , planes , spaces at infinity . The same also for Hyperbolic geometry with omega point , so ???
Since Natural logarithm of any complex number b, can be defined by any natural and real number as the power ,**w**, which represent the mapping to which a constant say e, would have to be raised to equal b , i.e. e^W= b and or e^ln(b) = e , [base e]^ ^{1}natural number w^1 = b ,Therefore , both fomula (d) and (e) represent the same mapping which is the regular polygonal exponentiation of unit complex monad ĀB = 1 on the two equal numbers , e and m , as the base of natural logarithms .
Using the generally valued equation of universe for zero work W = ds.© =∫A -B [P.ds] = 0 , [20] for primary Space and anti-Space on monad ĀB with the only two quantized quantities dš = |ĀB| and P = ȳ.∇i, then work is the action of the consecutive small displacements (shifts) along the unit circle caused by the application of infinitensimal rotations of AB =1 starting at 1 and continuing through the total length of the arc connecting 1 and -1 , in complex plane .

3..1.. Monads = Quantum = ds̄ = ĀB / (n = ∞ → 0) = [a ± b̄.i] = 0 → ∞ , are simultaneously (*actual infinity*) and also (*potential infinity*) in Complex number form , and this defines , infinity which exists between all points which are not coinciding (ds > 0) , and because **dš** comprises any two edge points with Imaginary part then this property differs between the infinite points. Plank length is a Monad **ds = 1,62x10¯35 ,** for two points A , B , and for the moment is accepted as the smallest possible size .This Monad is also infinitely divided because edge points A,B are not coinciding i.e. ..**|dš| = 1x10¯ N** < ∞ , where N is any number and this because *quantized* dimensions [ds̄ ,- P ,+ P] are the three Layers , where ***dš . dP = λ.p = constant k***$_{1,2,3}$ **,** wavelength = **λ .∇**

(- i) ↔ (0) = - P → [ds̄ , - P ,] → Black Holes Scale = k 1 → ds 1. dP 1 = k 1
(-i) ↔ (+i) = ∓ P → [ds̄ , - P , + P] → Planck Scale Matter = k 2 → ds 2. dP 2 = k 2
(+i) ↔ (0) = + P → [ds̄ , + P ,] → Dark Matter Scale = k 3 → ds 3. dP 3 = k 3

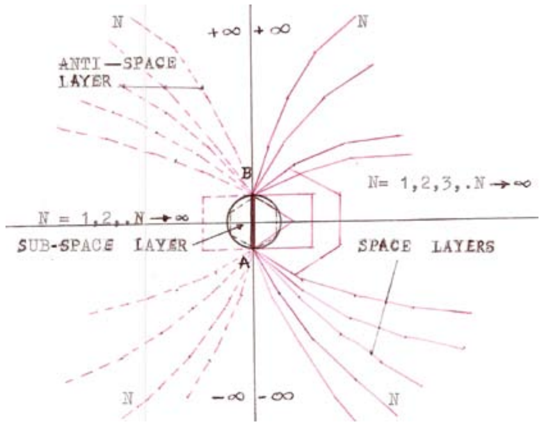
3..2.. **Spaces of Unit $\bar{A}B$** are (in Plane) the Infinite (+) Regular Polygons inscribed in the circles ***with AB as Side*** , (repetition of Unit AB) , the Nth Space , the Nth Unit Tensor of the **N** equal finite Elements **dš=z̄** , and for the ∞ Spaces , the line AB ↔ dš = (a+i.b)^{w}= |z̄|^{w}.[cos.wφ+i.sin.wφ]
The diameter of this circles extends to infinity (*it is of potential nature* dW/ds̄ = P∇) .

3..3.. **Anti - Spaces of Unit $\bar{A}B$** are (in the three dimentional space) the Symmetrically Infinite (-) Regular Solids inscribed in the Sphere ***with AB as side*** of the Solid , (The Harmonic Repetition of Unit BA , symmetrical to AB) , the Nth Anti-Space , the Nth Unit Tensor of the N equal finite Anti-Elements and for the ∞ Spaces , the Plane through line BA ↔ dš = (a+i.b)^{w}= - |z̄|^{w}.[cos.wφ+i.sin.wφ]
The diameter of this Spheres extends to infinity (*it is of potential nature* dW/ds̄ = - P∇) .

3..4.. **Sub- Spaces of Unit $\bar{A}B$** are (in Plane) the Infinite Regular Polygons inscribed in the circle ***with AB as diameter*** , (Harmonic Repetition of the Roots in Unit AB) and in Nth Sub-Space, the Nth Unit Tensor of the N finite Roots and in case of ∞ Elements are the points on the circle , *and for 3DSpace , the points on Sphere* AB) . The Superposition of Spaces , Anti - Spaces and Sub-Space Layers of Unit AB is shown in (F.3) .
Remark : (+) Spaces , (-) Anti -Spaces , (±) Sub-Spaces , of a unit **$\bar{A}B$** are between magnitude (Point = 0 = Nothing) , and the Infinite magnitude (↔ = ± ∞ = Infinite) which means that all Spaces are in one Space .Because in Spaces and Anti-Spaces , the ∞ Spaces of **Unit $\bar{A}B$** is line AB ↔ , and in Sub-Spaces the ∞ Sub-Spaces of **Unit $\bar{A}B$** are the points on the circle with AB as diameter , then this ordered continuum *for points on the circle* of Unit **$\bar{A}B$** and on *line AB* shows the correlation of Spaces in Unit **$\bar{A}B$** . **Monads ds̄ = 0 → ∞ are Simultaneously ,** ***actual infinity (because for n = ∞ then ds̄ = [$\bar{A}B$ / n = ∞] = 0 i.e. a point) and , potential infinity , (because for n = 0 then ds̄ = [$\bar{A}B$ / n = 0] = ∞ i.e. the straight line through AB . Infinity exists between all points which are not coinciding , and because Monads*** dš ***comprises any two edge points with Imaginary part , then this property is between infinite points ↔***
dš = (a+i.b)$^{1/w}$= ± |z̄|$^{1/w}$. [cos.wφ+i.sin.wφ] and potential ± dW/ds̄ = P∇.

3..5..The Superposition of Plane Space , Anti-Space Layers and Sub-Space Layers (F.3) **:**
The simultaneously co-existence of Spaces , Anti-Spaces and Sub-Spaces of any unit **$\bar{A}B$** , **Unit $\bar{A}B$ = 0 → ∞ ,** (A ≡ B) i.e. , Euclidean , Elliptic , Spherical , Parabolic , Hyperbolic, Geodesics , Metric and Non-metric geometries , exists in Euclidean Model as an Sub-case within . The Interconnection of Homogeneous and Heterogeneous bounded Spaces Anti-Spaces and Subspaces of the Universe as Unity of Opposites . This is also the *Quantized* property of Euclidean geometry < *all is one* > as it is , Discrete (for Monads $\bar{A}B$) and Continuous (for Points A , B) . For Primary Point **$\bar{A}$** , it is the only Space i.e. quaternion [**a+d̲.∇i]**

$$\mathbf{m^{\wedge}\pm({}^{a}+\underline{d}.\nabla i) = \bar{q}^{\,w} = (T\bar{q})^{w}. [\cos.w\varphi + \varepsilon .\sin w\varphi]}$$



(F.3) *where*

m = lim(1+1/w)^{w} for w = 1 → ± ∞ , e = m , q = z = ± (x+y.i)
sinφ = y / √x²+y² , cosφ = x / √x²+y² , |z| = √x²+y² ,
Tq = √ x²+y1²+y2²+ ….yn² , Ty = √ y1²+y2²+ ….yn²
ε = (ȳ.i/Ty) = [ȳ.∇i] / [Ty] = (y1.a1 + y2.a2+.) / (√ y1²+y2²+yn²)
(a+d̲.∇i) = The new exponent ,*vector,* corresponding to the ,w, power

Change (motion) and plurality are impossible for points of Absolute Space [PNS] and since it is composed only of Points , that consist an Unmovable Space , then neither Motion nor Time exists i.e. a constant distance AB = **dš** anywhere existing is motionless . **dš = [AB / n] > 0** = *quantum* and for infinite continuous **n , dš** convergence to **0 .** [10]
Physical world is scale-variant and infinitely divisible , consisted of finite indivisible entities dš = AB > 0 → and infinite points (|dš| = 0) i.e. is Continuous with points and Discontinuous with |dš| > 0 . [13] . In PNS dt = 0 , so motion cannot exist at all .

4.. Motion : Since on Primary point A exists , *Principle of Virtual Displacements* [**dš . (** P_A+P_B)= 0] and Impulse **P = ($P_A + P_B$) = 0 → P → ∞** may have any Price , then for P = 0 exists [PNS] . **Since points Ā , B of [PNS] coincide with the infinite Points** of , Spaces , Anti-Spaces and Sub-Spaces **in [PNS**] , and since also motion may occur at all Bounded Sub-Spaces then , this ***Relative motion*** is happening between all points belonging to [PNS] and to those points belonging to the other Sub-Spaces (A ≡ B) . [11] **.** The position of points in [PNS] creates the infinite dipole and all quantum quantities which acquire Potential difference and an Intrinsic moment in the three Spatial dimensions (x , y , z) and the infinite dimensions of the (i) Layers at these points , existing from the other Layers of Primary Space , Anti-Space and Sub-Space , where i = 1 → ∞ .
Motion is Continuous and occurs on Dimensional Units , *dš* , and not on Points which are dimensionless , upon these Bounded States of [PNS] , *Spaces and Anti-Spaces , and because of the different Impulses P_A , P_B of points A , B* , a constant force dP is exerted on dipole AB = λ . On any dipole exists a total conserved energy ,*work* , W = λ.p = λ.Λ = k 1,2,3 either as λ or ± Λ .
It will be referred later that this property in [PNS] , *becomes from the two coplanar Static force fields of* (S),(AS) *which exert a force on the infinite , embedded with momentum* Λ *(spin) dipole* AB = λ .
Energy (motion) is conserved following Pythagoras theorem for the three components of W as the total energy , *for and in λ as kinetic energy and for the circular motion of Λ = m.v = w.r , as the two fields* perpendicular each other , which conserve linear motion to circular motion of the combined helical motion .
Because Total Energy is conserved in perpendicular fields , so all interactions are mediated with this angular Energy , spin , (quaternion's property is the wave motion) , (Photons etc) .
Following the dialectic logic of ancient Greek (Αναξίμανδρος) « τό μή Ον , Ον Γίγνεσθαι » ‘ The Non-existent , Exists when is done ’ , ‘ The Non - existent (*whatever that is*) becomes and never is ’ and the Structure of Euclidean geometry [6] in a Compact Logic Space Layer , as this exists in a known Unit (***the case of 90*** ▫ ***angle***) . ***Since Non-Existent may be at a point A (the Primary point) and also everywhere*** , ***then how Existence of Space is found and is done everywhere ??*** [***Answer : Because Primary Point is the only Space and by negation truth* (eo) *, momentum* ± p *is transferred everywhere in [PNS] points , as vorticity Λ = p = mv = m .wλ , → which is gravity .***
In Euclidean geometry points do not exist , but their position and correlation is doing geometry .
Since Geometry is consisted of Real (**dš**) , (*the quantization of points*) and Imaginary Parts (**P_A , P_B**) , (*the quantization of Energy*) the universe cannot be created , because becomes by quantization of the one point which is nothing i.e. **from Monads = dš = ĀB / (n = ∞ → 0)= [ds ,** P∇i **]= [a ± b.i] = 0 → ∞** .

Since Points in [PNS] may exist with P = 0 and also with P ≠ 0 , and ($P_A + P_B$ = 0 for points in Spaces and Anti - Spaces) , therefore [PNS]* is self created** , ***and because at each point may exist P ≠ 0 this [PNS] is a Scalar Field with infinite points which have a Charge P = 0 →P →∞ , i.e. a Dynamic Tensor with absolute dimensions Xo , Yo , Zo and Charges Pi = 0 →Pi → ∞ , i = x , y , z

On *Monad* **ĀB = 0 ↔ AB ↔ ± ∞ exists** < ***a bounded State for each of the Infinite Spaces and Anti - Spaces*** > and the **[Dipole ĀB = Matter]** is the communicator **of Impulse [P] *of Primary Space* , with the Bounded Impulses (P_A^- , P_B^-) of Dipole .** or [10] ,

[P] ↔ [FMD = ĀB – P_A^- , P_B^-] → P_A^- , P_B^-
on **↓ Communicator = Medium ↓**
Impulse P → [Bounded *Primary Space- Anti-Space*] → Bounded Impulse P_A^-
P_A^- = P_B^-
A → . . . . ← O → . . . .← B

Monad **ĀB** is the ENTITY and [**A , B – P_A^- , P_B^-]** is the LAW , so Entities are embodied with the Laws .

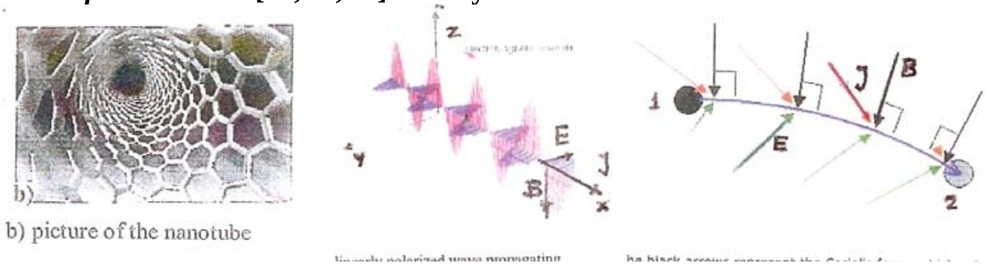
It is proved that this relation belongs to quaternion's property where monad **ĀB = λ** (*wavelength of particles*) and Impulse **P**$_A$ **, P** $_B$ = ± Λ (*the spin of dipole = angular momentum vector* p) **.**
Since motion occurs on Dimensional Units , *dš* **, and not on Points which are dimensionless** ***and because of the different Impulses P$_A$, P $_B$ of points A , B and that of Impulses Pi $_A$, Pi $_B$, of*** **Sub-Spaces , it is on straight line AB or on tracks of Spaces , Anti -Spaces , and Sub-Spaces of AiBi . [10-11]** ***. All particles act as wave (wave-particle vector duality) because of the Spaces domain and the Total energy conservation law of Pythagoras ,*** [16] and because , Λ , - Λ , are in inverted order of rotation and vice-versa .
Points A , B of AB (A≡B) carry the bounded Impulses P$_A$, P $_B$ which achieve Stable State in [PNS] so this allows other Primary Spaces to exist differently in the same Space because , (dP x dš) = ± (PB -PA) x |dš| , (*i.e. it is possible at the same monad ĀB= ds of [PNS] to exist infinite dP monads as Electric field → Magnetic field → PA , P B*) **and for every dipole dš = ĀB in [PNS]** ***two properties called wavelength***, **| dš =λ |** , ***and ± Spin*** **, (dPxdš =±Λ** *λ***)** , ***of it*** and for any constant moving Vector **dš = Āi Bi = [a ± b.i] = OP** *with J,E,B module on any Orthogonal moving vector* where

J = The **x** coordinate of **dš** = OP , representing the Strength magnitude of **dš** in **x-x** principal direction .
E = The **y** coordinate of **dš** = OP , representing the Strength magnitude of **dš** in **y-y** principal direction .
B = The **z** coordinate of **dš** = OP , representing the Strength magnitude of **dš** in **z-z** principal direction .

which is a [*Cuboid J,E,B*] = **[** ds , P∇i **]** *and* → $| d\check{s} | = \sqrt{|OP|^2} = \sqrt{J^2 + E^2 + B^2}$

It is proved in quaternions that , ***negation truth Unit*** **eo** = [-∇λ , ±∇xΛ] = 0 is the mould that instantly transfers Inertial mass , ***momentum*** **p = ϖ. λ = m.v =Λ** to all Inertial or not frames Layers **K**$_{1,2,3}$ = **λ.p .**

The three fictitious *forces Fields* [J , E , B] of any motion .



Linear motion *Non-Linear motion* (F.4)

On any single particle of wavelength AiBi = λ = ds and p = momentum exists :
dš . dP = λ.p = constant = h → is the reduced Planck constant for each Energy Layer .
Since dP = (∂P/∂x) **x** + (∂P/dy) **y** + (∂P/∂z) **z** = [(∂/∂x) + (∂/dy) + (∂/∂z)] **©** P **= - ∇P** , *the unit gradient of* **dš = a** ± **bi** = 1 → 1 = λ.∇ ± k.∇**.** (**i**) = λ.∇ ± k.∇(**©**) = **λ .∇ ± k.∇i** where .
λ .∇ = the wavelength , **Λ = k**$_{1,2,3}$ **.∇ i .** = the momentum **.**

Since Particles (wave like) do not enter a Space smaller than their wave length , therefore photons ***Light*** *do not enter the“ Gravity Space = Graviton” which is a Space smaller than wave length of light. In Black Holes also , where* ***ds is nere zero and P*** = ∞ , ***is needed a new type of light to see what is happening below Planck length Level .***

Since Vectors follow Pythagoras conservation law [16] , so exists a constant relation between them . In Vector Calculus are quantified the different Aspects and Physical Terms for the four properties ***Gradient , Divergence , Curl , Laplacian*** and generally ***Quaternions ,*** for spatial geometry and Spaces which relate Spaces as Scalar (ds) and Vector part (Imaginary) , *which very shortly are as* ,

1.. Operation ***Gradient*** → Notation **grad(f) = ∇f → Del (multiply)Sf**
Measures the Rate and Direction of Change in a Scalar Field (Sf) ,(by multiplying) *and* Maps Scalar Fields (Sf) to Vector Fields (Vf). → *Vector Flux per unit volume*
2.. Operation ***Divergence*** → Notation **div (F) = ∇. F → Del (dot) Vf**
Measures the Magnitude of a Source or Sink at a given Point in a Vector Field (Vf), (by multiplying) *and* Maps Vector Fields (Vf) to Scalar Fields (Sf). → *Vector Flux density per unit area*

3.. Operation ***Curl*** → Notation **curl(F) = ∇xF → Del (cross)Vf**
Measures the Tendency to Rotate about a Point in a Vector Field (Vf) ,(by rotating) *and* Maps Vector Fields to (pseudo) Vector Fields . → *Vector change of Flux*

4.. Operation ***Laplacian*** → Notation **$\Delta(f) = \nabla^2 f = \nabla.\nabla f$ → Del(dot)Del(multiply)Sf**
A Composition of the Divergence and Gradient operation (by multiplying and rotating) *and* Maps Scalar Fields (Sf) to Scalar Fields(Sf). → *The Divergence of the gradient of the Tensor (Scalar and Vector)*

5.. Operation ***Quaternion*** → Notation **$\bar{z} = [\, s , \bar{v} .\nabla i \,]$ → Scalar + Vector(dot)(multi-Del .v̄) $\bar{z}$**
A Composition of Scalar Fields (s) and Vector Fields (v̄) of a Frame , to a new Unit which maps the alterations of Unit by rotation only and transforms scalar magnitudes to vectors and vice-versa.

5.. Quaternions :

Euler's rotation in 3D space is represented by an axis (vector) and an angle of rotation , which is a property of complex number and defined as **$\bar{z} = [\, s \pm \bar{v}.i \,]$** where **s,|v̄|** are real numbers and **i** the imaginary part such that $i^2 = -1$. Extending imaginary part to three dimensions $v_1 i , v_2 j , v_3 k \rightarrow \bar{v}\, \nabla i$ becomes quaternion which has 1+3 = 4 degrees of freedom . .

Properties **:**

Quaternion **:** $\bar{z} = s + \bar{v} = [\, s + \bar{v}.i \,] = s + [v_1 + v_2 + v_3].\nabla i = [\, s + \bar{v}\, \nabla i \,]$, where s is the Scalar part and $\bar{v} = [v_1 + v_2 + v_3]$ the Imaginary part of it , equal to $\bar{v}\, \nabla i$.

Decomposition of $\bar{z}$ into exponential form is $\bar{z} = [\, s+\bar{v}\, \nabla i \,] = |\, \bar{z} \,|.e\text{^}(\theta/2).\bar{u}\, \nabla i = \sqrt{\bar{z}'\bar{z}}\, .[\cos(\theta/2) - \bar{u}.\sin(\theta/2)]$

where θ =ArcCos(s / |z̄|) , is the rotation angle and $\bar{u} = (\bar{v}.\ \nabla i\ )/|\ (\bar{z})|$ is the rotation unit axis ($\bar{u} = -1$)

where Unit axis $\bar{u} = e\text{^}(\theta/2) = \cos(\theta/2) - i.\sin(\theta/2)$ called also Rotor and $\bar{z} = e\text{^}\bar{u}\, .\nabla i$ the Translator.

If $\bar{u}$ is Unit quaternion then $\bar{u} = [\, s+\bar{v}\, \nabla i \,] = \cos\varphi + \sin\varphi.\nabla i$ where φ = ArcCos(**s**) , $s^2+v_1^2+v_2^2+v_3^2 = 1$

and vector $\bar{v} = [\, v_1 / \sin\varphi + v_2 / \sin\varphi + v_3 / \sin\varphi \,]$ and exponentially $\bar{u} = e\text{^}[\, (\theta/2).\bar{u}\, \nabla i] = \cos(\theta/2)+\sin(\theta/2)$

Quaternion Conjugate **:** $\bar{z}' = s - \bar{v}.i = s - [v_1 + v_2 + v_3]\,].\nabla i = s - \bar{v}.\nabla i$, which is defined by negating the vector part of the quaternion .

Quaternion Conjugation **:** $\bar{z}.\bar{z}' = (s + \bar{v}).(s - \bar{v}) = s^2 - \bar{v}^2$

Quaternion magnitude : The magnitude (Norm) is defined by $|\bar{z}| = \sqrt{\bar{z}.\bar{z}'} = \sqrt{s^2+|\bar{v}|^2} = \sqrt{s^2+|\bar{v}\nabla i|^2}$,

and for two $\bar{z}_1, \bar{z}_2$ → $|\, \bar{z}_1, \bar{z}_2 \,| = |\bar{z}_1|.|\bar{z}_2| = (\sqrt{\bar{z}_1\, \bar{z}_1'}).(\sqrt{\bar{z}_2\, \bar{z}_2'}) = (\sqrt{s_1^2 + |\bar{v}_1|^2}\,).(\sqrt{s_2^2 + |\bar{v}_2|^2}\,) =$

$(|s_1|^2+|\bar{v}_1|^2).(|s_2|^2+|\bar{v}_2|^2) = [\sqrt{s_1^2 +|\bar{v}_1\nabla i\,|^2}].[\sqrt{s_2^2 +|\bar{v}_2\nabla i\,|^2}]$

The Normalized (the versor) $\bar{z}v = \bar{z} / |\bar{z}|$, and the Inverse is $\bar{z}^{-1} = \bar{z}' / |\bar{z}|^2 = \bar{z}'/(s^2+\bar{v}^2) = z'/(\, s^2+|\bar{v}\nabla i\,|^2)$

Quaternion Unite : For norm $|\bar{z}| = 1$ then $\bar{z}$ is the unit quaternion , and inverse $\bar{z}^{-1} = \bar{z}' / |\bar{z}|^2 = \bar{z}'$ i.e the inverse of a unit quaternion equals to the conjugate of the unit quaternion .

Quaternion Multiplication **:** $\bar{z}_1.\bar{z}_2 = (s_1+\bar{v}_1\nabla i).(s_2+\bar{v}_2\nabla i) = [\, s_1 s_2 - \bar{v}_1.\bar{v}_2 + \bar{v}_1 x \bar{v}_2.\nabla i^2 + s_1\bar{v}_2.\nabla i + s_2\bar{v}_1\nabla i \,]$

where $\bar{v}_1.\bar{v}_2$ is the ,dot, *inner*, product and $\bar{v}_1 x \bar{v}_2$ the ,cross, *outer*, product of vectors $\bar{v}_1, \bar{v}_2$.

Multiplication of a quaternion with its conjugate , represent two equal and opposite spinning states .

Quaternion Assumptions **:** $\bar{z} = s + \bar{v} = s + [v_1 + v_2 + v_3].\nabla i$,and if so then exists an angle θ = arcos(s) and the unit vector $\bar{v}\nabla i = (\bar{v}_1 /\sin\theta),(\bar{v}_2 /\sin\theta),(\bar{v}_3 /\sin\theta)$ such that $\bar{z} = [\, \cos\theta + \bar{v}\nabla i.\sin\theta \,]$.

A unit quaternion $\bar{z} = s + z_x\, i+z_y\, j+z_z\, k$ in Spherical coordinates with three angles $\varepsilon,\varphi,\theta$ is as $z_x = \sin\varepsilon.\sin\varphi.\cos\theta$, $z_y = \sin\varepsilon.\sin\varphi.\sin\theta$, $z_z = \sin\varepsilon.\cos\varphi$, $s = \cos\varepsilon$ and the inverse unit quaternion as $\varepsilon = \cos^{-1} s$, $\varphi = \cos^{-1}(\, z_z /\sin\varepsilon)$, $\theta = \cos^{-1}[\, z_y /(\sin\varepsilon.\sin\theta)]$.

The Spatial interpretation of a point P is [P = a+v̄.i **,** where $\tan\theta = \bar{v}/a$, in *Complex System*] , and in 3D space *Cartesian System* **P** (a,x,y,z) , where $a^2+x^2+y^2+z^2 = 1$, and then point P represents a rotation around the axis directed by the vector (x , y, z) of the unit sphere by an angle $\varphi = 2.\cos^{-1} a = 2.\sin^{-1} .\sqrt{x^2+ y^2+ z^2} = 2 / [\sqrt{x^2+ y^2+ z^2}]$, and its Conjugate **P'**[a ,-(x,y,z)]

Since $\nabla i\, .\nabla i^{-1} = 1$, then all unit vectors are perpendicular between them .

Quaternion Actions :

Action (©) of a quaternion $\bar{z} = s + \bar{v}.i = s + \bar{v}\nabla i$, on point **P** (a,x,y,z) is $z_a = \bar{z}p\bar{z}^{-1}$ (*screw motion*) and for $a \neq 0$ then z and a©z = az have the same action $\bar{z}p\bar{z}^{-1}$, meaning that quaternion is homogeneous in nature . Action of a Unit quaternion on a scalar **s** is $\bar{z} = \bar{z}s\bar{z}^{-1} = s\bar{z}\bar{z}^{-1} = s$ **.**

Action of a Unit quaternion $\bar{z}$ on a vector ($\bar{v}\nabla i$) is $\bar{z}v\bar{z}^{-1}$ i.e another vector $\bar{v}'$(quaternion) $\bar{v}' = (0,\bar{v}'\nabla i\ )$ and of vector type $\bar{v}'\ \nabla i\ ) = \bar{z} + 2\bar{v}\, (\bar{v}\ x\ \bar{z}) + 2\bar{v}\ x\ (\bar{v}\ x\ \bar{z})$ **.**

When the components of a vector $\bar{w}$ (w_x i+w_y j+w_z k) are expressed in terms of the three Euler angles ε,φ,θ then is as quaternion z(z_o+z_x i+z_yj+z_z k), where z_o =cos(ε+θ)/2.cos(φ/2), z_x = -cos(θ-ε)/2.sin(φ/2), z_y = sin(θ-ε)/2.sin(φ/2), z_z = - sin(ε+θ)/2.cos(φ/2) , and the time derivative as [dz/dt] = (**z** / 2) **x** w .

Action of a Unit quaternion on a point **p** (a,x,y,z) is **p' = $\bar{z}$p$\bar{z}^{-1}$** , i.e. another point [takes point P(s, $\bar{v}_1\nabla i$) to point P'(s, $\bar{v}_2\nabla i$)] and if the point is on the unit axes , then the unit quaternion is representing rotation through an angle **θ** about the unit axis **v** and it is **p' =**(p±sin(θ/2)**v**) .
Generally all monads ĀB = [AB - P^{-}_A , P^{-}_B] which are entities embodied with their Laws are quaternions .

Example **1 :** The conjugation operation of a function (*quaternion*) **$\bar{z}$** = s + **$\bar{v}_n$**.∇i **= $\bar{z}$** [s +v_x i+v_y j+v_z k], (*when s =0 then z is a vector and i,j,k are unit vectors representing the three Cartesian axes*)→ on vector **ū**(0,v) of quaternion **ū** as ū = u_x i+u_y j+u_z k , **is ū$\bar{z}$ū$^{-1}$** . First consider the rotation of **$\bar{z}$** around the common axis v = i+j+k with rotation angle **θ** (*angle* θ *in rad*).
If the length (Norm) of the common axis is | ū | = √1+1+1 = √3 , then conjugation is done by norm unit quaternion **ū** = [cos(θ/2) + sin(θ/2) . {u**(**i+j+k**)** / |ū|**}**] and by the conjugate of unit quaternion **ū$^{-1}$** = [cos(θ/2) - sin(θ/2).{u**(**i+j+k**)** / |ū|**}**] . The rotation function (quaternion) is then **z' = uzu^{-1}** or **$\bar{z}$ '** = [cos(θ/2) **+** sin(θ/2).{u**(**i+j+k**)** /|ū|**}**]**.**(s + v_x i+v_y j+v_z k)**.**[cos(θ/2) **-** sin(θ/2).{u**(**i+j+k**)** / |ū|**}**]
For three ordinary vectors , p = p_x i+p_yj+p_zk with their unit quaternion is **pūū(pū)$^{-1}$** = **p(ūuū$^{-1}$)** **p^{-1}** *.i.e.*

The conjugation operation (The Action of $\bar{z}$ on ū) , of any quaternion $\bar{z}$ = [a.cosθ + ū .sinθ] = **e**^ (θ/2).[ª + **ū**∇i] ***on any homogenous point P (a', x,y,z) is the Rotation of the Normalized unit quaternion*** [ª + **ū**∇i] ***which is the versor ,*** *by the rotation angle 2θ* ***, around the rotation unit axis*** *ū(a',u)* ***through point P .*** The normalized unit Quaternion is [cos(θ/2) +sin(θ/2) .{u**(**i+j+k**)**∇i/|ū| **]** .

Example **2 :**
When vector **$\bar{w}$** (w_x i+w_y j+w_z k) is the angular velocity vector , in the absence of applied torques , **L** (L_x i+L_y j+L_z k) = I.w = r̄**x**m$\bar{v}$ = r̄p , is the angular momentum vector (*where r =lever arm distance and m.$\bar{v}$ = p , the linear or translation momentum*) and I = (I_1,I_2,I_3) are the Principal moments of Inertia then angular kinetic energy **T** = ½. **w L** = ½. $I_1.w_1{}^2$ + ½. $I_2.w_2{}^2$ + ½. $I_3.w_3{}^2$.
Since both L and T are conserved as $L^2 = L_1{}^2 + L_2{}^2 + L_3{}^2$ and $T = L_1^2/2J_1 + L_2^2/2J_2 + L_3^2/2J_3$ and by division becomes 1 = [$L_1^2/2TJ_1$] + [$L_2^2/2TJ_2$] + [$L_3^2/2TJ_3$] , [***Poinsot's ellipsoid construction***]

i.e. T + L = *constant and so are preserved*, and when $L^2 / 2TJ = r^2p^2 /2T.2T/(w^2) = w^2. r^2\mathbf{x}p^2/(4T^2)$ = $w^2 / [2T/ \bar{r}\mathbf{x}p]^2$, then this is a Kinetic-Energy Inertial ellipsoid dependent on the total kinetic energy T, and the translational momentum p, with axes **a** =[2T/ r̄**x**p_1] , **b** = [2T/ r̄**x**p_2] , **c** = [2T/ r̄**x**p_3].
(*For Ellipsoid* $x^2/a^2+ y^2/b^2+z^2/c^2 = 1$ *and when* $a^2= b^2 = c^2 = r^2$ *the sphere* $x^2+ y^2+z^2 = r^2$) .

Considering $L^2/2J = J.J.w^2/2.J = Jw^2/2$ = r̄.p.w/2 then $T = |\sqrt{r_1p_1w_1}|^2 + |\sqrt{r_2p_2w_2}|^2 + |\sqrt{r_3p_3w_3}|^2$ or $a^2 = (L_1\sqrt{2J_1})^2$, $\sqrt{T^2}= \sqrt{a^2+b^2+c^2}$) , i.e. a cuboid (**a**x**b**x**c**) , rectangular parallelepiped ,with dimensions $a = |\sqrt{r_1p_1w_1}|$, $b = |\sqrt{r_2p_2w_2}|$, $c = |\sqrt{r_3p_3w_3}|$ and the length of the space diagonal $T = \sqrt{a^2+b^2+c^2}$.

Because the ***position velocity*** of a quaternion z̄ = s+$\bar{v}$∇i is [dž/ds] = (dž/ds ,0) (s + $\bar{v}$.∇i) = (1+dv/ds. ∇i^{-1}) and ***acceleration*** [d^2z/ds^2] = (d/ds ,0) (1+d$\bar{v}$/ds.∇i^{-1}) = (0, $d^2\bar{v}/ds^2$.∇i) and in Polar plane coordinates where angular momentum L = Iw = r̄**x**m$\bar{v}$ = r̄p , then ***acceleration*** [$d^2\bar{z}/ds^2$] = (dz̄/ds,0$)^2$ (s , r.cosθ, r.sinθ, 0) = (0, L^2/m^2r^3 + r , 2Ldr/ds / mr^2 , 0) where ,
L^2/m^2r^3 + r = the acceleration in the (**r̄**) radial direction ,
2Ldr/ds / mr^2 = the acceleration in **θ** direction . Since Points are nothing and may be anywhere in motionless space , so Position quaternion is referred to this space only , *and generally the velocity and acceleration in a non-Inertial , rotating reference frame is as ,*

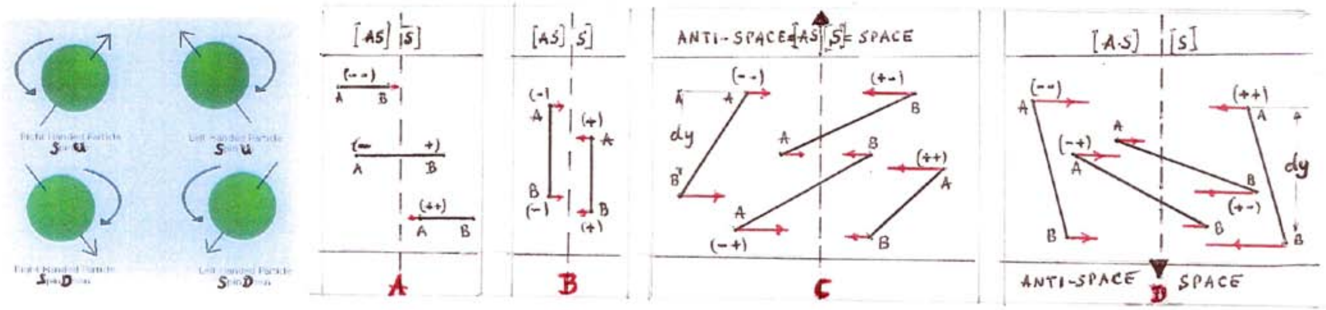
Velocity → **[dž/dt] = (d/dt , ϖ) (0 ,ž) = (- ϖ.ž , ϖ x ž + dž /dt)** and
Acceleration → **[d²ž/dt²]** = (d/dt , ϖ) (- ϖ.ž , dž /dt + ϖ x ž) =
= (- dϖ/dt .ž , d²ž /dt²+2 ϖ x **dž /dt + dϖ/dt** x **ž – ϖ.žϖ)** where

→ $-d\varpi/dt\,.\check{z}$ = the intrinsic acceleration of quaternion ,
→ $d^2\check{z}/dt^2$ = the translational alterations (they are in the special case of rotational motion where rotation on two or more axes creates linear acceleration in , *one only different rotational axis J*) ,
→ $2\,\varpi \times d\check{z}/dt + d\varpi/dt \times \check{z}$ = the coriolis acceleration , a centripetal acceleration is that of a force by which bodies (*of the reference frames*) are drawn or impelled towards a point or to a centre (*the hypothetical* motionless *non-rotational frame*) ,
→ $-\varpi.\check{z}\varpi) = -\varpi x(\varpi x\check{z}) + \varpi^2\check{z}$ = the azimuthal acceleration which appears in a non-uniform rotating reference frame in which there is variation in the angular velocity of the reference point .
Time **t** does not interfere with the calculations in the motionless frame .

i.e. ***The conjugation operation (The Action of ž on ū) is a constant rotational Kinetic–energy*** (**T**) , ***which is mapped out*** *,by the nib of angular velocity vector* ***ϖ , as the Inertia ellipsoid in space which instantaneously rotates around*** *vector axis* ***ϖ (the composition of all rotations) with the constant polar distance*** ϖ.L/|L| ***and the constant angles*** *θs ,* *θb, traced* ***on Space cone and on Body cone which are rolling around the common axis of*** *ϖ vector.*

and if the three components of T are on a cuboid with dimensions a,b,c then (Action of ž on any ū) corresponds to the composition of all rotations by the rotation of unit vector axis *ū(0,u⃗)* ***by keeping a unit cuboid (J,E,B) held fixed at one point of it , and rotating it ,θ, about the long diagonal of unit cuboid through the fixed point (*** *the directional axis of the cuboid on ū).*

6.. Spin Modeling *, of Dipole [AB] =[ĀB – PA⁻ ,PB⁻]* = **[a ± b.i] in microscopic description** : [20]



(F.5).

A. Force dP is applied in direction AB and is acting in the same straight line so moment lever is 0 .

a..(- -) Charge = dP = P B - PA = - dP Spin = - dP . (dy/2) = 0
b..(- +) Charge = dP = P B - PA = **0** Spin = dP . (dy/2) = 0
c..(++) Charge = dP = P B - PA = + dP Spin = + dP . (dy/2) = 0

B. Force dP = 0 or ≠0 applied in direction ⊥ AB and so moment lever = AB .

a..(- -) Charge = dP = P B - PA = ± **2.** dP Spin = ± dP . (AB/2) → ± 1 ↕ UP- Down
b..(++) Charge = dP = P B - PA = ∓ **2.** dP Spin = ∓ dP . (AB/2) → ∓ 1 ↕ Down-UP

C. Force dP is not applied in direction AB and not acting in the same line so moment lever≠ 0 .

a..(- -) Charge = dP = P B - PA = - dP Spin = - dP . (dy/2) → - ½ ↑ UP
b..(++) Charge = dP = P B - PA = + dP Spin = +dP . (ds/2) → + ½ ↑ UP
c..(+-) Charge = dP = P B - PA = - dP Spin = - dP . (dy/2) → - ½ ↑ UP
d..(-+) Charge = dP = P B - PA = + dP Spin = + dP . (dy/2) → + ½ ↑ UP

D. Force dP is not applied in direction AB and not acting in the same line so moment lever≠ 0 .
a..(- -) Charge = dP = P B - PA = - dP Spin = - dP . (dy/2) → - ½ ↓ Down
b..(++) Charge = dP = P B - PA = + dP Spin = + dP . (dy/2) → + ½ ↓ Down
c..(+-) Charge = dP = P B - PA = - dP Spin = - dP . (dy/2) → - ½ ↓ Down
d..(-+) Charge = dP = P B - PA = + dP Spin = + dP . (dy/2) → + ½ ↓ Down

7.. The method .

Equilibrium presupposes Homogenous Space and the Symmetrical Anti-Space .
For two points A , B which coincide , exists *Principle of Superposition* ($A \equiv B$) which is a Steady State containing Extrema for each point separately . i.e. In three dimentional Space , the infinite points exist as Space because of the Equal and opposite Impulses (*Opposite Forces PA+PB=0*) which is a new ***Notion , for Mass and Energy in AB distance .*** (Points A ,B are embodied with Opposite Forces). Dipole $\bar{A}B$ on the infinite others Spaces in [PNS] carry all quantum quantities and from this point laws of Physics start using the ***Quaternions Conformation manifold*** . (F.5-6)

1.. Primary Quaternion of equilibrium dipole $\bar{A}B = \bar{r} = \lambda$ in Space , Anti-space and of Force P :
The couple P , - P creates Work as moment $M = P.(r/2).\sin\theta = [r .P] .\sin\theta/ 2 = W = [P.\lambda]$. *As logarithm maps points on the sphere into points in the tangent sphere* , so work $P.\lambda$ is transformed as angular momentum $\bar{L} = p \times \lambda$ where then $\mathbf{P} = \mathbf{p}$ and it is the amount of rotation $\boldsymbol{\Lambda}$ i.e. $P = p = \Lambda$ and the work W, for the infinite points on the two tangential to **r** planes is equal to $W = [r.P] = [\lambda.\Lambda] = k$ *where*

> λ = displacement of A to B and it is a scalar magnitude called wavelength of dipole AB .
> Λ = the amount of rotation on dipole AB (this is angular momentum L^- and it is a vector).

i.***e*** $\boldsymbol{\lambda . \Lambda}$ = ***constant for all dipole*** λ , *and* ***since*** λ ***is constant*** $\boldsymbol{\Lambda}$ ***is also constant , and from*** equivalent formula of $\Lambda = L^- = \bar{r} \times p = I.w = [\lambda.p] = \bar{r}.(P.\sin\theta) = P.\bar{r} \sin\theta$ = constant → ***Spin of*** $\boldsymbol{\lambda}$ **.**
so the infinite dipole $A^-B,(\lambda.p) = A^-B,(\lambda.\Lambda)$ in Primary Space [PNS] are quaternion as ,

$\bar{\mathbf{z}}_{\mathbf{o}} = [\mathbf{s} , \bar{v}_n.\nabla i] = [\boldsymbol{\lambda} , \pm \boldsymbol{\Lambda}^- .\nabla i] = [\boldsymbol{\lambda} , \pm \mathbf{L}^-.\nabla i]$ ……………..(m1) → is the quaternion of the Primary Space dipole , ***where***

$\boldsymbol{\lambda}$ = ***the length of dipole (wavelength) which is a scalar magnitude ,***
$\boldsymbol{\Lambda}$ = ***the spin of dipole , equal to the angular momentum vector*** $p = \mathbf{L}^-$ *and exponentially*

$\mathbf{z}_{\mathbf{o}} = [\mathbf{s} , \bar{v}_n.\nabla i] = [\boldsymbol{\lambda} , \pm\boldsymbol{\Lambda}^-.\nabla i] = [\boldsymbol{\lambda} ,\pm \mathbf{L}^-.\nabla i] = | \bar{\mathbf{z}}_{\mathbf{o}} |.\mathbf{e}\wedge \arccos|\lambda/\bar{\mathbf{z}}_{\mathbf{o}}|.[\Lambda^-.\nabla i / |\Lambda|] = | \bar{\mathbf{z}}_{\mathbf{o}}|.\mathbf{e}\wedge\theta. \Lambda^- \nabla i$ .. (m2)

The conjugate quaternion is $\bar{\mathbf{z}}'_{\mathbf{o}} = (\lambda , + \Lambda^-.\nabla i) (\lambda ,- \boldsymbol{\Lambda}^-.\nabla i) = [\boldsymbol{\lambda^2 - |\Lambda|^2}]$
Repetition quaternion is $\bar{z}_{o1} = (\lambda , + \Lambda^-.\nabla i) (\lambda ,+ \Lambda^-.\nabla i) = [\lambda^2 - |\Lambda^-|^2 + 2.\lambda x\Lambda.\nabla i] =$
$= [\lambda^2 -\Lambda^2] = [\lambda^2 - (i^2+ j^2+k^2) |\Lambda^2|]$ since λ , Λ^- are $\perp$ axially.
In Polar form $\bar{z}_{o1} = (\lambda , \Lambda^-.\nabla i) = | \sqrt{\bar{z}'_o} . \bar{z}_o |e\wedge \{\mathrm{ArcCos} (\lambda / |\sqrt{\bar{z}'_o . \bar{z}_{o1}}|)\}. \Lambda^-.\nabla i / \sqrt{\Lambda'\Lambda^-}]$ …..(m3)

i.e. Primary Space quaternion $\bar{z}_o$ ***multiplied by its conjugate*** $\bar{z}'_o$***, is cancelling the vector*** $\Lambda . \nabla i$ ***and leaving the scalars*** $\lambda , |\Lambda^-|$ ***only .***

> ***Repetition quaternion's property (*** $\bar{z}_{o1}$ ***) is a new quaternion by transforming the scalar magnitudes*** *wavelength* λ *and spin magnitude* $|\Lambda|$ **,** ***to vectors*** $\Lambda^-\nabla i$ which is the velocity in a perpendicular plane) **.**
> ***In particles (*** *it is a spherical rotation in opposite directions for the Space Anti-space equilibrium)* ***consists the source motionless frame and it is velocity*** $\bar{v}$ ***in the two crossed fields*** $E \perp B$ **.**

Primary Space does not depend on Time because λ and Λ are constants and $(\partial/t , 0) (\lambda,\Lambda^-\nabla i) = 0$
The Binomial quaternion $q_B = \nabla q = \nabla^{-1}[\boldsymbol{\lambda} , \pm \boldsymbol{\Lambda}.\nabla i]$ and the Tangential $q_T = -\nabla^{-1}\nabla^1 q = -\nabla^o q =$
$- \nabla^o.[\boldsymbol{\lambda} , \pm \boldsymbol{\Lambda}.\nabla i] = - \nabla^o q = - q = -[\boldsymbol{\lambda} , \pm \boldsymbol{\Lambda}.\nabla i]$ ..….(m4)

i.e. The equilibrium Anti-Space , (*on the roll axis , equilibrium Spin of Space and Spin of Anti-Space*).

Using the quaternion multiplication rule ($\check{z}\varpi = -\check{z} \,.\, \varpi , \check{z} \times \varpi$) and since also $\nabla i.\nabla i = - 1$ then , on any reference frame (BF) , Euclidean product for two quaternion $\bar{\mathbf{z}}_{\mathbf{o}} , \bar{\mathbf{z}}$ is $\bar{\mathbf{z}}_{\mathbf{o}} © \bar{\mathbf{z}}$ →

$\bar{\mathbf{z}}_{\mathbf{o}} © \bar{\mathbf{z}} = [\boldsymbol{\lambda} , \boldsymbol{\Lambda}.\nabla i] © [\mathbf{s} , \mathbf{v}.\nabla i] = \boldsymbol{\lambda} \mathbf{s} - \boldsymbol{\Lambda} \bar{\mathbf{v}}.\nabla i , \boldsymbol{\lambda} \bar{\mathbf{v}}.\nabla i - \boldsymbol{\Lambda} \mathbf{s} .\nabla i - \boldsymbol{\Lambda} \times \bar{\mathbf{v}}.\nabla i$ ..……… (m5)
exponentially

$\bar{\mathbf{z}}_{\mathbf{o}} © \bar{\mathbf{z}} = [\boldsymbol{\lambda} , \boldsymbol{\Lambda}.\nabla i] © [\mathbf{s} , \bar{\mathbf{v}}.\nabla i] = | [\bar{\mathbf{z}}_{\mathbf{o}} © \bar{\mathbf{z}}] |.\mathbf{e}\wedge (\boldsymbol{\pi}½).[\bar{\mathbf{z}}_{\mathbf{o}} * \bar{\mathbf{z}}]. \boldsymbol{\Lambda}\nabla i / |[\bar{\mathbf{z}}_{\mathbf{o}} * \bar{\mathbf{z}}]|$ ………. (m6)

and for **w** power as they are the Spaces $(\bar{\mathbf{z}}_{\mathbf{o}})^W = (\boldsymbol{\lambda} , \boldsymbol{\Lambda}.\nabla i)^{\mathbf{w}} = | \bar{\mathbf{z}}_{\mathbf{o}}|^{\mathbf{w}} . \mathbf{e}\wedge [\bar{\mathbf{u}}\, w\theta] =$
$| \bar{\mathbf{z}}_{\mathbf{o}}|^{\mathbf{w}} . \mathbf{e}\wedge \{ [\Lambda^-\nabla i/\sqrt{\Lambda'\Lambda^-}].[\mathrm{ArcCos} (w\lambda/ |\sqrt{\bar{z}'_o} . \bar{z}_o|] \}$ → i.e. → *The Physical Universe seems to be of Infinite simple harmonic oscillators.*

. ***and represents the extrinsic rotation** , which is equal to the intrinsic rotation and by the same angles* **, of any quaternion of Spin Λ ,** *but with inverted order of rotations and vice-versa* **.**
Imbedded Events (Time) on (m5,6) are now the flipped signs in reverse order alterations , related to the Primary Space quaternion and not the substance .

where

λ s - Λ v̄.∇i = The Intrinsic alterations of the reference body frame , BF,
λ v̄.∇i = The Translational alterations , which are in the special case of rotational motion where rotation on two or more axes creates linear acceleration , *in one only different rotational axis J* .
Λ s.∇i = The Coriolis alterations , a centripetal acceleration is that of a force by which bodies (*of the reference frames ,BF,*) are drawn or impelled towards a point or to a centre (*the Primary Space is the hypothetical motionless non-rotational global frame* ,GF,),
Λxv̄ .∇I = The Azimuthal alterations , which appears in a non-uniform rotating reference frame in which there is variation in the angular velocity w̄ of the reference frame ,BF,.

Time **T** interfere with the calculations in reference frame only and ***does not*** with the motionless frame. Since work in [PNS] is $W=\int_{A-B} = [\ P.d\bar{s}\ ] = 0$ and is stored on points A,B as **quaternion $\bar{z}_o$ =[λ, Λ∇i]** then forces (***the spin Λ***) ***are conservative and because work from conservative forces between points is independent of the taken path and on a closed loop is zero , curl = 0 and Force becomes from the Potential function gradient*** , and also from the equilibrium of Spaces Anti-spaces , ***where then Spin rotations , Λ̄, - Λ̄, are in inverted order of rotation and vice-versa ,*** then even function f(Λ) = f(-Λ) and odd function is - f(Λ) = f(-Λ) and their sum f(Λ) + f(-Λ) = 0 (F.6) i.e.

Mapping (graph) of Even function f(Λ) , is always symmetrical about Λ axis (i.e. a mirror) and of Odd symmetrical about the origin and this is the interpretation of the Wave Nature of Spaces **[PNS].**

Differential operator of *even* order quaternion ***plus*** differential operator of *odd* order quaternion is zero: ***It is the Mapping (graph) of Even function* f(Λ) *and of Odd* f(-Λ) *and is the interpretation of the Wave nature of Spaces and all the others (i.e.The Physical Universe behaves as a simple harmonic oscillator*) .** Because functions f(Λ) , f(-Λ) are Stationary and only their sum creates their *conjugation operation* through ***mould*** $\bar{z}_o$, therefore their sum is zero independently of time (*negation truth*) as ,

even function	f(Λ)	→	(∂/∂t ,∇) © (λ=0 , Λ∇)	= - ∇.Λ , ∇ **x** Λ	= **eo**	
odd function	f(-Λ)	→	(∂/∂t ,∇) © (λ=0 , -Λ∇)	= ∇.Λ , - ∇ **x** Λ	= **eo**	... (m7)
even + odd =	0	→	(-∇Λ , ∇ **x** Λ) + (∇Λ , -∇ **x** Λ)	= **[0 , 0 + 0] = 0**		
even - odd =	0	→	(-∇Λ , ∇ **x** Λ) - (∇Λ , -∇ **x** Λ)	= 2 .**[-** Λ∇ , ∇**x**Λ]	i.e. is doubled .	

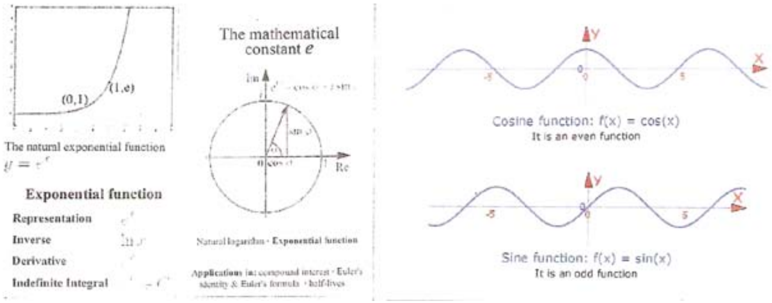
Quaternion of the Primary Space dipole is $\bar{z}_o$ **= [s,v̄**$_n$∇i] **= [λ , Λ**∇i] and it is the only one Physical existing truth monad ($\bar{z}_o$=1) and (eo = 0) the only Physical non-existing equilibrium monad , *This negation truth = the equilibrium of the two equal and opposite momentum p* = ± Λ̄ , on points and by using the additive form of Binary quaternion [∇ λ , 0] + [0 , -∇xΛ̄] = [∇ λ , -∇ x Λ̄] = 0 *then* , ***non-existence* (0) *becomes existence with* [PNS] *motionless dynamic mould* ($\bar{z}_o$=1) *, and it is Done everywhere*** , following Boolean logic operations with all combinational rules and laws , as follows ,

Element [z̄o =1]	Element [eo = 0]	Conjunction [z̄o → 0]	Conjugation [z̄o © 0]	Quaternion [z̄o ≡ eo]
0	0	zo	0	zo
zo	0	0	zo	0
0	zo	zo	zo	0
zo	zo	zo	0	zo

Quaternions (m1),(m3),(m7) ↔ **$\bar{z}_o$ = [λ , ±** Λ∇i] **, $\bar{z}'_o$** = **[λ² - |Λ̄|²] , eo** = **[** Λ∇ **, - ∇** x **Λ̄]**

are the three fundamental equations of [PNS] ,unifying the ***homogenous Euclidean geometry*** **(λ=λ∇)** and the ***source term Energy*** (dš.dP = λ.Λ = constant $K_{1,2,3}$ with motion Λ) ,and imbedding in them all conservation physical laws with the only two quantized magnitudes **λ , Λ̄** on Monad ĀB *which are*

λ = ***the length of geometry primary dipole*** (wavelength of dipole AB) ***which is a scalar magnitude*** ,
Λ̄ = ***the spin of dipole , source term ,*** the amount of rotation on dipole Λ̄B , ***equal to the angular momentum vector* p = Λ = ϖ. λ = m.v̄** = d/ds{ ∫A-B [P.ds] } **.**



e function Even – Odd (F.6).

2.. Metric (distance) of Euclidean Spaces

The Spatial interpretation of a point P is [P = r +v̄.i **,** where tan**θ** = v̄/r , in *Complex System*] , and in 3D space *Cartesian System* P = (r, r̄.∇i) = **P** [r, (i.x+j.y+k.z) , where $r^2+x^2+y^2+z^2 = 1$, and then point P represents a rotation around the axis directed by the vector (x,y,z) of the unit sphere by an angle φ as , cosφ = r/||p̄|| and in Polar form point q = (r+ v̄ i) = √q'q.e^[|r/√q'q|.|v̄ i/√v̄'.v|] . For two points where P_1 = r $_1$+$\bar{v}_1$i , P_2 = r $_2$+$\bar{v}_2$.i

ds̄(P_1,P_2) =ds̄= Normalized (P_1+P_2) = (P_1+P_2)'©(P_1+P_2) = [√|P'$_1$.P_1|+|P'$_2$.P_2|] = [√|P'$_1$.P_1|+|P'$_2$.P_2 |] ….(N1)

FUNCTIONS : *Polar* decomposition of point P = ||P ||.Up where Up =P/ ||P || = P [r,(i.x+j.y+k.z)] / √ $r^2+ x^2+y^2+z^2$) = (1/q') = P(r, r̄.∇i) / ||q || , *Reciprocal quaternion* q⁻ ' = q '/||q ||^2 , *Norm* of q = ||q || = √qq' = √ $r^2+ x^2+y^2+z^2$) , *Conjugate* q' = q(r, - r̄.∇i) = q [r,- (.x+.y+.z)] = 1 / unit quaternion , *the Unit quaternion* qu(r,r̄.∇i) = q(r, r̄.∇i) /||q|| = cosθ + ū sinθ = e where [($r^2+ x^2+y^2+z^2$) / ||q||^2] = 1 ,
θ = ArcCos (r/||q ||) = [r,(i.x+j.y+k.z)] / √ $r^2+x^2+y^2+z^2$ and ū = [(i.x+j.y+kz)/||q||sinθ] ,
Exponential q → e^(q) = e^(r,r̄) = e^r. [cos||r||+ r̄ /|| r̄||.sin||r̄||] , *Natural Ln of* q → = ln||q|| + (r̄/||r̄||) .ArcCos (r̄/||r̄||) , and the ,w, *Power* decomposition of q→ $q^W = |q|^W.e^{\wedge a}(^W\theta)$ = $|q|^W$.(cos.wθ + ā.sin.wθ) .

Conjugating (N1) on point P then [0, Λ] © [r + r̄.i **]** = 0 -Λr̄ , 0+ Λr+ Λxr̄ = **- |Λ̄|.|r̄| ,** r**Λ̄ + Λ̄xr̄** ….(N2)
i.e.
Quaternion (N2) instantly transfers momentum **Λ̄** = (Intrinsic **Spin** of [PNS]) to all points of the energy Frames Layers **K** 1,2,3 = **λ . Λ** and onto

Black Holes Scale conjugation → In Microscopic scale as Gravity for lower spin ???
Planck Scale Matter conjugation → In Inertial Frames as Gravity
Dark Matter Scale conjugation → In Macroscopic scale as Gravity for faster spin ???

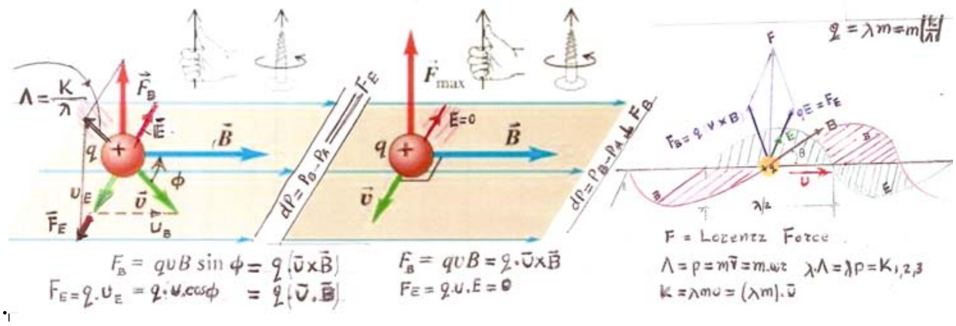
This embodied conjugation is done between the alterations of ,*negation truth Unit* , [0, Λ̄] onto points of [PNS] where P[|r| , r̄∇i] = P[|r̄| +(x+y+z).i] , |r| *is referred as the distance of any origin to the point P and angles φ,θ, are the* **r̄** *orientations and* x-z *axis respectively, x=|r|.sinθ.cosφ ,y=|r|sinθ.sinφ ,z=|r|cosθ.*

The ***quaternion's directional dipole*** **z̄o** = [0, Λ̄] © [|r|+r̄.∇i **]** = **- |Λ̄ |.|r̄| ,** |r|**Λ̄ +Λ̄xr̄** = [**so+v̄o .∇i**]
meaning that [0, Λ] is rotating around unit axis r̄ = i + j + k with a rotation angle 2ω , *in radians* and a unit length of | r̄| = √1+1+1 = √3 , and on the new directional dipole quaternion around unit axis , where [**so+ v̄o .∇i**] = cosθo + v̄o.sinθo∇i = {cosθo + v̄o.∇i .sinθo} = cosθo + [v̄o.∇i / Λ²|so²-vo²|]. sinθo ……(N3)

where θo = arc cos(so) , Λ² = the normalized [0, Λ] , |so²-vo²| = the normalized directional dipole
with

so = - |Λ̄ |.|r̄| → The gauge as Spin's magnitudes .
|r|**Λ̄** → The Coriolis alterations from the centripetal accelerations of forces on point P [the reference point frame , BF] which are drawn or impelled towards this , from
v̄o = ↕ a centre of the Primary Space which is the motionless and non-rotational GF frame.
Λ̄xv̄ → The Azimuthal alterations which appear on point from the non-uniform rotating reference frames in where there are variation in angular velocity . i.e.

Directional dipole **z̄o ,** is the rotation corresponding to keeping a cube held fixed at the point , and rotating it 2θo about the long diagonal through this fixed point , where the three axes are permuted cyclically, and influence on any other quaternions **zn** [s+v̄.i**]** equal to **eo** = [|r|+r̄.i **]** ©[0, Λ̄] © [s + v̄.i**]** following rotations at static points P[|r| , r̄∇i] of [PNS] , in the two , *perpendicularly interchanged and conserved*, equilibrium states **- |Λ̄ |.|r̄|** = |r| **. Λ̄** and **- |Λ̄ |.|r̄| = Λ̄ xr̄** where |r|**Λ̄** ⊥ **Λ̄xr̄ .** Example : (F.5) – (F7)



(F.7)

Dipole $\bar{A}B$ = [λ ,Λ] in [PNS] are composed of the two elements λ , Λ which are created from points A , B **|AB| = λ** = wavelength (*dipoles*) and from W= (r.dP) = r̄xp = I.w = [λ.p] = λ.Λ = k2 → **Λ** = **p** = momentum and Forces **dP =** P^{-}_{B} –P^{-}_{A}) which are the sources of Space field .(*the moving charges is velocity* **v̄** *created from dipole momentum* ±Λ^{-} *when is mapped on the perpendicular to Λ plane as* → **v̄** E || dP and **v̄** B ⊥ dP). Since (dP ⊥ ± $\bar{\Lambda}$) the work occurring from momentum p̄ = mv̄ = Λ (Spin) acting on force **dP** is zero , so ***momentum*** $\bar{\mathbf{\Lambda}}$ **= mv̄** ***only , is exerting the velocity vector*** $\bar{v}$ ***to the dipole , λ ,*** with the generalized mass **m** (*the reaction to the motion*) which creates the component forces , **F**E || dP^{-}.v̄ and **F**B ⊥ dP^{-}**x** v̄.

Forces dP // (parallel) *to the parallel of Space Anti-Space lines* { [S] ≡ [AS] } , create a Static force field **E** in (dP , λ) plane where E^{-} ⊥ ± $\bar{\Lambda}$ and so E^{-}, v̄ E^{-}, dP are co plane .
Forces dP ⊥ (perpendicular) *to the parallel of Space Anti-Space lines* { [S] ≡ [AS] } , create a Static force field **B** which is perpendicular to **E** force field and perpendicular to dP , λ plane also .

Velocity vector **v̄** (**v̄** E , **v̄** B) is in [$\bar{v}$**.** B^{-}] plane forming an angle θ < 180° to the force field **B⁻** .
The oriented parallelogram spanned by the cross product of the two vectors **v̄** and **B** is the bivector **B⁻**^ **v̄** which cross product is vector v̄ **x** B^{-} **,** therefore is created a force in a vector product → **F**B = (λm). **v̄** x **B** (*because velocity vector* **v̄** *=(p/m) = (Λ /m) = [k2 / λm] and* → *(λm).*$\bar{v}$ *= k 2*) and for the coplanar force field **E⁻** the vector **v̄ . E⁻** = (**v̄** E). **E⁻** which experience a force → **F** E = (λm).**E .** i.e.
The two perpendicular Static force fields **E** and Static force field **B** of Space-Anti-Space ,experience on any moving dipole $\bar{A}B$ = [λ , Λ] with velocity **v̄** a total force **F = F** E + **F** B = (λm). **E** + (λm). **v̄** x **B** which combination of the two types result in a helical motion , with stability demand → E = -(v̄**x**B) = **-(**v̄.B)⊥ …(N4)
which is the alternative conservation of momentum [k 2= Λ²/2λm] *in the two perpendicular fields E , B .*
In case (λm) = **q** then force **F** = **F** E+**F** B = q.E + q.v̄ x B = **q**.[**E + v̄ x B**] → ***which is Lorentz force, in the Electromagnetic crossed fields*** *E* ***and*** *B* ***with electric charge*** *q* = λm ***and it is the fundamental interpretation cause (effect) of motion , in small and large scales . !!!***

3.. Gravitational field and Newton's 2nd Law in a Non-inertial rotating Frame :
When conjugation is done between eo = [-∇ λ ,∇ x Λ] = 0 , and a quaternion of the differential time operator ∂/∂t and 3D angular speed vector ϖ then , → (∂/∂t , ϖ) © (-λ∇ ,∇ x Λ) =
d/dt(-∇ λ) + ϖ.∇xΛ **,** d/dt(∇xΛ) + ϖ**.**∇λ - ϖ x∇ x Λ = 0 - ϖ.Λ **,** 0 + ϖλ + ϖ.Λ = **0 , ϖ.λ** or ,

(∂/∂t , ϖ) © (- λ∇ ,∇ x Λ) = 0 , ϖ. λ = [0 , Λ] ………………(N5)

Equation (N5) implies that the new quaternion which maps the alterations of , *negation truth Unit* , by rotation only , transforms only vector term magnitudes and since ϖ is velocity ϖ.λ is momentum p , i.e. ***negation truth Unit*** **eo** = [- λ∇,∇xΛ] = 0 is a machine that instantly transfers Inertial mass as ***momentum*** **Λ = ϖ. λ = p = m.v̄ = m.(w̄ r̄)** = (mr̄).**w̄** = (mλ)**w̄ = (J).w̄** to all points , in Inertial or not , frames Layers **K**1,2,3 = **λ.Λ** and over spaces , ***NOT as said with Big-Bang*** but of this reason only .

Wavelength **(λ)** may be equal to **0 ,** where then angular velocity ϖ → ∞ meaning that , this is also happening to all Inertial or not Frames . Label 'gravity' probably is referred to something heavy .
Conjugation between the quaternion of the differential time operator ∂/∂t and 3D angular speed vector ϖ and the Position quaternion z̄ = (r = 0, ž) is the velocity (∂/∂t, ϖ) © (0 ,ž) = (- ϖ.ž , ϖ x ž + dž /dt) and (∂/∂t , ϖ) © (- ϖ.ž , ϖ x ž + dž /dt) = (- dϖ/dt .ž , d²ž /dt²+2 ϖ x dž /dt + dϖ/dt x ž – ϖ.žϖ **)** .....(N6)
and it is the acceleration which transforms both scalar and vector parts .
Time (t) ,which is a phenomenological reference concept of alterations and it is the only element in the scalar of an event of a Position quaternion , does not exists in **eo** unit where Energy is related as momentum **Λ** , **so** → ***Universe is a Space-Energy Configuration Frame*** *and not* Space-Time as believed **.**

Force field is the derivative of the potential of Newton's scalar field equation $\nabla^2\Phi = 4\pi G\rho$ and for vacuum $\Phi = GM / \sqrt{(x^2+y^2+z^2)}$ which is the same as the square quaternion of **eo** → $(eo)^2 = [\nabla\lambda, \Lambda x\nabla]^2 = [\lambda^2 - \Lambda^2 \pm 2\lambda.\Lambda\nabla^2] = \lambda^2 - \Lambda^2 \pm 2\lambda.\Lambda = \lambda^2-\Lambda^2\pm 2.\lambda x\Lambda = [\lambda^2 - \Lambda^2] = [\lambda^2 - (i^2+j^2+k^2)|\Lambda^2|]$ since λ , Λ are axially .

The upper relation was used by Special relativity [*Minkowski metric* ($g\mu\nu$)] *for Events to be represented as 4-vectors for all scalar operations*, ***without knowing that this is*** (**eo**)^2 , ***which equilibrium the two opposite and spherical rotations of Space Anti-space transforms vorticity magnitudes*** ± |**Λ**| ***to all other , 4- vectors*** $\bar{z} = [s, \bar{v}.\nabla i]$. Conjugation of $\bar{z}_n$ © $(eo)^2 = [s_n, \bar{v}_n.\nabla i].[\lambda^2-\Lambda^2] = \lambda^2.s_n - \Lambda^2.\bar{v}_n.\nabla i, - s_n.\Lambda^2 - \lambda^2.\bar{v}_n.\nabla i - (\bar{v}_n x\Lambda^2)\nabla i$ may give an explanation to linear central force and to Newton's inverse square law .

Newton law of motion for a material point is - Force = mass **x** acceleration (a) - in an Inertial frame $a = d^2r/dt^2$

By twice applying the transformation from stationary to rotating frame , absolute acceleration **a** is written as $a = d^2r/dt^2 = |d/dt|.|dr/dt| = |d/dt|.[(dr/dt) + w\mathbf{x}\, r] = (d^2r/dt^2)+|dw/dt|\mathbf{x}\, r + 2w\mathbf{x}(dr/dt)+w\mathbf{x}(w\mathbf{x}\, r)$ or

$$|dw/dt|\mathbf{x}\, r + 2w\mathbf{x}(dr/dt) + w\mathbf{x}(w\mathbf{x}\, r) = [a - d^2r/dt^2] \quad \rightarrow \quad \text{i.e.}$$

Apparent acceleration's terms of , **a** ,

are independent of mass and a first interpretation is because *acceleration is the hypothetical* ***external reaction to the motion*** which does not happen in stationary [PNS] but of action on points only.

Momentum p= mv on points in [PNS] is expressed by constant **m** and angular velocity **v** and since **m** is hypothetical magnitude representing the reaction to the motion , then p = m **x** angular velocity = m **x** $\bar{w}$.

Rearranging (N2) then $[0, \bar{\Lambda}]©[r + \bar{r}.i] = |\bar{\Lambda}|.|\bar{r}|+|r|.\bar{\Lambda} + \bar{\Lambda}x\bar{r} = |r|.\bar{\Lambda} + |\bar{\Lambda}|.|\bar{r}|+\bar{\Lambda}x\bar{r} = \mathbf{\bar{w}.|r| + |\bar{\Lambda}|.|\bar{r}| + \bar{\Lambda}x\bar{r}}$ ...(N7)

because by replacing $\Lambda = m\bar{w}$ then Velocity on Points $P(r, \bar{r}.\nabla i) = m[\bar{w}.|r| + \bar{w}.\bar{r} + \bar{w}x\bar{r}]$ and since for points in motionless Frame [PNS] momentum Λ and the position vector of point vector $\bar{\mathbf{r}}$ apply on the same stationary point then $\bar{w}.|r|=0$ (*in motionless* [PNS] *only*) and the variation at point which is the same as the variation of $\bar{r}$, therefore, $|\bar{\Lambda}|.|\bar{r}|$ is equal to $d\bar{r}$,else $|\bar{\Lambda}|.|\bar{r}|$ **,** Angular velocity $\bar{w} = |\bar{\Lambda}|/|\bar{r}| = \mathbf{k}/(\lambda m)$ i.e (N7) is the effect of momentum $\Lambda = m\bar{w}$ on points as velocity magnitude ,and so quaternion of the differential time operator $\partial/\partial t$ to 3D angular speed vector $\bar{w}$ is the Apparent acceleration at Point as , $\gamma = \partial/\partial t[d\bar{r} + |\bar{\Lambda}|.|\bar{r}| + \bar{\Lambda}x\bar{r}] = \partial/\partial t[d\bar{r}/dt + \bar{w}.\bar{r} + \bar{w}x\bar{r}].m = m.(d/dt)[d\bar{r}/dt + \bar{w}.\bar{r} + \bar{w}x\bar{r}] = m.(d/dt)[d\bar{r}/dt + \bar{w}x\bar{r}]$,

which is → $\gamma = \mathbf{m.}\, d/dt)[d\bar{r}/dt + \bar{w}x\bar{r}] = \mathbf{m}.[(d^2\bar{r}/dt^2) + |d\bar{w}/dt|\mathbf{x}\,\bar{r} + 2\bar{w}\mathbf{x}(d\bar{r}/dt) + \bar{w}\mathbf{x}(\bar{w}\mathbf{x}\,\bar{r})]$ ......(N8)

where ,

$m.(d\bar{r}/dt).\bar{v}$ = Centrifugal Energy $(m.w^2r)$ **,** $\bar{v}$ = Position Velocity $= [d\bar{r}/dt + \bar{w}.\bar{r} + \bar{w}x\bar{r}]$

$m.(d\bar{r}/dt)\, x\, \bar{v}$ = Coriolis Energy (2m.wv) **,** $\bar{a}$ = Position Acceleration = $d\bar{v}/dt$

$m.\bar{r}\, x\, \bar{a}$ = Euler's Energy (m.w.r) **,** m = Constant ⇉ *The hypothetical Reaction to the motion*

$m.\bar{r}.\bar{a}$ = Any other Energy

$(d^2\bar{r}/dt^2)$ = Linear acceleration of position point

$|d\bar{w}/dt|\mathbf{x}\,\bar{r}$ = Euler intrinsic acceleration of position point

$2\bar{w}\mathbf{x}(d\bar{r}/dt)$ = Coriolis intrinsic acceleration of position point

$\bar{w}\mathbf{x}(\bar{w}\mathbf{x}\,\bar{r})$ = Centrifugal intrinsic acceleration of position point

Remarks :

Effection (γ) **,** *is The Gravity in Inertial Frames of Planck's Scale matter conjugation* **,** ***of momentum*** ($\mathbf{\Lambda = m\bar{v}}$) ***on Points*** $\mathbf{P(r, \bar{r}.\nabla i)}$ **of [PNS]** → $\mathbf{z_o = [\lambda, \pm\Lambda\nabla i] = |\bar{z}_o|.e}^{\wedge arc.cos|\lambda/\bar{z}_o|.[\bar{\Lambda}.\nabla i / |\Lambda|]} = |\bar{z}_o|.e^{\wedge\theta.\bar{\Lambda}\nabla i}$ ***is the ,*** **Gravity in Scale Frames** , which is harmonically oscillated to all points of infinite spaces in [PNS] as *the New Quaternion* $(-|\bar{\Lambda}|.|\bar{r}|, r\bar{\Lambda} + \bar{\Lambda}x\bar{r})$ and oriented on the directional axis of the points **(** ***is on the unit quaternion of the points*** **)** as the Diagonal of an Energy Cuboid (*Poinsot's ellipsoid*) or Cube , which decomposition follows Pythagoras conservation law, *Total Energy*→$W = \int_{A-B}[P.ds] = T = \sqrt{J^2 + E^2 + B^2}$ where the three orthogonal magnitudes (J,E,B) , $\mathbf{J} \approx d\bar{r}/dt = \bar{v}$, $\mathbf{E} \approx \bar{w}.\bar{r} = \bar{\mathbf{v}}_E$, $\mathbf{B} \approx \bar{w}x\bar{r} = \bar{\mathbf{v}}_B$ of Energy-state follow Cuboidal , Plane , or Linear Diagonal direction as the normal quaternion $[-|\bar{\Lambda}|.|\bar{r}|, r\bar{\Lambda} + \bar{\Lambda}x\bar{r}] / (\Lambda r\sqrt{3})$.

Stability is obtained by the opposite momentum $-\Lambda^{-}$ where $E = -(\bar{v}\mathbf{x}B) = -(\bar{\mathbf{v}}.B)\perp$ → or $\mathbf{B} \perp \mathbf{E}$

The two perpendicular Static force fields **E** and Static force field **B** of Space-Anti-Space , ***experience*** on any moving dipole $\bar{\Lambda}$ B = [λ , Λ] with velocity $\bar{\mathbf{v}}$ (*only momentum* $\bar{\Lambda} = m\bar{v}$ *is exerting the velocity vector* $\bar{\mathbf{v}}$ *to the dipole* **λ**) a total force $\mathbf{F} = \mathbf{F}_E + \mathbf{F}_B = (\lambda m).\,\mathbf{E} + (\lambda m).\,\bar{\mathbf{v}}\, x\, \mathbf{B}$ which combination of the two types result in a helical motion and generally to any Space Configuration (*Continuum*) Extensive property , as ***Kinetic*** (3-current motion) and ***Potential*** (*the perpendicular Stored curl fields E , B*) ***energy , by displacement*** (*the magnitude of a vector from initial to the subsequent position*) ***and rotation*** .

8.. Applications .

8..1 - Cauchy equations of equilibrium in Classical Mechanics .

Notation :

When external forces (F_i) are applied to objects made of Isotropic Elastic materials (E , G) , they produce change in shape and size (the deformation) of the objects .

Stress (σ) = Force / Area , → It is the Internal force (per unit area etc) associated with a strain (deformation) . The Internal average forces per unit of surface acting within a deformable body .

Force = Stress . Area → F = (σ) . A

Strain (ε) = change of length / length → It is the relative change in shape or size of an object due to externally-applied forces . Young modulus (E) = tensile Stress / tensile Strain

Stress (σ) = E. Strain = E. (ε) , Strain = Stress / E = **ε** = **ε** (u , v , w)

G = shear modulus = E.m/2(m+1) where m = Poisson's ratio = $1 / \mu \approx 10/3$

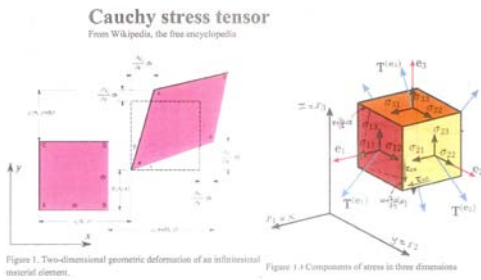
Displacement (vector) = δ ($\delta x,\delta y,\delta z$) = **dš** → It is the shortest distance (length) from the initial to the final position of a point P and has not any width .

γ_{xz} = the change in angle of plane xz and equal to strain τ_{xz} / G.

In Physics , ***Energy*** is a conserved extensive property of a physical system which cannot be directly observed but can be calculated , by its ability to ***perform work*** (W) .

Potential Energy is the Energy stored in a body or system due to its position in a force field or due to its configuration , and is absorbed by the system undergoing deformations and when released this *Potential Strain Energy* (Up) perform Mechanical Work, and when violence released *Potential Energy* is transformed to ***Kinetic Energy*** (Uk) and so on , i.e the Mechanical system will then vibrate at once or more of its "natural frequency" and damp (Force friction is transformed into heat) down to zero .

Strain Energy (W) is defined as the energy absorbed by a system undergoing deformations .



(F.8)

(σ) = Stress tensor = [(σ_{11} , σ_{12} , σ_{13}) / (σ_{21} , σ_{22} , σ_{23}) / (σ_{31} , σ_{32} , σ_{33})] ≡
= [(σ_{xx} , σ_{xy} , σ_{xz}) / (σ_{yx} , σ_{yy} , σ_{yz}) / (σ_{zx} , σ_{zyz})] ≡
= [(σ_x , τ_{xy} , τ_{xz}) / (τ_{yx} , σ_y , τ_{yz}) / (τ_{zx} , τ_{zy} , σ_z)] .

(ε) = Strain-displacement = [(ε_x ,γ_{xy},γ_{xz}) / (γ_{yx} , ε_y , γ_{yz}) / (γ_{zx} ,γ_{zy} , ε_z)] .

$u_i(x,y,z)$ = In index notation , displacement field in three dimensions = (u , v , w)

$\delta u_i(x,y,z)$ = Virtual displacement field in three dimensions = (δu , δv , δw)

The theory of Elasticity is based on the following principles ,

1.. The demand of conservation of stability between the exerted forces (and gravity F) , and the tensions that are applied on an infinitesimal unit volume and satisfy the equilibrium equations.
2.. The conservation demand of the geometrical continuum (ρ = mass density) during the elastic (E=Young modulus) deformation and also the principle of conservation of the angular momentum.
3.. The elastic constitutive equations , where Hooke's law represents the material behavior which relates the unknown stresses and strains (Cauchy-Navier equations of Virtual work for solids) .

1.1 The first demand is enclosed in equations (1) as follows ,

$$\partial \sigma_x / \partial x + \partial \tau_{yx} / \partial y + \partial \tau_{zx} / \partial z + F_x = 0$$
$$\partial \tau_{xy} / \partial x + \partial \sigma_y / \partial y + \partial \tau_{zy} / \partial z + F_y = 0$$
$$\partial \tau_{xz} / \partial x + \partial \tau_{yz} / \partial y + \partial \sigma_z / \partial z + F_z = 0 \qquad (1)$$

2.1 The second demand is enclosed in equations (2) where the parameters of strain $\varepsilon_x,\varepsilon_y,\varepsilon_z$, γ_{xz} ,γ_{zx} ,γ_{xy} are depended on (2a) ,(2b) as follows ,

$$\varepsilon_x = \partial u / \partial x \ , \quad \varepsilon_y = \partial v / \partial y \ , \ \varepsilon_z = \partial w / \partial z \quad \text{and} \quad \mathbf{e} = \varepsilon_x + \varepsilon_y + \varepsilon_z \qquad (2)$$
$$\gamma_{yz} = \partial w / \partial y + \partial v / \partial z \ , \quad \gamma_{zx} = \partial u / \partial z + \partial w / \partial x \ , \gamma_{xy} = \partial v / \partial x + \partial u / \partial y$$

$\sigma x = 2.G.[e/(m-2) + \varepsilon x]$ $\tau yz = G.\gamma yz$ (2a)
$\sigma y = 2.G.[e/(m-2) + \varepsilon y]$ $\tau zx = G.\gamma zx$
$\sigma z = 2.G.[e/(m-2) + \varepsilon z]$ $\tau xy = G.\gamma xy$
$\sigma x = 2.G.[e/(m-2) + \partial u / \partial x]$ $\tau yz = G. [\partial w / \partial y + \partial v / \partial z]$ (2b)
$\sigma y = 2.G.[e/(m-2) + \partial v / \partial y]$ $\tau zx = G. [\partial u / \partial z + \partial w / \partial x]$
$\sigma z = 2.G.[e/(m-2) + \partial w / \partial z]$ $\tau xy = G. [\partial v / \partial x + \partial u / \partial y]$
where $e = \partial u / \partial x + \partial v / \partial y + \partial w / \partial z$ the cubical dilatation.

3.1 The third demand is enclosed in equations (3) as follows ,
$\varepsilon x = [\sigma x - \mu.(\sigma y + \sigma z)] / E = [\sigma x - (\sigma y + \sigma z) / m] / E$ (3)
$\varepsilon y = [\sigma y - \mu.(\sigma z + \sigma x)] / E = [\sigma y - (\sigma z + \sigma x) / m] / E$
$\varepsilon z = [\sigma z - \mu.(\sigma x + \sigma y)] / E = [\sigma z - (\sigma x + \sigma y) / m] / E$
$\gamma yz = \tau yz / G$, $\gamma zx = \tau zx / G$, $\gamma xy = \tau xy / G$, $\varepsilon x + \varepsilon y + \varepsilon z = (1 - 2.\mu).[\sigma x + \sigma y + \sigma z] / E$
and finally (3a) the six equations for normal stresses and normal strains ,

$\sigma x = [E/(1+\mu)] . [e\mu/(1-2.\mu) + \varepsilon x] = 2.G.[e\mu/(1-2.\mu) + \varepsilon x] = 2.G.[e/(m-2) + \varepsilon x]$ (3a)
$\sigma y = [E/(1+\mu)] . [e\mu/(1-2.\mu) + \varepsilon y] = 2.G.[e\mu/(1-2.\mu) + \varepsilon y] = 2.G.[e/(m-2) + \varepsilon y]$
$\sigma z = [E/(1+\mu)] . [e\mu/(1-2.\mu) + \varepsilon z] = 2.G.[e\mu/(1-2.\mu) + \varepsilon z] = 2.G.[e/(m-2) + \varepsilon z]$
$\tau yz = G. \gamma yz$, $\tau zx = G. \gamma zx$, $\tau xy = G. \gamma xy$,
introducing (3a) in Cauchy equations (1) then becomes

$2G[\partial^2 u/ \partial x^2 +(1/m-2).(\partial e/\partial x)] + G.[\partial^2 u/\partial y^2 +(\partial^2 v/\partial x \partial y)] + G[\partial^2 u / \partial z^2 +(\partial^2 w/\partial x \partial z)] + Fx = 0$ or
$[\partial^2 u/ \partial x^2 +\partial^2 u/\partial y^2 +\partial^2 u / \partial z^2] + (\partial/\partial x).[\partial u/\partial x+\partial v/\partial y+\partial w/\partial z] + [(2/m-2).\partial e/\partial x] + Fx/G = 0$ where

$\partial u / \partial x + \partial v / \partial y + \partial w / \partial z = e$ and introducing Laplace operation
$\partial^2 / \partial x^2 + \partial^2 / \partial y^2 + \partial^2 / \partial z^2 = \nabla^2$ we have the fundamental equations of Elasticity .

$\boldsymbol{\nabla^2 u +[m /(m-2)]. \partial e / \partial x + Fx / G = 0}$ (4)
$\boldsymbol{\nabla^2 v +[m /(m-2)]. \partial e / \partial y + Fy / G = 0}$
$\boldsymbol{\nabla^2 w +[m /(m-2)]. \partial e / \partial z + Fz / G = 0}$ where
u , v , w are the three normal components of strain **ε** , **e** = $\partial u / \partial x + \partial v / \partial y + \partial w / \partial z$
Fx , Fy , Fz the three components of force **F** , **m** and **G** are constants and in *Differential form*

$\boldsymbol{\nabla^2 u +[m /(m-2)]. \nabla x.(\nabla.u) + Fx / G = 0}$ (4a)
$\boldsymbol{\nabla^2 v +[m /(m-2)]. \nabla y.(\nabla.v) + Fy / G = 0}$
$\boldsymbol{\nabla^2 w +[m /(m-2)]. \nabla z.(\nabla.w) + Fz / G = 0}$ and for **ε = ∇dš (u , v , w)**
$G.\nabla^2. \varepsilon + [m.G / (m-2)] . \nabla [\nabla. \varepsilon] + F = 0 \rightarrow G.\nabla^2. \nabla dš + [m.G / (m-2)] . \nabla [\nabla. dš] + F = 0$ **… (4b)**

i.e (4b) is a Static Equilibrium (zero acceleration) System of equations that relates the displacement field **ε** ($\varepsilon x=\partial u/\partial x,\varepsilon y=\partial v/\partial y,\varepsilon z=\partial w/\partial z$) with force field **F(** Fx,Fy,Fz **)** and constants **m ,G** derived , considering tractions a) across the surface of a volume element correspond to the stress Tensor which is a physical quantity independent of the coordinate system with scalar magnitude for the vector, resolved into two components .
[The one component is normal to the plane , ***the normal stress σ*** , and the other component parallel to this plane called the ***shearing stress τ*** ($\tau yz =G.\gamma yz$,$\tau zx =G.\gamma zx$, $\tau xy=G.\gamma xy$)] and b) to the ***body forces* F** .
According to the principle of conservation of angular momentum , the Tensor is symmetric where then $\tau xy = \tau yx$ $\tau yz = \tau zy$, $\tau zx = \tau xz$, and all deformed magnitudes lie in perpendicular planes to the normal stresses direction .
For dipole **ÃB** then **(dš) =[a ± b.i]** = |∇ε (u ,v,w) ± b.i| where **∇ε (u ,v,w)** = **dš**.© and © is the conjugation Contents of **(dš)** under Stability → G.**∇². ∇(a)** ± **[**m.G / (m-2)**] .∇** [**∇b.i**] = **F** → ***Gravity force .***
Solving equations (4) , functions u , v , w are defined and from (2) then $\partial^2\varepsilon y/ \partial z^2 = \partial^2 v/\partial y \partial z^2$, $\partial^2\varepsilon z/ \partial y^2 = \partial^2 w/\partial z \partial y^2$, $\partial^2\gamma yz /\partial y\partial z = \partial^2 v /\partial y \partial z^2 + \partial^2 w/\partial z \partial y^2$ and from equation $\partial^2\gamma yz /\partial y\partial z = \partial^2\varepsilon y /\partial z^2 + \partial^2\varepsilon z /\partial y^2$ then elastic deformation $\partial^2\gamma zx /\partial z\partial x = \partial^2\varepsilon z /\partial x^2 + \partial^2\varepsilon x /\partial z^2$, $\partial^2\gamma xy /\partial x\partial y = \partial^2\varepsilon x /\partial y^2 + \partial^2\varepsilon y /\partial x^2$

Saint Venant equation :
It has been proved that linear distribution of tensile stresses by Navier coincides with that of Bernoulli
For further definition of u , $\partial^2 u / \partial y^2 + \partial^2 u / \partial z^2 = 0$ and from Cauchy equations ,
$(\partial u / \partial y).dz - (\partial u / \partial z).dy + y.dy + z.dz = 0$ which means that all surfaces perpendicular to main axis x turn as a disk and points follow $u = u(y,z)$ function and for u =constant then $y.dy + z.dz = 0$ and by integration $y^2 + z^2$ = constant , meaning that only circular Plane envelope may exist . Any line of symmetry is a principal axis and Plane of symmetry perpendicular to the principal axis .

8..2.. VIRTUAL WORK OF INTERNAL FORCES dFi

Castigliano's method :

First theorem for Forces in an elastic structure . *States that if the strain energy of an elastic structure can be expressed as a function of Generalized displacement* ***δi*** *(U is a scalar function)then the partial derivative of the strain energy with respect to generalized Displacement gives the generalized force Fi and in equation form Fi* = ∂U / ∂δi , and for all forces **∇σ + Fi = ∂U / ∂δi , ε = [∇σ +∇ū] / 2 .**

Second theorem for Displacements in an elastic structure .*States that if the strain energy of an elastic structure can be expressed as a function of Generalized force* ***Fi*** *then the partial derivative of the strain energy with respect to generalized force gives the generalized displacement* ***δi*** *in the direction of Fi .*
It is necessary to calculate the Virtual Work of the internal forces as a function of the components of the stress tensor , by isolating an infinitesimal element (dx ,dy ,dz) of volume dV and assuming that the components of the stress are constant throughout its volume . If this infinitesimal element is subjected to a Virtual displacement **δi** , it is easy to calculate the virtual Work (**W**) performed by the external forces acting on its surface .Mechanical Energy of a System is the work being performed by all External and Internal Forces and is equal to zero as ,

$W = (1/2). \sum_{1}^{n} Fi.\delta i$, $dW / dFr = \delta r$, $\partial W / \partial Fr = \delta r$,

For Principal stresses and stress invariants work W (in unit volume) and after substituting (3) in work then becomes ,

$2.W / \delta x.\delta y.\delta z = [\sigma x.\varepsilon x + \sigma y.\varepsilon y + \sigma z.\varepsilon z + \tau yz.\gamma yz + \tau zx.\gamma zx + \tau xy.\gamma xy]$ and in unit volume

$\mathbf{W = [\sigma x^2 + \sigma y^2 + \sigma z^2 - (2/m).(\sigma y.\sigma z + \sigma z.\sigma x + \sigma x.\sigma y] / 2E + [\tau yz^2 + \tau zx^2 + \tau xy^2] / 2G}$

$\mathbf{W = [\sigma x + \sigma y + \sigma z]^2 / 2E - [(\sigma y.\sigma z + \sigma z.\sigma x + \sigma x.\sigma y - \tau yz^2 - \tau zx^2 - \tau xy^2] / 2G}$

$\mathbf{W = G. [\varepsilon x^2 + \varepsilon y^2 + \varepsilon z^2 + (e^2/(m-2) + (\gamma yz^2 + \gamma zx^2 + \gamma xy^2) /2]}$ where $\mathbf{e} = \varepsilon x + \varepsilon y + \varepsilon z$

and for normal stresses and normal strains , then work W is ,

$W = [m.(\sigma 1^2 + \sigma 2^2 + \sigma 3^2) - 2.(\sigma 2.\sigma 3 + \sigma 3.\sigma 1 + \sigma 1.\sigma 2] / 2mE$

$W = [G.(\sigma 1 + \sigma 2 + \sigma 3)^2 - E.(\sigma 2.\sigma 3 + \sigma 3.\sigma 1 + \sigma 1.\sigma 2] / 2EG$

$W = G. [\varepsilon 1^2 + \varepsilon 2^2 + \varepsilon 3^2 + (e^2/(m-2]$

The partial derivative of the strain energy (**W**) *with respect to generalized force* (**Fi**) *gives the generalized displacement* ***δi*** , and (**W**) with respect to generalized Displacement (**σ**) gives the generalized force **Fi** which is Castigliano's theorem where ,

Ws = The work on surface (done by surface forces)
Wv = the work in Volume (by body forces).

$\partial W/ \partial \sigma x = \varepsilon x$, $\partial W/ \tau yz = \gamma yz$, $\partial W/ \partial Fx = \delta x$, $\partial W/ \partial \varepsilon x = \sigma x$, $\partial W/ \gamma yz = \tau yz$, $\partial W/ \partial \delta x = Fx$

For $e = (m-2/Em).[\sigma x+ \sigma y+ \sigma z] = 0$ then

$Ws = [\sigma 1 + \sigma 2 + \sigma 3]^2 (m-2) / 6mE = (m-2) . [\sigma 1 + \sigma 2 + \sigma 3]^2 / (12G.(m+1))$

$Wv = W – Ws = [(\sigma 2 - \sigma 3)^2 + (\sigma 3 - \sigma 1)^2 + (\sigma 1 – \sigma 2)^2] / 12G$

SPECIAL CASES OF VIRTUAL WORK

a). Volume stress :
A material is said to be under Plane stress if the stress vector is zero across a particular surface
i.e $\sigma 3 = 0$ or $\sigma z = \tau yz = \tau xz = 0$ and then work ,

$W = [(\sigma 1^2 + \sigma 2^2) / 2 - (\sigma 1. \sigma 2) / m] / E$

$Ws = [(\sigma 1 + \sigma 2)^2. (m-2) / [12.G.(m+1)]$, $Wv = [(\sigma 1^2 + \sigma 2^2 - \sigma 1. \sigma 2) / 6.G]$

b). Plane stress :
$\sigma y = \sigma z = \tau yz = \tau xz = 0$ and $\sigma x \neq 0$, $\tau xy = \tau \neq 0$ then ,

$W = \sigma x^2 / 2.E + \tau^2 / 2.G$

$Ws = (m-2).\sigma x^2 / [12.G.(m+1)]$, $Wv = (\sigma x^2 + 3.\tau^2) / 6.G$

c). Linear stress : $\sigma 1 \neq 0$, $\tau = 0$ then $W = \sigma 1^2 / 2.E$,$Ws = (m-2).\sigma 1^2/ [12.G.(m+1)]$,$Wv = \sigma 1^2 /6.G$

d). Shear stress :
$\sigma 2 = - \sigma 1$, $\sigma 3 = 0$, $\tau = \sigma 1$ then , $W = \sigma 1^2 / 2.E = \tau^2 / 2.G$, $Ws = 0$, $Wv = \sigma 1^2 / 2.E = \tau^2 / 2.G$

.

REMARKS :
A→↔← B = ∞ *Principle of Virtual Displacements.* **W** = ∫ P.ds = 0 , **ds** = ∂W / ∂P , **δr** = ∂W / ∂Fr
P A + P B = 0 *Principle of Stability*

In Physics , ***Energy is a conserved Extensive property*** , *of a Physical System (Configuration) which cannot be directly observed but can be calculated* , ***by its ability to perform work*** (W*) and in our case, Equations (4b) represents work equal to zero* , becoming from the action of forces Fi on an ,*Elastic material,* Configuration as Strain energy and absorbed as displacement field (u ,v ,w) upon the deformed placement of the Configuration (*It is a linear Elastic continuum medium*) on which is formed *the constant Stress state* (This is of distributed forces and couples as Kinetic energy = The stress Tensor T = n.σ) *as ellipsoid,* centered at the origin of the coordinate system and directed on principal axes ,with the lengths of the semi axis of ellipsoid equal to the magnitudes of the principal stresses which is → [Poinsot's ellipsoid construction] . Part of the Strain Energy is generally transformed on the Support Reactions , *and if* the three components of T are transformed on a cuboid with dimensions a,b,c then , (Action of Fi on any ž) corresponds to the composition of all distributed forces and couples as rotations only , by the rotation of unit vector axis ū(0, ž) , by keeping a unit cuboid held fixed at the origin of it and rotating it ,θ, about the long diagonal of unit cuboid throught origin (directional axis of cuboid on ū) , or
[(ÃB = (dž) = [a±b.i] = [∇**ε** (ū ,v̄,w̄) ± bi] = dž © , where © is the Contents of (dš)] .

8..3 - CAUCHY STRESS TENSOR .
Notation :
The Cauchy stress tensor is a second order tensor (a linear map) with nine components **σij** that define the state of stress at a point inside a material in the deformed placement or configuration ,and units the unit length direction **n** to the stress vector **T**(^{n}) across an imaginary surface perpendicular to **n** or ,
T(^{n}) = **n** . **σ** → T(^{n})j = σij . ni where
.
(**σ**) = *Stress tensor* = [(σ11 , σ12 , σ13) / (σ21 , σ22 , σ23) / (σ31 , σ32 , σ33)] ≡
= [(σxx , σxy , σxz) / (σyx , σyy , σyz) / (σzx , σzyz)] ≡ (a)
= [(σx , τxy , τxz) / (τyx , σy , τyz) / (τzx , τzy , σz)]

and because of the conservation of angular momentum ,stress tensor is symmetric i.e. six independent stress components .Three invariants associated with the stress tensor (the three eigenvalues of the stress tensor) are defined independently of any coordinate system (neither covariant nor contra variant). Because of continuity upon any surface (*real or imaginary*) that divides the body , the action of one part on the other , is equivalent to the system of distributed forces **Fi** and couples on the surface dividing the body , and is represented by the field **T**(^{n}), (stress vector) , defined on the surface **S** and dependent on unit-length vector **n** and equal to **dFi/dS ,** resolved into two components , the Normal and the Shear stress where , **σ$_n$** = **dF$_n$ / dS , τ$_s$** = **dF$_s$ / dS .**

Using on *Cauchy tetrahedron* Euler's law of motion , then the four equilibrium forces give ,
T(^{n})dA - **T**(e$_1$)dA$_1$ - **T**(e$_2$)dA$_2$ - **T**(e$_3$)dA$_3$ **= ρ.(**h.dA/3).**a** where **ρ** is the density , **a** is acceleration
dA$_1$ = (n.e$_1$)dA = n$_1$.dA , dA$_2$ = (n.e$_2$)dA = n$_2$.dA , dA$_3$ = (n.e$_3$)dA = n$_3$.dA , the projected to **n** faces and by cancelling dA then Equilibrium of Stress vector **T**(^{n}) -**T**(e$_1$)n$_1$ -**T**(e$_2$)n$_2$ -**T**(e$_3$)n$_3$ **= ρ.(** h/3).**a,** and for tetrahedron shrinking to 0 (h=0) then **T**(^{n}) = **T**(e$_1$)n$_1$ + **T**(e$_2$)n$_2$ + **T**(e$_3$)n$_3$ **,** and by analyzing to a Cartesian system then in index notation **T**(e$_i$) =**T**(e$_i$)j.ej = **σ$_{ij}$** .e$_j$ for j=1,3 ,where then **σ** = **σ$_{ij}$** →(b) and using the components of the stress tensor then **T**(^{n}) j = **σ**ij.n$_i$,
Normal and Shear forces :
The magnitude of the normal stress component **σ$_n$** of any stress vector **T**(^{n}) acting on a plane with normal unit vector **n** at a point, in terms of the components **σ$_{ij}$** of the stress tensor **σ** is the dot product of the stress vector and the normal unit vector **σ$_n$** = **T**(^{n}).**n** = **T**(^{n}) i. n$_i$ = **σ$_{ij}$ .n$_i$ .n$_j$** . ….(c)
and the magnitude of the shear stress component **τ$_n$** acting in the plane of the two vectors **T**(^{n}) and **n** is

Pythagoras theorem **τ$_n$** = √(**T**(^{n}))^2- **σ$_n$**^{2} =√(**T**(^{n})).(**T**(^{n})-**σ$_n$**^{2} = √ (**σ**ij.**n$_j$**).(**σ**i$_k$.**n$_k$**)= **σ**ij**σ**i$_k$.**n**$_j$ **n**$_k$ - **σ$_n$**^{2} ...(d).
Equations of motion :

Defined traction or Surface forces $\mathbf{T}^{(n)}{}_i$ per unit area acting on every point of the body surface **S** , and body forces **Fi** per unit of volume , on every point within the volume **V** , equilibrium if the resultant force acting on the volume is zero .**i.e**. $\int_V [\boldsymbol{\sigma}_{ji.j} + \mathbf{F}_i].dV = 0$ or $\boldsymbol{\sigma}_{ji.j} + \mathbf{F}_i = 0$ ……(e)

Principal stresses and stress invariants :

At every point in a stressed body there are at least three Principal Planes with normal vector **n** with the principal directions where the corresponding stress vector is perpendicular to the plane , without shear stresses , $\boldsymbol{\tau}_n = 0$, *called Principal stresses* . The components $\boldsymbol{\sigma}_{ij}$ depend on the orientation of the coordinate system at the point under consideration , but since stress tensor itself is a physical quantity , (a scalar) and as such it is independent of the coordinate system , and so similarly every second rank tensor (the stress and the strain) has three independent invariant quantities (**J,E,B**) associated with it. A set of invariants are the principal stresses of the stress tensor which are the eigenvalues of the stress tensor and their direction vectors are the principal directions or eigenvectors, or $\mathbf{T}^{(n)} = \lambda\mathbf{n} = \boldsymbol{\sigma}_n .\mathbf{n}$ ..(f) where $\boldsymbol{\lambda}$ is a constant of proportionality corresponding to the magnitudes $\boldsymbol{\sigma}_n$ of the principal stresses . Expanding determinant $| \boldsymbol{\sigma}_{ij} – \lambda_{ij} | = 0$ leads to the characteristic equation = $– \lambda^3 + I_1\lambda^2 - I_2\lambda + I_3 = 0$ where stress invariant $I_1 = \sigma 11+\sigma 22+\sigma 33$, $2I_2 = [\boldsymbol{\sigma}_{ii}\boldsymbol{\sigma}_{jj}-\boldsymbol{\sigma}_{ij}\boldsymbol{\sigma}_{ji}]$, $I_3 = \det [\boldsymbol{\sigma}_{ij}]$ and has three real roots λ_i and the Principal stresses at a point are $\boldsymbol{\sigma}_1 = \max(\lambda_1, \lambda_2, \lambda_3)$, $\boldsymbol{\sigma}_2 = I_1- \boldsymbol{\sigma}_1 – \boldsymbol{\sigma}_3$, $\boldsymbol{\sigma}_3 = \min(\lambda_1, \lambda_2, \lambda_3)$ which are independent of the orientation and the coordinate system . The stress invariants become
$I_1 = \sigma 1+\sigma 2+\sigma 3$, $I_2 = \sigma 1\sigma 2+\sigma 2\sigma 3+\sigma 3\sigma 1$, $I_3 = \sigma 1\sigma 2\sigma 3$. (g)

Maximum and minimum Principal shear stresses :

It is equal to one-half the difference between maxima and is oriented 45° from the principal stress planes as , $\boldsymbol{\tau}_{max} = [\boldsymbol{\sigma}_{max} - \boldsymbol{\sigma}_{min}] / 2 \rightarrow \boldsymbol{\tau}_{max} = [\boldsymbol{\sigma}_1 – \boldsymbol{\sigma}_3] / 2$ and $\rightarrow$ $\boldsymbol{\sigma}_n = [\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_3] / 2$.
Using the Lagrange's multiplier for stationary maximum values where the gradient of $\boldsymbol{\tau}_n^2$ is parallel to the gradient of surface forces **F ,** then $\partial \mathbf{F} / \partial \mathbf{n}_1 = 0$, $\partial \mathbf{F} / \partial \mathbf{n}_2 = 0$, $\partial \mathbf{F} / \partial \mathbf{n}_3 = 0$ and analytically
$\partial F/\partial \mathbf{n}_1=[\boldsymbol{\sigma}_1^2-2\boldsymbol{\sigma}_1\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_n^2-\boldsymbol{\tau}_n^2]\mathbf{n}_1= 0$, $\partial F/\partial \mathbf{n}_2=[\boldsymbol{\sigma}_2^2-2\boldsymbol{\sigma}_2\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_n^2-\boldsymbol{\tau}_n^2]\mathbf{n}_2= 0$, $\partial F/\partial \mathbf{n}_3=[\boldsymbol{\sigma}_3^2-2\boldsymbol{\sigma}_3\boldsymbol{\sigma}_n+\boldsymbol{\sigma}_n^2-\boldsymbol{\tau}_n^2]\mathbf{n}_1=0$
with three sets of solutions :

[n1= 0 , n2= 0 , n3 = ± 1,τ_n = 0] , [n1= 0 , n2= ± 1 , n3= 0, τ_n = 0] , [n1= ± 1, n2= 0 ,n3= 0 ,τ_n = 0]
[n1= 0 , n2≠ 0 , n3 = ≠0,τ_n = 0] , [n1≠ 0 , n2≠0, n3= 0, τ_n = 0] , [n1= 0, n2≠ 0 , n3≠ 0, τ_n = 0]
and in all cases $\rightarrow$ $\boldsymbol{\tau}_{max} = [\boldsymbol{\sigma}_1 – \boldsymbol{\sigma}_3] / 2 = [\boldsymbol{\sigma}_{max} – \boldsymbol{\sigma}_{min}] / 2$ ………… (h)

The work W has been reverted as the deformation of the Configuration by the principal stresses (σ) and on the principle strains direction (n) .
W = $[m.(\sigma 1^2 + \sigma 2^2 + \sigma 3^2) - 2.(\sigma 2.\sigma 3 + \sigma 3.\sigma 1 + \sigma 1.\sigma 2] / 2mE$ …. (j)
W = $[G.(\sigma 1 +\sigma 2 +\sigma 3)^2 - E.(\sigma 2.\sigma 3 +\sigma 3.\sigma 1 +\sigma 1.\sigma 2] /2EG$, W = G. $[\varepsilon 1^2 +\varepsilon 2^2 +\varepsilon 3^2 + (e^2/(m-2]$ *where*
m = Poisson's ratio , E = Young modulus , G = Shear modulus = E.m/2(m+1)

Stress deviator tensor :

Volumetric stress tensor or mean normal stress tensor is $\boldsymbol{\pi\delta}_{ij}$, and stress deviator tensor is $\mathbf{s}_{ij}$ where $\boldsymbol{\sigma}_{ij} =\mathbf{s}_{jj} + \boldsymbol{\pi\delta}_{ij}$ and $\boldsymbol{\pi} = \boldsymbol{\sigma}_{kk} / 3 = [\sigma_{11} ,\sigma_{22} ,\sigma_{33}] / 3 = I_1 / 3$ and pressure $p = \nabla.\hat{u} – \pi = \lambda [\partial u_k / \partial x_k] – \pi$, $\hat{u}$ is the velocity , λ is a proportionality constant , u_k the k;th Cartesian component of $\hat{u}$, so Cauchy stress tensor $\rightarrow$ $\mathbf{s}_{jj} = \boldsymbol{\sigma}_{ij} - \boldsymbol{\sigma}_{kk} . \boldsymbol{\delta}_{ij} / 3 = [(\sigma_{11}–\pi , \sigma_{12} , \sigma_{13}) / (\sigma_{21} ,\sigma_{22} –\pi ,\sigma_{23}) / (\sigma_{31} ,\sigma_{32} ,\sigma_{33}-\pi)]$ ..(i)

Lame's stress Ellipsoid is $\mathbf{n}^2= [\mathbf{T}^{(n)}{}_n]^2/\boldsymbol{\sigma}_n^2 \rightarrow \mathbf{n}_1^2+\mathbf{n}_2^2+\mathbf{n}_3^2 =[\mathbf{T}_1^2/\boldsymbol{\sigma}_1^2]+[\mathbf{T}_2^2/\boldsymbol{\sigma}_2^2]+[\mathbf{T}_3^2/\boldsymbol{\sigma}_3^2]= 1$
and Cuboid $\mathbf{n} = [\mathbf{T}^{(n)}{}_n] / \boldsymbol{\sigma}_n \rightarrow\sqrt{} \mathbf{n}_1^2+\mathbf{n}_2^2+\mathbf{n}_3^2=\sqrt{}[\mathbf{T}_1^2/\boldsymbol{\sigma}_1^2]+[\mathbf{T}_2^2/\boldsymbol{\sigma}_2^2]+[\mathbf{T}_3^2/\boldsymbol{\sigma}_3^2]$

which is mapping the stress state at a point of the continuum , on the stress ellipsoid surface .
When external (Fi) are applied to objects made of Isotropic Elastic materials (E ,G) , they produce change in shape and size (the deformation) of the objects and always forming , Lame's stress Ellipsoid independently of the orientation and the coordinate system ,

If the Homogeneous and Isotropic Elastic Continuum is a quaternion as $\rightarrow$ $d\check{s} = \check{z} = s + \hat{u} = s+\hat{u}.i$ = $s + [u_1+ u_2+u_3].\nabla i = \check{z}^W. e\wedge \pm [(s/\sqrt{}|\check{z}'z| \,.\, |\hat{u}/\hat{u}'u+|.\nabla i$, where scalar part ,**s,** are the points of P(x,y,z) and imaginary part ,û, forces F following Hook's law F = k.ds̄ **,** ž',û' are the normalized ž,û and rotation angle $\varphi = |s / \sqrt{}|\check{z}'z||$ **,** then ,

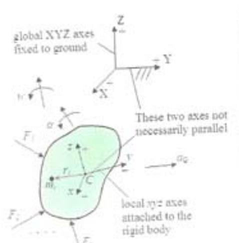
The Action of Fi on ž is a constant rotational Stress state in Continuum (Kinetic–energy = The stress tensor **T = n .σ**) , ***which is mapped out*** *, by the nib of vector* $\varpi = \omega r\varphi = \mathbf{r}\omega\varphi$ ***, as the Inertia ellipsoid [Poinsot's ellipsoid construction] in Continuum (medium) which instantaneously rotates around*** *vector axis* ϖ, φ ***with the constant polar distance*** ϖ.T/|T| ***and the constant angles*** θs ***,*** θb *,traced* ***on ,*** *Reference Frame cone* ***and*** *on Continuum cone* ***, which are rolling around the common axis of*** ϖ *vector.* ***and if the three components of T are on a cuboid with dimensions a,b,c then (Action of Fi on any ž) corresponds to the composition of all rotations only , by the rotation of unit vector axis*** *ū(0,u)* ***, by keeping a unit cuboid held fixed at one point of it , and rotating it ,θ, about the long diagonal of unit cuboids through the fixed point (*** *the directional axis of the cuboid on ū).*

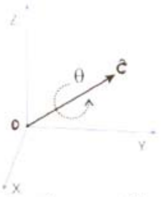
Generally , The dynamics of any system = Work = Total energy , ***is transferred as force to →***

1.. *To Elastic material Configuration* , as ***Strain energy*** and is absorbed as displacement field [**∇ε** (ū ,v̄,w̄)] upon the deformed placement , (where these alterations of shape by pressure or stress is equilibrium state of the Configuration $G.\nabla^2.\,\boldsymbol{\varepsilon} + [m.G/(m-2)]\,.\,\nabla\,[\,\nabla.\,\boldsymbol{\varepsilon}\,] = F$) ,
2.. *To Solid material Configuration* , as ***Kinetic*** (*Energy of motion* v̄) and ***Potential*** (*Stored Energy*) energy by displacement (the magnitude of a vector from initial to subsequent position) and rotation , on the principal axis of the Solid (the unit quaternion of point of mass rotating 2θo about the long diagonal) as the Diagonal of an Energy Cuboid , *which decomposition follows Pythagoras conservation law* , where the three magnitudes (J,E,B) of Energy-state follow Cuboidal , Plane, or Linear Diagonal direction,
3.. *To Quaternions Extensive Configuration* , as ***New Quaternions*** (with Scalar and Vector magnitudes). Points in Spaces carry A priori the work $W = \int_{A\text{-}B} [\,P.ds\,] = 0$, where magnitudes P, ds̄ can be varied leaving work unaltered .

8..4 Euler's Rigid Body Equations :

They study the movements of the ,*not deformed,* bodies of the local (x ,y ,z) reference Body Frame (BF) attached to the body , relative to the Global (X,Y,Z) reference frame (GF) by the External vector sum forces Fe acting and measured on the (BF) .




A rotation represented by an Euler axis and angle.


Euler angles, one of the possible ways to describe an orientation

(F.9)

Applying Newton's second law to mass points then $\Sigma Fe = \Sigma F + \Sigma Fi = m\bar{a}_i$ *where*

ΣFe=The vector sum of the forces,ΣF=The resultant force on point,ΣFi=The resultant Torque on point
m = mass , it is a constant equal to the Reaction to the motion or as Inertia (I) .
$\bar{a}_i = \bar{a}_c + \bar{a}x\bar{r}_i + \bar{w}x(\bar{w}x\bar{r}_i)$ = The acceleration vector of the vector mass point Pi with respect to GF .
$\bar{a}_c = a_{cx}\,\bar{i} + a_{cy}\,\bar{j} + a_{cz}\,\bar{k}$ = The acceleration of the Center of Mass , point C .
x , y , z „ $\bar{i}$, $\bar{j}$, $\bar{k}$ = The components and Unit vector of point P along the orientation vector .
$\bar{a} = a_x\,\bar{i} + a_y\,\bar{j} + a_z\,\bar{k}$ = The angular acceleration of point C with respect to GF ,resolved along BF.
$\bar{r}_i = x\,\bar{i} + y\,\bar{j} + z\,\bar{k}$ = The orientation vector of position point Pi.
$\bar{w} = w_x\,\bar{i} + w_y\,\bar{j} + w_z\,\bar{k}$ = The angular velocity of the body with respect to GF , resolved along BF .
and by substitution

$$\Sigma Fe = m\,.\,(\,\bar{a}_c + \bar{a}x\bar{r}_i + \bar{w}x(\bar{w}x\bar{r}_i)\,) = mi\,.\{\,\bar{r}_i\,\mathbf{x}\,\bar{a}_c + \bar{r}_i\,\mathbf{x}\,[\bar{a}\mathbf{x}\bar{r}_i] + \bar{r}_i\,\mathbf{x}[\bar{w}\mathbf{x}(w\mathbf{x}\bar{r}_i)]\,\}$$
$$= \{\,\bar{r}_i\,\mathbf{x}\,\bar{a}_c + \bar{r}_i\,\mathbf{x}\,[(dw/dt)\mathbf{x}\bar{r}_i] + \bar{r}_i\,\mathbf{x}[\bar{w}\mathbf{x}(w\mathbf{x}\bar{r}_i)]\,\} = \bar{r}_{c1}\,\mathbf{x}\,F_1 + \bar{r}_{c2}\,\mathbf{x}\,F_2 + \bar{r}_{c3}\,\mathbf{x}\,F_3 = \Sigma Fe_x + \Sigma Fe_y + \Sigma Fe_z$$
$$= (\sqrt{\Sigma Fe_x})^2 + (\sqrt{\Sigma Fe_y})^2 + (\sqrt{\Sigma Fe_z})^2 = J^2 + E^2 + B^2$$ *where*

$\bar{r}_{c1,2,3}$ = The position vectors from the center of mass C to its Principal orientation of the rigid body and for one material point the orientation vector of the point P .

i.e. The dynamics of any system = Work = Total energy , ***is transferred as generalized force Fe to →***
$Fe = \partial W/\partial(\delta\bar{r}_c)$, $(\delta\bar{r}_c) = \bar{v}i.\delta t = [\,\bar{v}_c + \bar{w}.\bar{r}_n]\,\delta t$ = (Translational + rotational velocity) . δt
$Fe = \bar{v}_c\,.\,(\partial W/\delta t) + \bar{w}.\bar{r}_n].(\,\partial W/\delta t)$ → *Translational kinetic energy+ Rotational kinetic energy*

1..*To Elastic material Configuration* , as ***Strain energy*** and is absorbed as *Support Reactions* and displacement field [$\nabla\boldsymbol{\varepsilon}$ ($\bar{u}$,$\bar{v}$,$\bar{w}$)] upon the deformed placement , (where these alterations of shape by pressure or stress is the equilibrium state of the Configuration $G.\nabla^2.\ \boldsymbol{\varepsilon}\ + [m.G / (m\text{-}2)]\ .\ \nabla\ [\ \nabla.\ \boldsymbol{\varepsilon}\ ]\ =\ F$) ,

2.. *To Solid material Configuration* , as ***Kinetic*** (*Energy of motion* $\bar{v}$) and ***Potential*** (*Stored Energy*) energy by displacement (*the magnitude of a vector from initial to subsequent position)* and rotation , on the principal axis (*through center of mass of the Solid*) *as ellipsoid* , which is mapped out , by the nib of vector ($\delta\bar{r}_c$) = [$\bar{v}_c + \bar{w}.\bar{r}_n$] δt , as the Inertia ellipsoid [Poinsot's ellipsoid construction] in (GF) which instantaneously rotates around vector axis $\bar{w},\varphi$ with the constant polar distance $\bar{w}.Fe/|Fe|$ and the constant angles θs , θb , traced on , Reference (BF) cone and on (GF) cone , which are rolling around the common axis of $\bar{w}$ vector , *without slipping* , and if Fe , is the Diagonal of the Energy Cuboid with dimensions a,b,c *which follow Pythagoras conservation law* , then the three magnitudes (J,E,B) of Energy-state follow Cuboidal , Plane , or Linear Diagonal direction ,

If Potential Energy is zero then vector $\bar{\mathbf{w}}$ is on the surface of the Inertia Ellipsoid .

3.. *To Quaternions Extensive Configuration* , as ***New Quaternions*** (with Scalar and Vector magnitudes). Points in Space carry A priori the work $W = \int_{A\text{-}B}\ [\ \mathbf{P}.ds\ ]\ =\ 0$, where magnitudes P, $d\bar{s}$ can be varied leaving work unaltered (N4). Diffusion (*decomposition*) of Energy follows Pythagoras conservation law where the three magnitudes (J,E,B) of Energy-State follow *Cuboidal* , *Plane* , or *Linear Diagonal* [18].

4.. *To Space conserved Extensive property Configuration (Continuum)* , as ***Kinetic (3-current motion) and Potential (perpendicular Stored curl fields)*** energy by displacement (*the magnitude of a vector from initial to the subsequent position*) and rotation . Energy is conserved in *E and B* fields .

8..5 The Equations of Equilibrium in Classical Mechanics .

A . Lagrange , by applying Inertia Forces (constraints) [$P = m.d\bar{v}/dt$] on stationary Points , developed the following General equations of Equilibrium :

$$\sum_{i=1}^{i=n} [\ P_i + \bar{H}_i\ ].\delta\bar{r}_i = 0 \quad \text{or} \quad \sum_{i=1}^{i=n} [\ P_i - m_i.d\bar{v}_i / dt\ ].\delta\bar{r}_i = 0 \quad \text{and in rectangular Cartesian coordinates} \qquad (1)$$

$$\sum_{i=1}^{i=n} \{\ [\ X_i - m_i.\ d^2x_i / dt^2\ ].\delta x_i + Y_i - m_i.\ d^2y_i / dt^2\ ].\delta y_i + Z_i - m_i.\ d^2z_i / dt^2\ ].\delta\bar{z}_i\ \} = 0 \qquad \text{where :}$$

i = 1 , 2 ……….……. n	:	The material points .
x , y , z	:	The position in Cartesian coordinates (*degrees of freedom*)
m i (m1 , m2 , …mi..mn)	:	The mass on every point $i = 1 \rightarrow n \rightarrow \infty$
P i (Xi ,Yi , Zi)	:	The applied Resultant Forces .
$\delta\bar{r}_i$ (δx_i , δy_i , δz_i)	:	The Virtual displacement (*the possible motion from point A to B*)
$d\bar{v}_i / dt$ = ……………...v	:	The time derivatives of velocities
$d^2\,\delta\bar{r}_i / dt^2$ ……….…	:	The time derivatives of acceleration .
n , j	:	An Integer label corresponding to a generalized coordinate .
Hi …………………….…	:	Inertia Forces (Newton’s second law) equal to $m\ddot{r}$
$\Sigma\ P_i .\delta\bar{r}_i$ = V	:	Potential Energy
$\Sigma\ [\ m_i.d\bar{v}_i / dt\ ].\delta\bar{r}_i$ = T	:	Kinetic Energy , Lagrangian $\rightarrow$ L = T - V

For i = 1 , rewrite equation (1) as [P1 + H1] $.\delta\bar{r} = 0$, or [P + H] . $d\bar{r} = 0 \rightarrow$ **$d\bar{r}$.[P + H] = 0** …..(**2**)

Since (for i = 1) Primary Point is the only Space , then this point to exist in this Space and somewhere else , must move from the Initial Position , say A , to another position , say B .This Equilibrium for points A and B presupposes in Mechanics the Principle of Virtual Displacements ,

$\rightarrow$ *work done* $W = \int P.ds\ =\ 0$, *or when ds = distance AB* , **[ds . ($P_A + P_B$) = 0]** , …(**s**)

i.e. The two equations (2) , (s) are the same and quantities **$d\bar{r} \equiv d\bar{s}$, (P + H) $\equiv$ ($P_A + P_B$) ,** satisfy the two equations when one of them is zero , ***and so*** , equation is holding .

Since forces **P = - H** $= 0 \rightarrow P\ n \rightarrow \infty$, *in [PS] and [PaS]* , are equal and opposite , ***then Resultant force $\bar{A}$ is zero ,*** and according to the three mathematical condition for Field Forces , *issues* ,

a). The Curl of $\bar{A}$ is **$\nabla \times \bar{A} = 0$** **b).** The net work through a closed trajectory **$\bar{A}.d\bar{s}$ = gc$\bar{A}$d$\bar{s}$ = 0**

c). Opposite forces **AP , AH** can be written as the negative gradient of the same potential **$\bar{A}PH = -\nabla U$** so *then* ***the*** **n** th ***Space and Anti-Space is a Conservative Force Field*** **$\bar{A}n$** corresponding to the **n** th ***Potential energy function $\breve{U}n(\tilde{a})$*** or **$\bar{A}n(\tilde{a}) = -\nabla\ \breve{U}n(\tilde{a}) = -\nabla\ [\ \nabla\ \breve{U}n\ ] = \nabla^2\ \breve{U}n$** , where **$\tilde{a}$ = x o , y o , z o ,** *the coordinates of Initial Space* **.**

8..5 Principles and generalized forces in [PNS] .

In Field Theory , the Position vector $d\bar{s} = AB$ of two points A , B in a Standard coordinate System is related to the generalized coordinates by transformation equation $\bar{r}_i = d\bar{s} = d\bar{s}(q_i, \bar{p}_i)$, where

q_i = The **i** number of degrees of freedom (x , y , z)

$\bar{p}_i$ = A set of variables or constant magnitudes (time , forces , momentum , etc)

FA = (X A , Y A , Z A) = Generalized forces **F A** with the components **X A , Y A , Z A** .

FB = (X B , Y B , Z B) = Generalized forces **F B** with the components **X B , Y B , Z B** .

$\bar{\mathbf{r}} = \mathbf{d\bar{s}(\bar{r}_1 , \bar{r}_2 \ldots \bar{r}_n)}$, $\mathbf{r_i (dx_i , dy_i , dz_i)}$ = Incremental distance dx , dy , dz

m = (mass) , A constant which is considered as the hypothetical Reaction to the Motion (Inertia) .

$\mathbf{T = \frac{1}{2}\sum_{i=1}^{i=n} m_i . \bar{r}_i . \bar{r}_i}$ = The Kinetic Energy for **n** points in a system .

8..6 Lagrange´s equation (2 nd kind) .

For any material System with **n** degrees of freedom , the position vector $\bar{\mathbf{r}}$ in a Standard coordinate System is related to the generalized coordinates by *the transformation equation* $\bar{r} = \bar{r}(t, \bar{q}_n)$ and depends on $\bar{\mathbf{q}}_\mathbf{n}$ (xi , yi ,zi ... n = number of degrees of freedom in the system) coordinates at Time **t** and for **n** generalized velocities → $\bar{r} = \bar{r}(t, \bar{q}_n, \dot{q}_n)$

The expression for the Virtual displacement $\boldsymbol{\delta\bar{r}_i}$ of the system for , *velocity depended constraints* , is the same form as a total differential .

$$\delta\bar{r}_i = \sum_{n=1}^{n} [\, \partial\bar{r}_i / \partial\dot{q}_n \,] . \delta\bar{q}_n = \nabla\bar{\mathbf{r}} .© = [\, \nabla.\bar{r} , \nabla x\, \bar{r} \,]$$

where $\bar{\mathbf{q}}_\mathbf{n}$ → **n** Independed generalized coordinates (*are the number of degrees of freedom in the system* or *the spatial coordinates*) and **Qn** the Total Work done by the applied forces **P i** on one of the Virtual displacement $\boldsymbol{\delta\bar{q}_n}$. The Total Kinetic energy **T** for the system of Point particles is defined by ,

$\mathbf{T} = (1/2).\sum_{i=1}^{n} m_i \dot{r}_i^{\,2}$ and $Q_n = (d/dt).(\partial T/\partial\dot{q}_n) - (\partial T/\partial q_n)$ the generalized forces . Action $S = \int_{t1}^{t2} L\, dt.$ where Lagrangian L= T - U and U = The Total Potential energy of the system. and work

$$Q_n = (d/dt).(\partial T/\partial\bar{q}_n) -- (\partial T/\partial q_n) = (d/dt)(\nabla. T) -- \nabla^2 T \qquad \ldots\ldots\ldots (3)$$

i.e. The dynamics of any system = Work = Total energy, is transferred as generalized force Qn .

$Q_n = \partial W/\partial(\delta\bar{q}_n)$, $(\delta\bar{q}_n) = \bar{v}_n.\delta t = [\, \bar{v}_c + \bar{w}.\bar{r}_n] \,\delta t$ = (Translational + rotational velocity).δt

$Q_n = \bar{v}_c . (\partial T/\delta t) + \bar{w}.\bar{r}_n].(\partial T/\delta t)$ → *Translational kinetic energy + Rotational kinetic energy .*

Applying the fundamental equations on two points of stationary [PNS] , $\bar{z}_o = [\lambda , \pm\Lambda\nabla i]$, $\bar{z}'_o = [\lambda^2 - |\Lambda|^2]$, $e_o = [-\lambda\nabla, \nabla x\Lambda] = 0$ then→ $e = \nabla x\Lambda = \nabla©\Lambda =$ $\mathbf{[-div\bar{\Lambda}, curl\, \bar{\Lambda}] = [0 , \pm \Lambda]}$ i.e. ***the points are incorporating the equilibrium vorticity ± Λ either as even or odd functions*** . Since $\bar{z}_o = [\lambda , \pm\Lambda\nabla i]$, then positive $\bar{z}_o$ $\bar{z}_o = [\lambda , \Lambda\nabla i]$ and $\bar{z}'_o = [\lambda , -\Lambda\nabla i]$ is the conjugate quaternion and because $\bar{z}_o$ is a unit quaternion then Action on point is → A = New quaternion $\mathbf{z} = \bar{z}_o© \bar{o} = \bar{z}_o . \bar{o} . \bar{z}'_o = [\lambda , \Lambda\nabla i].[\, 0 , \Lambda\nabla i] .[\lambda, -\Lambda\nabla i] =$ $= [\, -\Lambda^2, \lambda\Lambda + \Lambda x\Lambda \,].[\, \lambda , -\Lambda] =$

$$[-\lambda\Lambda^2 - \Lambda^2\Lambda + \lambda\Lambda^2 + \Lambda^2\Lambda , -\Lambda^2\Lambda + \lambda^2\Lambda - \lambda\Lambda\Lambda + \Lambda(\Lambda\mathbf{x}\Lambda) - \lambda\Lambda\Lambda + \Lambda(\Lambda\mathbf{x}\Lambda)] = \mathbf{[}\, 0, (\lambda^2 - \Lambda^2).\Lambda + 2\Lambda(\Lambda\Lambda) + 2\lambda(\Lambda\mathbf{x}\Lambda)\mathbf{]} \quad \ldots(4)$$

Since $\mathrm{div}\Lambda = 0 = |\Lambda|.\mathrm{div}\bar{\Lambda} + \bar{\Lambda}. \nabla|\Lambda| = |\Lambda|.\mathrm{div}\bar{\Lambda} + \bar{\Lambda}. d|\Lambda|/ds$ then $\bar{\Lambda}.\nabla = d/ds$, which is the arc-length derivative of Λ direction showing that ***on points exists directional vorticity as*** ,

$(\lambda^2-\Lambda^2).\Lambda$ =	Euler	vorticity	↺
$2\,\Lambda(\Lambda\Lambda)$ =	Coriolis	vorticity	↻
$2\lambda(\Lambda\mathbf{x}\Lambda)$ =	Centripedal	vorticity	↻↺

and for unit $\bar{v} \perp \Lambda$ then ,

choosing $\lambda = \pm\cos(\theta/2)$ and $\Lambda = \bar{\mathbf{v}}.\sin(\theta/2)$ then $\mathbf{z} = \mathbf{[}\, 0 , \Lambda.\cos\theta + (\bar{v}x\Lambda).\sin\theta \,\mathbf{]}$ which is the *Euler-Rodrigues* formula for the rotation by an angle θ , of the vector Λ about its unit normal $\bar{\mathbf{v}}$ **.** Comparing (3) with (4) then

Conjugation of $\bar{\mathbf{o}}$ on point **P** is → $[\, 0, \Lambda] © [\, r + \bar{r}.i \,\mathbf{]} \mathbf{=} 0 - \Lambda\bar{r} , 0 + \Lambda r + \Lambda x\bar{r} = \mathbf{- |\bar{\Lambda}|.|\bar{r}| \;,\; r\bar{\Lambda} + \bar{\Lambda}x\bar{r}}$ ….(5)

and for $\Lambda \perp \bar{r}$ then $A = [\, 0 , r. \bar{\Lambda} + \bar{\Lambda}x\bar{r} \,]$ i.e. *A Potentially Rotational kinetic energy (mr.w) as above .*

The normalized quaternion is $= (-|\bar{\Lambda}|.|\bar{r}|, r\bar{\Lambda} + \bar{\Lambda}x\bar{r}) / (\sqrt{\Lambda^2 r^2 + \Lambda^2 r^2}) = [-|\bar{\Lambda}|.|\bar{r}|, r\bar{\Lambda} + \bar{\Lambda}x\bar{r}\,] / (\Lambda r\sqrt{3})$ ….(5a)

Hamiltonian function $H=T+U=2T-L$, Force $F= -\nabla U \equiv \dot{p}= - (\partial H/\partial q)$ and $\bar{u} =(1/2).(\partial mv^2/\partial p)=(\partial H/\partial p)$
q_n , $\dot{q}_n$ = The generalized coordinate (GC) with **n** degrees of freedom and velocities (GV) .
T , U = Kinetic (*expressed in generalized momenta ,mv,*) and Potential energy in terms of (GC) .
Substituting Hamiltonian function into the Lagrangian equation of motion and derivation of a system of the 2n first-order differential equations to be solved , is the solution of the equations of motion .

Lagrange Equations :
Lagrangian function is $L = (1/2).\Sigma_i m_i (\dot{x}_i^2 + \dot{y}_i^2 + \dot{z}_i^2) - U(\bar{\mathbf{r}})$ where $\bar{\mathbf{r}} =(\bar{r}_1 ,\bar{r}_2 ..\bar{r}_n)$, $r_i (x_i ,y_i ,z_i)$.
Force $F_i = m_i . \ddot{\mathbf{r}}_i = (F_{x,i} , F_{y,i} , F_{z,i}) = -\partial U(\bar{\mathbf{r}}) / \partial \bar{\mathbf{r}}_i$. Potential energy is given as

$U(\bar{r}1 ,\bar{r}2 ..\bar{r}n) = \Sigma_i .W1(\bar{\mathbf{r}}_i) + \Sigma_{.j>i} W2(\bar{\mathbf{r}}_i , \bar{\mathbf{r}}_j) + \Sigma_{.k>j} W3(\bar{\mathbf{r}}_i , \bar{\mathbf{r}}_j , \bar{\mathbf{r}}_k)$ *where*

$\Sigma_i .W_1(\bar{\mathbf{r}}_i)$ = The external Forces as fields and constraining fields ,
$\Sigma_{j>i} W_2(\bar{\mathbf{r}}_i , \bar{\mathbf{r}}_j)$ = The dependency of Forces relative to border forces $W_2(\bar{r}_i,\bar{r}_j) = W_2(\bar{r}_i - \bar{r}_j)$
$\Sigma_{k>j} W_3(\bar{\mathbf{r}}_i , \bar{\mathbf{r}}_j , \bar{\mathbf{r}}_k)$ = The dependency of Forces relative to the geometry of forces →
$W_3(\bar{r}_i , \bar{r}_j , \bar{r}_k) = W_3 (\cos\theta_{ijk})$ where $(\cos\theta_{ijk}) = (r_{ji} . r_{jk}) / | r_{ji} |.| r_{jk} |$

Passing from set $(q,\dot{q})$ to equal (q,p) where $p_a =\partial L/\partial \dot{q}_a$=general momentum ,then Lagrangian $L= L(q,\dot{q})$
$L= \Sigma_a \dot{q}_a dp_a - \Sigma_a ,\dot{p}_a dq_a$ and Hamiltonian $H(p,q,t) = \Sigma_a ,p_a\dot{q}_a - L= \Sigma_a ,p_a\dot{q}_a -\Sigma_a ,\dot{q}_a dp_a +\Sigma_a ,\dot{p}_a dq_a$ …(6)

For free material point (*in generalized coordinate* (GC) , $L= (m/2).\dot{q}^2 =(m/2).\dot{\mathbf{r}}^2 =(m/2). (\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$
For Interacting free material point (*in generalized coordinate* (GC) , $L = (m/2).\dot{q}^2 - U(q)$
For conservative systems $L = T(\dot{q}) -U(q)$ where **T** depends on velocities only and **U** on coordinates only.

9.. General Remarks .

9.1. *Properties of Space-Energy Configuration .*

1..All universe is Isotropic and Homogenous because of points (*in spatial and temporal domain*) and work (W) is quantized on points as spin ± (p) and from this equilibrium of the quantized angular momentum , independently of time, is capable of forming the wave nature of Spaces , following the Boolean logic and distorting momentum p =Λ as energy, *on the intrinsic orientation position of points,* on all points of the microscopic and macroscopic homogeneity as ($\partial/\partial t$, ϖ) © ($-\lambda p$,$\nabla \times \Lambda$) = [0 , Λ].
2.. Momentum p = Λ on the infinite dipole AiBi with a momentum lever equal to zero (0) or equal to wavelength λ create ***linear motion*** , while with a momentum lever ≠ 0 creates the ***rotational motion*** (Euler , Coriolis , Centrifugal) → $\mathbf{m} .[(d^2\bar{r}/dt^2)+ \mathbf{m} . [|d\bar{w}/dt|\mathbf{x}\, \bar{r} +2\bar{w}\mathbf{x}(d\bar{r}/dt) +\bar{w}\mathbf{x}(\bar{w}\mathbf{x}\, \bar{r})]$ where momentum p = mw and mass **m** is a constant equal to *, the Reaction to the motion* , or as Inertia (I) which are a natural property of dipole and both are conserved vice versa .
Forces $dP = P_A-P_B$ parallel to the Space , Anti-Space lines [S]=[AS] , create a Static force field **B ,** and when Forces dP are perpendicular to the Space Anti-Space lines , create a Static force field **E ,** which experience *Lorentz force* and it is the fundamental interpretation cause of motion , in small and large scales .
On all dipole of wavelength λ and momentum Λ , their product ***λ . Λ = k*** $_{1,2,3}$ constant for each energy level. The fundamental force in universe is the total kinetic energy $T =1/2wL = \Sigma(L^2/2I)$, a repulsive force following Pythagoras conservation law such that both T and L be conserved (when T decreases then this lost energy is transferred to angular momentum L and vice versa , *in L by changing angular velocity vector* , differently is needed a speed faster than that of light *, which may happen in other scale levels . Energy is conserved on three perpendicular fields J,E,B, on dipole such that the total kinetic energy to be the diagonal of the cuboid.*
3.. The action of a quaternion on point is equivalent as -*energy density and pressure*- the state of stress at a point on the deformed placement or new configuration which is on the directional axis of the point .
Gravity exists upon the point axis as $[|d\bar{w}/dt|\mathbf{x}\,\bar{r} + 2\bar{w}\mathbf{x}(d\bar{r}/dt) + \bar{w}\mathbf{x}(\bar{w}\mathbf{x}\,\bar{r})]$ where angular velocity is $\bar{w} = |\Lambda|/|\bar{r}|= k/(\lambda m)$ and so exerts a direct action between two events, i.e. Stationary points of [PNS] are rotating dipole and may be pictured as wave existing in the infinite points of Spaces and exerting an action (pressure) on the moving Spaces , dipole . The Stability is achieved by the Anti-space.
4.. In Black hole Energy scale (***λ . Λ = k*** $_1$) there are infinite high frequency small amplitude vacuum fluctuations at Planck energy density of 10^{113} J/m3 that exert action (pressure) on the moving Spaces dipole and their Stability is achieved by Anti-space also.

5.. Dipole vectors are quaternions (versors) of waving nature , i.e., one wavelength in circumference in energy levels , that conserve energy by transferring Total kinetic energy T into angular momentum $L= \bar{r} m\bar{v} = \bar{r}p = \bar{r}\Lambda$, where mass m = is a Constant . Different versors with different Energy (scalar) possess the same angular momentum . A Composition of Scalar Fields (s) and Vector Fields ($\bar{v}$) of a frame , to a new unit which maps the alterations of Unit by rotation only and transforms scalar magnitudes (particle properties) to vectors (*wave properties*) and vice-versa , and so , has all particle-like properties of waves and particles .In Planck Scale , when the electron is being accelerated by gravity which exists in all energy levels as above , the gravity is still exerting its force . Matter is built out of the primary dipole AiBi .

6… Dark matter Energy (***λ . Λ = k* 3**) is supposedly a homogeneous form of Energy that produces a force that is opposite of gravitational attraction and is considered a negative pressure ,or antigravity with density $6x10^{-10}$ J/m3 and G = gravitational constant = L^3/MT^3 = $6,7x10^{-11} m^4/ N.sec^4$, Planck force= $Fp = c^4/g = 1,21x10^{44}$ N and dynamic Plank length = $\sqrt{h.G / c^3} = 1,616x10^{-35}$ m , and the reduced wavelength $\bar{\lambda} = \lambda/2\pi = c / w$.

10.. Acknowledgment .

Because , as Engineer , my deep intuition contradicts to some very acceptable conceptions , then instead of the written below , *are altering as* → ***but ...***

Time comes first and nothing changes without time → *Time* ***is not existing*** *and it is a meter of changes in the movable Spaces and this springs from Primary Space creation ,* W = ∫ P.ds = 0 .

Casually is the cause for every effect or observation → *Conservation of* ***the A priori work*** *on Points and on all Dipole between the infinite points in PNS is ,* ***gravity of Spaces*** *, the only effective cause .*

Laws of Physics exist before the entities involved → ***Laws are*** : A=B *The Principle of the Equality* . A ≠ B = *Principle of the Inequality ,* A↔ B = ∞ *Principle of Virtual Displacements* W = ∫ P.ds = 0 , PA+ PB = 0 *Principle of Stability* , A≡B *Principle of infinite Superposition (extrema)* → ***and Entities*** (*Monads* $A\bar{B}$ ***) are embodied with the Laws*** (*A ,B - PA , P B*) because ***Entities = Monad*** $\bar{A}B$ **= *Quaternions*** [AB , $\bar{P}_A - \bar{P}_B$] and all of them built on the Euclidean logic.

Sequence that space was created before matter → *Human mind , in front of this dilemma created the outlet in Religious and Big-Bang.*

11.. References .

[1] MATRIX STRUCTURAL of ANALYSIS by J.L.MEEK library of Congress Catalog 1971
[2] DER ZWECK IM RECHT by RUDOLF V. JHERING 1935.
[3] The great text of J . L .Heiberg (1883-1886) and the English translation by
[4] ELEMENTS BOOK 1 Richard Fitzpatrick .
[5] WIKIPEDIA , org , THE FREE ENCYCLOPEDIA .
[6] GREEK MATHEMATICS by, Sir Thomas L .Heath - Dover Publications, Inc , New York. 63-3571.

ON SCRIBD & VIXRA

[7] A SIMPLIFIED APPROACH OF SQUARING THE CIRCLE . (**MELAN.doc**)
[8] THE PARALLEL POSTULATE IS DEPENDED ON THE OTHER AXIOMS (**EUCLID.doc**)
[9] MEASURING REGULAR POLYGONS AND HEPTAGON IN CIRCLE **(REGULAR.doc**)
[10] THE TRISECTION OF ANY ANGLE . (**TRISECTION.doc**)
[11] THE EUCLIDEAN PHILOSOPHY OF UNIVERSE . (**EUCLID .Spaces.doc**)
[12] UNIVERSE ORIGINATED **not** WITH BIG BANG (**EUCLID .not.BIG BANG .doc**)
[13] COMPLEX NUMBERS QUANTUM MECHANICS,SPRING FROM EUCLIDEAN UNIVERSE. (**Complex.Quantum.Euclid.doc**)
[14] ZENOS PARADOX,NATURE OF POINTS IN QUANTIZED EUCLIDEAN GEOMETRY.[**ZENO-PARADOX.doc]**
[15] THE DECREASING TUNNEL BY PR.FLORENDIN SMARANDASHE (**ORTHIC.doc**)
[16] THE SIX –TRIPLE CONCURENCY LINES - LINE POINTS (Six.TriplePoi.Line.**doc)**
[17] ENERGY LAWS FOLLOW EUCLIDEAN MOULDS (Pythagoras.**doc)**
[18] HIGGS PARTICLE AND EUCLIDEAN GEOMETRY (Higgs-Particle.**doc)**
[19] HIGGS BOSON AND EUCLIDEAN GEOMETRY (Higgs-Boson.**doc)**
[20] GEOMETRY- SPACE UNIVERSE (Space-Universe.**doc)**
[21] QUANTIZATION OF POINTS AND ENERGY (Space-Energy.**doc)**
[22] THE PARALLEL POSTULATE AND SPACES (IN SciEP)
[23] GRAVITY AS VORTICITY IN GEOMETRY (Gravity**.doc)**
[24] THE QUANTIZATION OF POINTS AND POTENTIAL , AND THE UNIFICATION OF SPACE AND ENERGY WITH THE UNIVERSAL PRINCIPLE OF VIRTUAL WORK , ON ***Geometry Primary dipole*** , ( )

by Marcos Georgallides .