Couplings in the deep infrared limit from M-theory - does one numerical formula deserve the benefit of the doubt?

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Abstract

In this note, we preliminarily discuss the possibility that the expression $\alpha_{em}^{-1} = 4\pi^3 + \pi^2 + \pi$ has a physical interpretation and can even be helpful in model building. If one interprets this expression in terms of the volumes of l_p - sized three-cycles on G_2 holonomy manifolds and requires that it also comprises effects of the running of the coupling, one can obtain the desired value, but only in a setup which is clearly different from the standard model of particle physics (SM). An understanding of the nature of the link between such putative model and SM is needed. Studying this issue could possibly shed some light on existing problems in model building within string theory (ST), particularly the hierarchy problem. Numerological "success", which can be achieved if one interprets the formula in terms of volumes of three-cycles on the compactification manifold, as we intend to do here, cannot change the fact that discussion in this note represents merely a heuristic estimate of the feasibility of further research in a certain direction.

Discussions regarding the plethora of simple numerical expressions for SM parameters periodically reappear both in printed publications and in blogs, even (or rather especially) in the "classical" case of α_{em}^{-1}. The reasons why such attempts are commonly considered to be meaningless and such claims preposterous are well known. A formula extracted directly from compactification geometry could be valid only on very high energies and Renormalization Group (RG) flow to the infrared and symmetry breaking effects should make it unrecognizable in the low energy limit. Therefore, for the sake of argument, it seems appropriate to outline at once why we have chosen to take one of the existing numerical observations as granted.

There exists a certain numerical expression which gives a value rather close to α_{em}^{-1} in the IR limit, which has the form of the sum of powers of π, namely $\alpha_{em}^{-1} = 4\pi^3 + \pi^2 + \pi$. This, in turn, suggests the familiar $\frac{1}{e^2} = \sum_i \frac{1}{g_i^2}$ structure. However, if one wants to have even a remotely realistic toy model, one or more (or possibly all) terms should stem from the running of the coupling(s). If this is the case, it is clear that in one-loop expression upper and lower integration limits should be related by an exponential suppression factor of the $\sim \exp(-m\pi^n)$ form , resembling an effect of suppression by instanton $\sim \exp(-\frac{1}{g^2})$. Therefore, this picture cannot be a part of SM for at least two reasons. First, in such a setup one has the $\sim m_e$ scale as an exponentially suppressed ultraviolet (UV) cutoff scale, and Higgs condensate is either absent from the picture or somehow identified with UV cutoff. Secondly, as there are only three terms, particles other than electrons and positrons are not present in the loops (or all of them are degenerated in mass). As the numerical value is clearly from SM, and should be significantly influenced by polarization loops containing particles other than electrons, everything looks like a strange coincidence. Is it possible to turn these problems to possible hints concerning the ST model building? If one intends to proceed, some additional conjectures are inevitable.

First, some rationale needes to be provided as to why a crucial SM parameter pops up in one clearly non-SM context. Our most importyant assumptions are the following: SM corresponds to some configuration of branes and other ST objects which either evolved from another configuration - one in which the displayed expression appears natural - or the values of the parameters in the IR limit were somehow forced upon the SM configuration, presumably by some interaction with hidden sector. However, keeping in mind the dependence of the fine structure constant value of the particle spectrum of SM, this would inevitably also mean that the particle content of the SM has been forced upon the system of branes where the gauge fields of SM reside. It would be illuminating to have an understanding of this at least on the toy-model level.

Another important idea is that the very presence of suppression by some non-perturbative instanton effect points towards the compactification upon some G_2 holonomy manifold, where hierarchies introduced in this way are generic (see e.g. the recent review [1] and references therein). Of course, in these developments a hierarchy exists between the Planck and gravitino scales, and, strictly speaking, a proper link between the observation we intend to display and compactification upon G_2 manifolds has yet to be determined.

Any non-oversimplified calculation of the running of coupling will also take into account the contributions of polarization loops besides the one with the electron and positron; in models beyond SM, there can be even more species of charged particles. Consequently, there is no reason to expect some simple formula here. At the same time, within the framework of ST or M-theory model building, a purely geometrical formula appears to be natural only if connected with the 11-dimensional Planck scale l_p. It can be argued that what we actually need are formulas for g and g' in the familiar SM expression $e = \frac{g'g}{\sqrt{g'^2+g^2}}$ and not

$\alpha = \frac{e^2}{4\pi}$ itself. In addition, if one is working with l_p-scale, then, *a priori,* masses are expected to be of the Stueckelberg type, and not due to the electroweak Higgs effect.

We shall take as granted the expression $4\pi^3 + \pi^2 + \pi = 137,036...$ (which was probably proposed for the first time in the early 1970s). Further, when, for example, β-function of Quantum Electrodynamics or specific G_2 holonomy manifolds and so on are used, they play role of illustrative tools at hand and should not be understood as claims. Here, admittedly, the numerical coincidence substitutes the phenomenological input usually used in preliminary dimensional and numerical estimates.

One of the possibilities is that values of the couplings were somehow forced upon the system of branes that actually hosts SM fields, for example, by the SUSY breaking mechanism. Another possibility would be the existence of a "protostandard model" (ΠSM in further text), which naturally comprises the described expression and belongs to the same universality class as SM. Then, ΠSM would eventually evolve into SM (an idea somehow in spirit of brane realizations of duality cascades). In the latter case an obvious objection can be immediately raised. Formula is very simple, and if it describes some running, it is within some very simple model, with few (likely only one) particle pairs in the polarization loops. At the same time, it has a rather precise value. More charged matter in the abelian case means stronger polarization. Therefore, the concept of some class of models with the same abelian coupling in the IR limit, with the same coupling on unification/Planck scale, seems wrong and requires further modification. An attempt shall be made to address this issue.

Needed numerical values can be obtained if one substitutes l_p-sized length parameters (e.g. defined as inverse tension of the fundamental M-theory object, namely M2 brane) in the expression for gauge coupling for $D6$ branes wrapped upon three-cycles on G_2 holonomy manifold and in exponential suppression factors (of the instantonic type), as well depending upon volumes of the such cycles. Obviously, if presented observation is taken as granted, the problem of gauge coupling values essentially becomes a part of the hierarchy problem, as one starts from a model where the scale of the lightest charged particle equals the nonperturbatively suppressed UV cutoff scale.

A straightforward estimate of coupling using compactification of D6 branes and E2 instantons upon l_p-sized three-cycles on G_2 holonomy manifold rather naturally yields the desired value. Finally we shall make some guesses concerning the question how this can be reconciled with the actual content of SM. Eventually one will need both a constructive realization of ΠSM and understanding of its further evolution.

1 One-loop polarization versus the displayed formula

The mentioned formula renders a value only slightly outside the accepted interval of measurement results for the inverse of the electromagnetic fine structure constant:

$$\alpha_{em}^{-1} = 4\pi^3 + \pi^2 + \pi = 137,036... \tag{1}$$

One can observe that using the volume elements on the corresponding homology cycles on the manifold $S^1 \times S^3$, i.e. generators of de Rham cohomology of $U(2) \backsim SU(2) \times U(1)/Z_2$, $\omega_1, \omega_2, \omega_3 = \omega_1 \wedge \omega_2$ (not normalized to integer) it is possible to write[1]:

$$"137,036..." = \int_{S^1 \times S^3} \frac{1}{2}(2\omega_3 + \omega_1 + \omega_2) \tag{2}$$

[1] To our best knowledge expression (2) has never before been published in this form.

This may be interesting on its own, but will not be used in this note[2].

Such expressions seem to contradict the common knowledge about the running of α_{em}. One usually overlooks that, for example, formula (1) can be reconciled with the running of the coupling as such, but not with the running of the coupling in SM.

Let us suppose that one tries to match some expression like (1) with the 1-loop formula

$$\frac{1}{e_R^2} - \frac{1}{e^2} = \frac{N_f}{6\pi^2} \ln \frac{\Lambda}{Am_e} \tag{3}$$

where Λ stands for some high-energy cutoff scale and the denominator is connected to the threshold for creating the pairs of charged particles. It will be useful to express (3) as:

$$\frac{1}{e_R^2} - \frac{1}{e^2} = \frac{N_f}{6\pi^2} \ln \frac{\Lambda}{\frac{<v>}{\sqrt{2}} Y_e} \tag{4}$$

where $< v >$ is the electroweak Higgs condensate and Y_e stands for the Yukawa-coupling responsible for electron mass.

It seems that all we need to get one or two terms in (2) is to have scales in the logarithmic term, all way down from UV cut-off to the lightest charged pair threshold as resulting from exponential suppression factors of obvious form and only one charged particle in the loop. But a simple picture like that cannot be correct, keeping in mind that the running α_{em} is an effective coupling, corresponding to all charged particles in the loop, which have rather large contributions (otherwise, e.g. α_{em}^{-1} at m_{Z_0} would be $\approx 134,6$ and not $\approx 128,8$).

We may wonder if there could be some mechanism that would force the values of couplings in the deep IR region upon some class of models. (This would also mean that we live in a "predictive neighborhood" of ST landscape [3], where a considerable part of it has the same dimensionless couplings.)

Some recent developments may suggest that an idea like this is not entirely unthinkable. In ST model building, there are rather common situations where IR properties are encoded in geometry outside the brane stack in which SM fields reside. This can be done as sequestering in the Randall-Sudrum spirit, using the hidden sector, within the context of mediation of SUSY-breaking and so on. One should also mention here the work subsequent to the discovery of Seiberg duality [4] , where the use of quiver gauge theories in an ST context (which began with [5]) led to duality cascades in warped throats [6]. One can say that IR properties of the theory in these models are due to interesting interplay of different objects from ST. Quiver Gauge Theories also exhibit interesting periodic and quasi-periodic behavior of RG flow of gauge couplings, what was demonstrated already in the seminal paper [6].

Within the framework of ST model building, we have reason to believe that some of the Yukawa couplings in SM, those which are necessary from the viewpoint of phenomenology but prohibited by global symmetries , are due to instanton contributions[3], (see e.g. [7]). However, Y_e in (4) does not correspond to such coupling in these models. In addition, it describes the suppression of $< v >$ and not suppression of the UV cutoff Λ.

Nevertheless, (if we do not bother much with justification) the terms in (1) can be obtained if one straightforwardly uses formulae for D6 branes wrapping the l_p- sized three-cycles on manifolds with G_2 holonomy. One can hope that this might motivate an attempt to obtain IISM within the framework of

[2] Keeping in mind the role of the $S^1 \times S^3$ manifold in flux quantization in M-theory (starting from [2]), one can be tempted to speculate along this line.

[3] specifically the ones responsible for the Majorana neutrino masses

ST model building and further study it. However, one also need some additional conjectures, not least concerning the forcing of the coupling value upon the system that evolved from ΠSM.

2 G_2 manifolds, l_p- scale compactification and values of the coupling

In exploring the stabilized vacuum expectation values (*vevs*) of moduli fields, considerable successes have been achieved using compactifications with fluxes [8, 9, 10] or G_2 fluxless models [11]. Admittedly, realistic predictions are still lacking. As natural candidates for compactification manifolds we take singular G_2 holonomy manifolds (see e.g. [12] and references therein). For G_2 manifolds there are no-go theorems forbidding background fluxes (an issue explored starting from [13]). By background flux we refer to a generalized magnetic field in extra dimensions. But it is possible to have M5/NS5 branes as sources of the magnetic field, where they can play an important role in the stabilization of compactified volumes.

We are interested in Supersymmetric Yang-Mills (SYM) theory in four dimensions. This theory can be obtained from the SYM in seven dimensions, where three of them are compactified upon the three-manifold Q, the latter being the submanifold of a G_2 holonomy manifold. Witten and Friedmann worked out normalization factors in [14] and obtained for $\alpha = \frac{g_{4d}^2}{4\pi}$:

$$\alpha_{GUT} = \frac{(4\pi)^{\frac{1}{3}}\kappa_{11}^{\frac{2}{3}}}{V_Q} = \frac{(2\pi)^3 l_p^3}{V_Q} \tag{5}$$

The following unit conventions were used for tension of the $M2$ brane, 11-dimensional Planck length and 11-dimensional Newton constant:

$$T_2 = \frac{1}{4\pi^2 l_p^3} \qquad l_p = g_s^{\frac{1}{3}} l_s \qquad \kappa_{11}^2 = \frac{1}{2}(2\pi)^8 l_p^9$$

V_Q is the volume of the three-submanifold of the G_2 holonomy manifold, g_s string coupling and l_s string length. As in the example most elaborated on in [14], Q can be a lens space.

We want to estimate gauge couplings for the D6 branes wrapped on, possibly additionally modded, round Planck-scale spheres S^3. It is possible to motivate slightly different choices for values of S^3 radii, based only upon fundamental properties of M-theoretical objects (or better to say probes in M-theory). Using the notorious ambiguity in definition of UV cutoff, it is actually possible to obtain any of the three terms in (1), or all three of them if one is prepared to indulge in further speculations concerning the specific physical meanings of such choices.

In the most straightforward fashion, a characteristic length scale can be obtained from the tension of the M2 brane and substituted into the formula (5) for the gauge coupling. For the radius $\sim (\frac{1}{T_2})^{\frac{1}{3}}$ and $Q \sim S^3$ the result is

$$\alpha = \frac{1}{\pi}. \tag{6}$$

Keeping in mind that S^3 is expected to be modded and that we can have more than one brane in the corresponding stack, expressions for α in our estimates can also accommodate rational factors.

We shall use volumes of S^3 spheres in a slightly different context, as volumes wrapped by instantons. Again, we need to guess what "l_p-size" actually means. One natural cut-off comes from the resolvability by D0 branes, so it is natural to expect the diameter of S^3 not to be smaller than l_p. Another possibility appears if one recalls that M5/NS5 sources of the flux are expected to be generically present in a model.

The 11-dimensional supergravity metric describing N M5 branes can be written as:

$$ds^2 = f^{-1/3}(-dt^2 + \sum_{i=1}^{5} dx_i^2) + f^{2/3}(dr^2 + r^2 d\Omega_4^2) \tag{7}$$

$$f = 1 + \frac{\pi N l_p^3}{r^3}$$

and there is a four-form flux of N units on the S^4. The type IIA NS5 brane can be considered as M5 brane localized on the 11-dimensional circle, and the four-form flux is on $S^1 \times S^3$.

In the near-horizon limit, the geometry is $AdS^7 \times S^4$, with

$$R_{S^4} = (N\pi g_s)^{\frac{1}{3}} l_s \tag{8}$$

Here we shall not study embeddings of M5 brane in the G_2 holonomy manifold, but in one of the further estimates simply take the radius of curvature $\sim (N\pi g_s)^{\frac{1}{3}} l_s$ as characteristic size.

If we have an explanation for $\frac{1}{e^2}$ on some specific energy scale, the question is how the r.h.s. of (3) can be expressed by terms from (1). In our introductory considerations we were motivated by developments described, for example, in [7], but we cannot apply them in straightforward fashion. Specifically, in the model building, Yukawa couplings entirely given by nonperturbative contributions were used in cases when symmetries of the models does not allow for perturbative contributions (essentially because of Stueckelberg-type mass terms), but where Yukawa couplings must exist for phenomenological reasons.

We can expect that nonperturbative instantonic suppression factors in Yukawa couplings either have in exponent $\sim -\frac{8\pi^2}{g_{YM}^2}$ for gauge instantons, where g_{YM}^2 represents the gauge coupling of the $U(N)$ theory on the matter brane or instanton supression factor e.g. for the E2-instanton

$$\sim \exp -\frac{2\pi}{l_p^3} Vol\, E_2 \tag{9}$$

or, in some developments we have in exponent

$$S_\epsilon^{(0)} = \frac{8\pi^2}{g_a^2} \frac{Vol\, E_2}{Vol\, D6_a} \tag{10}$$

(e.g. expression (100) in [7]) where $Vol\, E_2$ and $Vol\, D6_a$ are respectively volumes of the three-cycles wrapped in the internal space by $E2$-instantons and $D6_a$-branes respectively, and g_a is the gauge coupling of the gauge theory on the corresponding $D6$ brane stack (not the same as the gauge coupling for the matter brane). Such ratios can introduce hierarchies.

In the present preliminary estimate we suppose that exponential suppression is given purely by factors of the (9) type (needless to say, we are referring to ΠSM here and not SM itself) and, in our preliminary considerations, use the r.h.s. of (3). In addition, the prefactor in (9) is set to 1. We suppose that the three-cycle is S^3 modded by Z_K.

Then, for $r_{S^3} = \frac{l_p}{2}$ one finds

$$\alpha_R^{-1} - \alpha_\Lambda^{-1} = \frac{4N_f\pi}{6\pi^2} \ln \frac{\Lambda}{\Lambda \exp -\frac{2\pi}{l_p^3} Vol\, E_2} = \frac{N_F}{3K}\pi^2 \tag{11}$$

or for $r_{S^3} = (N\pi g_s)^{\frac{1}{3}} l_s$,

$$\alpha_R^{-1} - \alpha_\Lambda^{-1} = \frac{4N_f\pi}{6\pi^2} \ln \frac{\Lambda}{\Lambda \exp -\frac{2\pi}{l_p^3} Vol\, E_2} = \frac{2N_F N}{3K} 4\pi^3 \tag{12}$$

If (11) and (12) pertain to different brane stacks, one can see that it is possible to obtain (1) by adjusting the number of brane stacks, and consequently, the number of $U(1)$ factors. However, $\frac{3K}{N_F}$and $\frac{3K}{2N_F N}$ need to be integers. As already stressed, the straightforward use of QED formulas in expressions above serves only illustrative purposes.

3 Generation of hierarchies

If the idea that some terms in (1) are due to exponential suppression of the UV cutoff scale, there is a problem how to recognize the electroweak or any other “phenomenological” scale in the model, as they simply do not exist in such aΠSM setup. Of course, one can postulate two consecutive suppressions by instantons of different sizes. However, again, in such a situation one has the “right answer in an obviously wrong setup”. Admittedly, value of $(4\pi^3 + \pi)^{-1}$ is not far from that which is expected on the electroweak scale. At the same time, as already stressed, α represents bookkeeping device for a rather complicated interaction.

The presented observation turns out to be about trading the problem of coupling values for both an aggravation of the hierarchy problem and an unacceptable particle spectrum. Nevertheless, we shall present a simple argument on the toy model level. Of course, placing this on firmer ground would require a better understanding of the interplay between Higgs and Stueckelberg mechanisms of mass generation and better understanding of SUSY breaking.

Let us consider one of the proposed models for SUSY-breaking mediation (see e.g. [15]). There is the possibility that stacks of branes could have common $U(1)$ fields, regardless of the distance between brane stacks, due to interactions with form-fields in the bulk (if they wrap the cycles in the same homology class on compactification manifold). In this way, $U(1)$ bosons from corresponding brane stacks have one massive and one massless combination. Expression for coupling constant of the massless orthogonal combination is as follows:

$$g' = \frac{g_H g_V}{\sqrt{g_H^2 + g_V^2}} \qquad\qquad A' = \frac{1}{\sqrt{g_H^2 + g_V^2}} (g_H A_V - g_V A_H) \tag{13}$$

with indices H and V referring to hidden and visible and A' being massless bosonic field. The expression is reminiscent of that from electroweak theory, but the massive combination has Stueckelberg-type mass, rather than mass due to the Higgs effect. Obviously, the value of the fine structure constant is shifted due to coupling of A_H bosons.

However, now we are dealing with at least two brane stacks. In this preliminary estimate, for the sake of argument, one can suppose that on one, say hidden stack, there exists UV cutoff scale $\widetilde{\Lambda}$, established by some mechanism, whatever it may be, and IR scale. The latter is UV scale suppressed by exponential factor described by (9), with the $E2$ instanton volume being of l_p size. At UV - limit of the QED-type theory on the hidden stack we expect very strong coupling, therefore, by virtue of (13), $g'(\widetilde{\Lambda}) \to g_V(\widetilde{\Lambda})$.

Then, for $r_{S^3}=\frac{l_p}{2}$ one finds

$$\alpha^{-1}_{Rhidden} - \alpha^{-1}_{\widetilde{\Lambda}} = \frac{4N_f\pi}{6\pi^2}\ln\frac{\widetilde{\Lambda}}{\widetilde{\Lambda}\exp - \frac{2\pi}{l_p^3}Vol\,E_2} = \frac{N_F}{3Z}\pi^2 \tag{14}$$

where Z is due to the possible modding of S^3. Therefore, in the limit $\alpha^{-1}_{\widetilde{\Lambda}} \to 0$ it is not difficult to obtain the needed term in expression (1). Due to mixing (13) there is a shift in the value of fine structure constant. As one naturally expect the running described by (14) to correspond to rather high energies, if there are no lighter charged particles in the loops, the running stops, and on lower energy scales the shift due to g_H is constant. For the fields residing on the hidden stack, we have assumed very strong coupling on high energies (may be even a kind of "Landau pole"). Now, due to mixing, on the visible stack we have a flow from the UV point, where we have, in the described limit, $g'(\widetilde{\Lambda}) \to g_V(\widetilde{\Lambda})$, to a value in the IR which is very close to the experimentally known fine structure constant.

Without additional shifting due to another $U(1)$ field, the running of the charge caused by the polarization of the vacuum is described by an expression like (3). After the mixing, there is a new coupling value in the IR limit. If the UV value remains unaffected (obviously, this condition can be weaker, it is enough for the running of the charge on the additional stack to be different), and if one does not permit a change of $\frac{\Lambda}{m_e}$, one needs additional terms (loops) on the r.h.s. of (3), that is, additional charged matter (that is not degenerate in mass).

Along these lines, one can obtain a toy-level rationale as to why the brane system stabilizes in a system resulting in SM. Number of flavors, Yukawa couplings and scalar condensate are stabilized on values which make it possible to have the RG flow consistent with the described mixing with another ("hidden") sector. Of course, the real challenge is to construct ΠSM and study its evolution.

4 Conclusion

A physical justification of the observed expression seems to require better understanding of the hierarchy of Stueckelberg and Higgs energy scales. This could also pave the way for trading the problem of the IR limit of gauge coupling for the problem of hierarchy of electroweak and UV cutoff scales. If conjecture about the evolution within same universality class is correct, our understanding of the stabilization of the moduli will change considerably.

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