Metrical Model of a Star with Thermal Profile

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Abstract

We consider the extension of the Schwarzschild metric to a counterpart that can describe an extended spherically symmetric stellar body with acceptable density distribution, and with a thermal profile. With a length parameter, apart from the Schwarzschild radius, the proposed metric can fit the values of density, pressure, and temperature, at the stellar surface, and give a complete profile down to the core. Such a metric extension seems to describe a central core with an "energy-producing, explosive core", as well as an inflationary "coronal windy layer", two regions where the mean pressure seems to acquire negative magnitudes. Our illustrative computations and graphical illustrations refer to the sun as a reference example. We discuss the Schwarzschild limit of such metrical model. We also discuss the interior gravitational potential and its repulsive central core.

1 Introduction

The spherical potential of Newtonian gravitation is usually associated, in Einstein's general relativistic theory, with the spherical metric of the line element:[1]

$$ds^2 = \left(1 - \frac{s}{r}\right) c^2 dt^2 - \left(1 - \frac{s}{r}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

Here $s = 2GM/c^2$ is the gravitational, or Schwarzschild, radius of a compact spherical source of mass M. This metric is merely a solution of Einstein equations, in spherical coordinates (r, θ, ϕ), and in the absence of energy-momentum sources. The above form was presented by Schwarzschild in terms of a radial variable $R = (r^3 + s^3)^{1/3}$, emphasizing the fact that the metric is really well-defined only in the region exterior to the gravitational radius $r > s$ of the source; our $r = s$ corresponds to Schwarzschild's $r = 0$. We are told[2] that the above form was presented by Hilbert, and that the distance s should be properly termed the Hilbert radius.

Einstein's equations in terms of the Ricci tensor ${R_\mu}^\nu$, the Ricci scalar R, and the energy-momentum tensor ${T_\mu}^\nu$ are given by:

$${R_\mu}^\nu - \frac{1}{2}{\delta_\mu}^\nu R = -\frac{8\pi G}{c^4}{T_\mu}^\nu \tag{2}$$

The energy density of a gravitational source is given by the time component ${T_0}^0$ of the energy-momentum tensor, while other components of the latter give pressure, momentum, and stress quantities associated with the material source.

It is of great interest to examine the sort of spherical metrics that extend the vacuum solution for all values of radial distance, without having singularities in the metric, which would describe some material distribution with a finite total mass, and which would give the Newtonian potential at large radial distance. For that purpose, we first consider a spherical line element in the form:

$$ds^2 = A(r)c^2dt^2 - A^{-1}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{3}$$

We shall examine whether there are acceptable non-singular forms of $A(r)$ that would lead to physically acceptable components of the energy-momentum-stress tensor. To compute the components of the latter we first obtain the non-vanishing components of the Ricci tensor:

$$\begin{cases} R_{00} = -\frac{1}{2r}A(2A' + rA'') \\ R_{11} = \frac{1}{2r}\frac{2A'+rA''}{A} \\ R_{22} = -1 + A + rA' \\ R_{33} = R_{22}\sin^2\theta = (-1 + A + rA')\sin^2\theta \end{cases} \tag{4}$$

Here, the prime denotes differentiation with respect to the radial coordinate r. For the Ricci scalar, we have

$$R = g^{\mu\nu}R_{\mu\nu} = -\frac{1}{r^2}(-2 + 2A + 4rA' + r^2A'') \tag{5}$$

The energy density $\rho(r)c^2$ is given by the ${T_0}^0$ component, and we get the mass density

$$\rho = -\frac{c^2}{8\pi G}\frac{1}{r^2}(-1 + A + rA') \tag{6}$$

We have three pressure parts P_1, P_2 and P_3. These are given respectively by ${T_1}^1$, ${T_2}^2$ and ${T_3}^3$. We obtain

$$P_1 = \frac{c^4}{8\pi G}\frac{1}{r^2}(-1 + A + rA') \tag{7}$$

$$P_3 = P_2 = \frac{c^4}{16\pi G}(2A' + rA'') \tag{8}$$

We define the pressure by the average value $P = (P_1 + P_2 + P_3)/3$, and obtain

$$P = \frac{c^4}{24\pi G}\frac{1}{r^2}(-1 + A + 3rA' + r^2A'') \tag{9}$$

The dimensionless ratio $\omega = P/\rho c^2$, we call the *thermal parameter*, is given by

$$\omega = -\frac{1}{3}\frac{(-1 + A + 3rA' + r^2A'')}{(-1 + A + rA')} \tag{10}$$

The value of ω is well-known for certain systems. For example, we have $\omega = 1/3$ for electromagnetic radiation. For a massive spherical object, like a star, we cannot say much about the meaning of this ratio in the deep core. However, near the surface of the star where the density and pressure are low, we can apply the ideal gas equation and write $\omega = kT/mc^2$. Here k is the Boltzmann constant, T is the absolute temperature, m is the average mass of the atoms constituting the gas (we can assume it is mainly hydrogen), and c is the speed of light.

In the following paragraphs, we shall consider some form for $A(r)$ which would extend the Schwarzschild metric and give acceptable expressions for the energy density. However, we shall see that the corresponding expression for ω cannot be acceptable as a physical description of a physical object like a star, since the ideal gas picture for large radial distances would not emerge. Hence we shall return, in the following section, with different formal extensions of the Schwarzschild metric that would be more acceptable.

Let us first consider a function of the form

$$A(r) = 1 - \frac{2G}{c^2}\frac{M(r)}{r} \tag{11}$$

This replaces the constant mass parameter in the Schwarzschild metric by a spherical mass distribution $M(r)$. From the expression for the mass density $\rho(r)$ we obtain

$$\rho = \frac{1}{4\pi r^2}M'(r) \tag{12}$$

Hence $M(r)$ describes the total mass contained within the radial distance r. The value of $M(r)$ at the surface of a star would give the total mass. In terms of $M(r)$, the thermal parameter is given by

$$\omega = \frac{P}{\rho c^2} = -\frac{1}{3}\left(1 + \frac{rM''}{M'}\right) \tag{13}$$

We shall consider the following forms for $M(r)$, which goes to zero as $r \to 0$, and give the total mass M when $r \to \infty$, and involve a length scale a:

$$M(r) = M\frac{r^n}{a^n + r^n} \tag{14}$$

where n is a positive integer. The corresponding expression for mass density is

$$\rho = \frac{na^n r^{-3+n}}{8\pi G(a^n + r^n)^2} \tag{15}$$

Notice that this expression is non-singular as $r \to 0$, only for $n \geq 3$, and would be positive for all r. However the thermal parameter $\omega = P/\rho c^2$ is given by

$$\omega = \frac{n}{3}\frac{(-a^n + r^n)}{(a^n + r^n)} \tag{16}$$

This shows that the pressure is negative for $r < a$ and positive for $r > a$. This can be interpreted as representing a spherical object with an 'energy-producing or explosive' core, and normal compressed outer layers, as for a normal star. However, as $r \to \infty$

the value of ω does not go to zero as the temperature of the outer layers of a gaseous star would do, but $\omega \to n/3$. This cannot describe the thermodynamic profile of a star.

We conclude that the metric which extends the Schwarzschild form by replacing the quantity $(1 - s/r)$ by a convenient function $A(r)$, which reduces to $(1 - s/r)$ for large distances, *cannot describe the thermal profile of a star*, even if it leads to an acceptable energy density. In the following section, we shall consider a different approach which introduces two functions $A(r)$ and $B(r)$ associated with the time and radial parts of the metric respectively. These two functions will be related by requiring that we must have an acceptable thermal profile for the outer gaseous layers of a star.

2 Spherical Metric with Thermal Profile

We now consider a metric whose line element takes the form

$$ds^2 = A(r)c^2dt^2 - B^{-1}(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

Here $A(r)$ and $B(r)$ are two functions each of which should reduce to the Schwarzschild form $(1 - s/r)$, with $s = 2GM/c^2$, for large radial distance $r \gg s$. Correspondingly, we compute the components of the Ricci tensor $R_\mu{}^\nu$, the Ricci scalar $R = g^{\mu\nu}R_{\mu\nu}$, then derive the non-vanishing components of the energy-momentum-stress tensor $T_\mu{}^\nu$ using Einstein's equations. The latter include the energy density ρ, and the three pressure parts P_1, P_2, P_3 whose average is the pressure P. For the mass density, we obtain

$$\rho = -\frac{c^4}{8\pi G}\frac{1}{r^2}(-1 + B + rB') \tag{17}$$

where the prime denotes differentiation with respect to the radial distance r. For three components of pressure, we obtain

$$P_1 = \frac{c^4}{8\pi G}\frac{1}{r^2}\frac{A(-1 + B) + rA'B}{A} \tag{18}$$

$$P_3 = P_2 = \frac{c^4}{32\pi G}\frac{1}{r}\frac{(-rB(A')^2 + 2A^2B' + A(rA'B' + 2B(A' + rA'')))}{A^2} \tag{19}$$

For the mean pressure, we have

$$P = \frac{c^4}{48\pi G}\frac{1}{r^2}\frac{(-r^2B(A')^2 + 2A^2(-1 + B + rB') + rA(rA'B' + 2B(2A' + rA'')))}{A^2} \tag{20}$$

The thermal parameter $\omega = P/\rho c^2$ is given by

$$\omega = -\frac{1}{6}\frac{(-r^2B(A')^2 + 2A^2(-1 + B + rB') + rA(rA'B' + 2B(2A' + rA'')))}{A^2(-1 + B + rB')} \tag{21}$$

In the following section, we shall consider an example which correspond to a model, and where the functions $A(r)$ and $B(r)$ are specified in ways that can produce acceptable energy density and thermal profile for stellar-like spherical body. We shall find that our viable example would require a single additional parameter. With reference to the sun, for example, we can relate the additional parameter to the solar surface temperature.

3 Stellar Model with Simple Mass Distribution (SMD)

Let us consider a model with a metric specified by the following functions:

$$\begin{cases} A(r) = 1 - \frac{s}{r}\left(\frac{r^3}{a^3+r^3}\right) \\ B(r) = 1 - \frac{s}{r}\left(\frac{r^3}{b^3+r^3}\right) \end{cases} \tag{22}$$

It is clear that both functions would reduce to the Schwarzschild form for $r \gg a$ and $r \gg b$ respectively. Substituting in the expressions for energy density and pressure components, we find the following expressions for large values of r,

$$\begin{cases} \frac{P_1}{\rho c^2} = \frac{1}{3}\left(1 - \frac{4a^3}{b^3}\right) + \frac{1}{3}\left(1 - \frac{a^3}{b^3}\right)\frac{s}{r} + \dots \\ \frac{P_3}{\rho c^2} = \frac{P_2}{\rho c^2} = -\frac{2}{3}\left(1 - \frac{4a^3}{b^3}\right) - \frac{1}{2}\left(1 - \frac{a^3}{b^3}\right)\frac{s}{r} + \dots \end{cases} \tag{23}$$

Here the dots represent higher-order terms in the power series for (s/r). In order to have an acceptable thermal profile for large r, we must set $b^3 = 4a^3$, so that the initial terms in the above series would not appear. Therefore our metric fucntions must take the form

$$\begin{cases} A(r) = 1 - \frac{s}{r}\left(\frac{r^3}{a^3+r^3}\right) \\ B(r) = 1 - \frac{s}{r}\left(\frac{r^3}{4a^3+r^3}\right) \end{cases} \tag{24}$$

With this, we have specified our stellar metric with just one new length parameter a, besides the Schwarzschild radius $s = 2GM/c^2$. Having specified the metric, we proceed to determine the parameter a with reference to the sun as an example.

The thermal parameter $\omega = P/\rho c^2$ can now be expressed using the above, and we obtain

$$\omega = \frac{\begin{pmatrix} -72a^{12} + 80a^9r^2s + 6a^6r^4(18r^2 - 7rs - 4s^2) + \\ 3a^3r^7(12r^2 - 15rs + 2s^2) + r^{10}s(-4r + 3s) \end{pmatrix}}{24(a^3 + r^3)^2(a^3 + r^2(r - s))^2} \tag{25}$$

But this can be equated to $\omega = kT/mc^2$ near the stellar surface. Putting $r = 6.96 \times 10^8$ m to be the solar radius, $M = 1.98892 \times 10^{30}$ kg the solar mass, $c = 2.99792458 \times 10^8$ m/sec the speed of light, $G = 6.67259 \times 10^{-11}$ the Newtonian constant, $m = 1.67264 \times 10^{-27}$ kg the mass of a hydrogen atom, $T = 5800$ K the solar surface temperature, and $k = 1.38066 \times 10^{-23}$ the Boltzmann constant, we find the solution

$$a = 5.41836 \times 10^6 \, \text{m} \tag{26}$$

The stellar density as a function of radial distance is now given by

$$\rho = \frac{3a^2c^2s}{2\pi G(4a^3 + r^3)^2} = \frac{3a^3M}{\pi(4a^3 + r^3)^2} \tag{27}$$

Using the value of a obtained above, and with values pertaining to the sun, we obtain for the solar density *at the solar surface* the value 2.65788×10^{-6} gm/cc. According to the above formula, the value of the density *at the center of the sun* would be 7.46217×10^{5} gm/cc. On the other hand, the pressure as a function of radial distance takes the following expression:

$$P = -\frac{\begin{pmatrix} a^3c^4s(72a^{12} - 80a^9r^2s - 6a^6r^4(18r^2 - 7rs - 4s^2) \\ -3a^3r^7(12r^2 - 15rs + 2s^2) + r^{10}(4r - 3s)s) \end{pmatrix}}{16\pi G(a^3 + r^3)^2(4a^3 + r^3)^2(a^3 + r^2(r - s))^2} \tag{28}$$

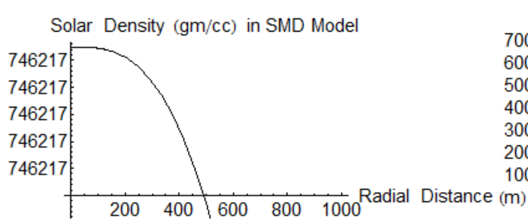
This gives the value of pressure *at the solar surface* to be 1.27247 atm. However the value of pressure *at the solar center* would be -2.012×10^{21} atm. This means that the pressure at the core is tremendous and negative — this reflects *the explosive and repulsive nature of the core.*

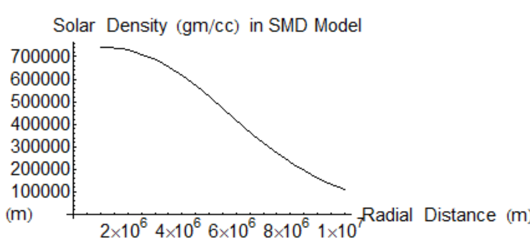
It is clear from the results obtained above, taking the sun as our example, and using the value $a = 5.41836 \times 10^{6}$ m for the length parameter that our simple metrical model is *not very far from reality.* The values obtained for the surface temperature, the surface density, and the surface pressure, are all reasonable and do not contradict well-known solar measurements. We now proceed to describe graphically the solar profile for density, and pressure as functions of radial distance.

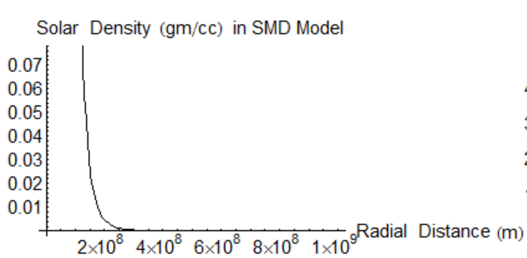
We obtain the following numerical expression for the solar density as a function of radial distance r. The radial distance must be in meters, and the density is in gram/cc.

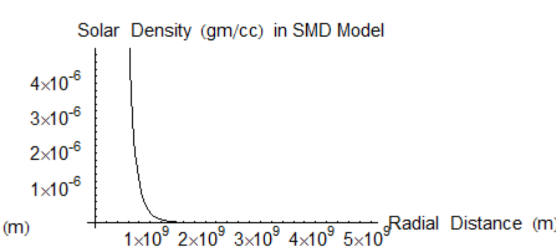
$$D = \frac{3.02128935567301182 \times 10^{47}}{(6.36302478675285865 \times 10^{20} + r^3)^2}$$

These are plots of the density in the central, intermediate, and superficial, regions of the sun:





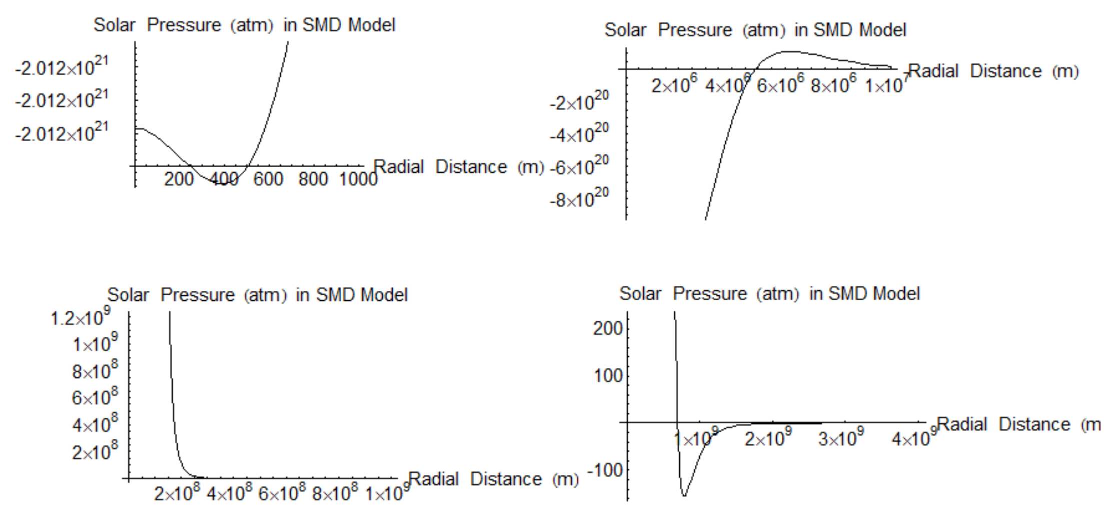




This is a numerical expression for the solar pressure as a function of radial distance. The radial distance must be in meters, and the pressure must be in atmospheres:

$$P = \frac{\left(\begin{array}{c} -1.33654 \times 10^{65}(-6.96262 \times 10^{8} + r)(-4.88321 \times 10^{6} + r)\times \\ (5.41762 \times 10^{6} + r)(7.57441 \times 10^{6} + r)(6.96263 \times 10^{8} + r)\times \\ (5.73778 \times 10^{13} - 7.57522 \times 10^{6} r + r^2)(2.93626 \times 10^{13} - 5.41984 \times 10^{6} r + r^2)\times \\ (2.38473 \times 10^{13} + 4.88353 \times 10^{6} r + r^2) \end{array}\right)}{(1.59076 \times 10^{20} + r^3)^2 (6.36302 \times 10^{20} + r^3)^2 (1.59076 \times 10^{20} - 2953.25 r^2 + r^3)^2} \tag{29}$$

This is a plot of the solar pressure as a function or radial distance near the core, in the intermediate, and in superficial regions:



The above plot shows that the pressure near the core is great and negative. This is a reflection, as we noted before, of the fact that the central region is an *explosive energy-producing active region*. The pressure changes sign and becomes positive at some point beyond the core, to be followed by monotically decreasing pressure, and finally showing a *remarkable effect*. Pressure *changes sign again* and gets to a negative minimum before going to zero.

At this point, let us examine the source of negative pressure in its parts P_1, the radial part, or P_2, the angular part. The respective expressions of P_1 and P_2 are given by:

$$\begin{cases} P_1 = \frac{3c^4}{8\pi G} \frac{a^3 s(-3a^3 + r^2 s)}{(a+r)(a^2 - ar + r^2)(4a^3 + r^3 - r^2 s)} \\ \\ P_2 = -\frac{3c^4}{32\pi G} \frac{a^3 s \left(\begin{array}{c} 48a^{12} - 54a^9 r^3 - 144a^6 r^6 - 42a^3 r^9 - 48a^9 r^2 s + \\ 90a^6 r^5 s + 63a^3 r^8 s + 6r^{11} s + 16a^6 r^4 s^2 - 16a^3 r^7 s^2 - 5r^{10} s^2 \end{array}\right)}{(a+r)^2 (a^2 - ar + r^2)^2 (4a^3 + r^3)^2 (a^3 + r^3 - r^2 s)^2} \end{cases} \tag{30}$$

From the above expressions we obtain that the central values ($r = 0$) of both P_1 and

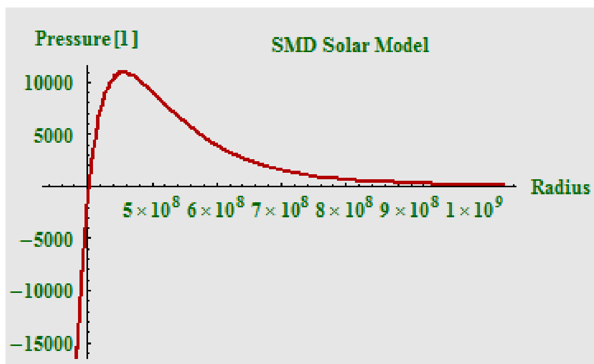
P_2 are equal and both given by negative expression

$$P_c = -\frac{9c^4}{32\pi G}\frac{s}{a^3} \tag{31}$$

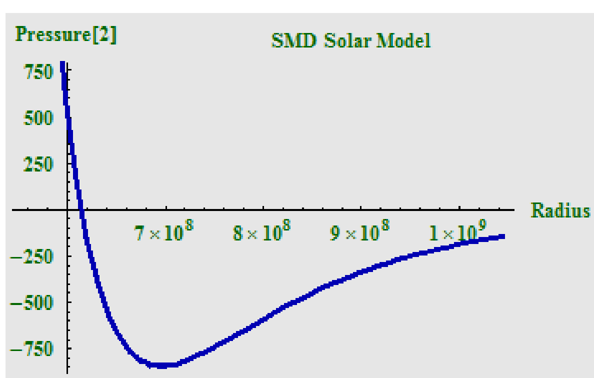
which for $s = 2GM/c^2$ becomes:

$$P_c = -\frac{9}{16\pi}\frac{Mc^2}{a^3} \tag{32}$$

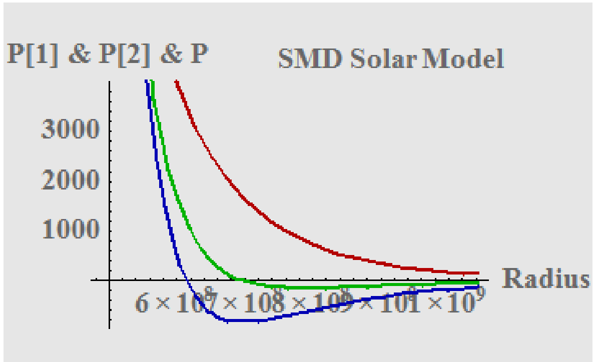
However, P_1 becomes 0 for $r = \sqrt{3}a\sqrt{a/s}$ and becomes positive afterwards reaching a maximum value, before decreasing monotonically towards 0. The following is a plot of P_1, showing it maximum value and monotonic decrease, with reference to the sun:



With regard to P_2, it also changes sign at some radial distance, however much later that P_1, reaches a maximum positive value, then goes back to zero, changes sign again to become negative, before increasing monotically to zero. The following is a plot of P_2, showing its ultimate decrease from positive to negative values and its minimum value, with reference to the sun:



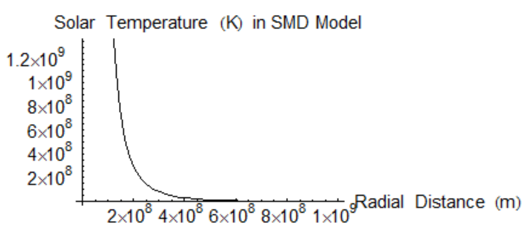
Hence, we can see clearly that the negative values of the mean pressure P, in the upper solar layers, come from the angular parts $P_2 = P_3$. The following shows comparative plots of P_1, P_2, and P in the relevant upper radial range:

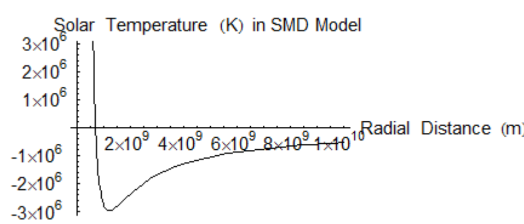


The following is a numerical expression for the temperature of the sun as a function of the radial distance (although the meaning of this parameter is not necessarily temperature in the deeper regions of the sun where is phase may not be that of an ideal gas). The radial distance is in meters, and the temperature in degrees Kelvin:

$$T = \frac{\begin{pmatrix} -5.35927\times10^{15}(-6.96262\times10^{8}+r)(-4.88321\times10^{6}+r)(5.41762\times10^{6}+r)\times \\ (7.57441\times10^{6}+r)(6.96263\times10^{8}+r)(5.73778\times10^{13}-7.57522\times10^{6}r+r^{2})\times \\ (2.93626\times10^{13}-5.41984\times10^{6}r+r^{2})(2.38473\times10^{13}+4.88353\times10^{6}r+r^{2}) \end{pmatrix}}{(1.59076\times10^{20}+r^{3})^{2}(1.59076\times10^{20}-2953.25r^{2}+r^{3})^{2}} \tag{33}$$

This is a plot of the solar temperature as a function or radial distance in the superficial regions:





The above plot shows the change of sign of temperature to negative values somewhere outside the solar surface. As we have mentioned before, negative temperatures mean negative pressures, and this would indicate some sort of explosive, or inflationary, tendency. Now noting the fact that the minimum value reached is of the order of millions of degrees, could this phenomenon provide a description or *an explanation of the coronal temperatures and the associated solar winds*? It would be interesting if there is such a fundamental connection. We must be cautious at this point.

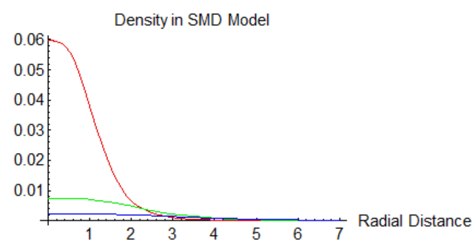
4 The Schwarzschild Limit

The functions $A(r)$ and $B(r)$ comprising our metrical model do become the Schwarzschild factor $(1 - s/r)$ in the limit $a \to 0$. Let us see how physical quantities would behave in such a limit. The density as well as the pressure would vanish for all radial values, but their thermal ratio becomes

$$\omega = \frac{P}{\rho c^2} \to \frac{s(-4r+3s)}{24(r-s)^2}$$

This limit is nonsingular for $r \neq s$. However, before getting to the value $a = 0$, the pressure and the thermal ratio would pass through a singular point. The latter corresponds to the zero in radial coordinate of the factor $(a^3 + r^2(r-s))$ in the denominator. Actually, as the following graphical analysis shows, there are low values of the parameter a for which the metric and the pressure could have singular points in radial coordinate.

Whereas the density is nonsingular throughout the radial domain whatever is the value of $a > 0$, let us examine the plot of the density for a few values of a. The following is a plot of the density for three values of $a = (1, 2, 3)$, in suitable units, and with corresponding colors (red, green, blue).

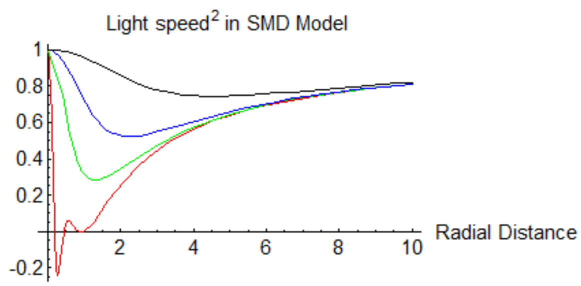


The above plot shows that as the parameter a becomes smaller and smaller, the density becomes more and more concentrated at the center. Hence, we might view the parameter a tending towards smaller values as associated with stellar evolution towards collapse.

In considerations of the Schwarzschild metric, we know that the radial coordinate speed of light squared is given by $c^2(1-s/r)^2$. This is the value obtained from the null geodesic equation. This shows that the photon speed would vanish at the Schwarzschild surface $r = s$. Now for our SMD model, the radial coordinate speed of light squared is given by

$$v^2 = c^2 \left\{ 1 - \frac{s}{r} \left(\frac{r^3}{a^3 + r^3} \right) \right\} \left\{ 1 - \frac{s}{r} \left(\frac{r^3}{4a^3 + r^3} \right) \right\}$$

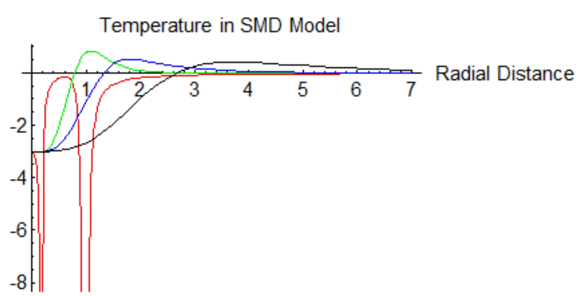
It should be clear that for large enough values of a compared to the Schwarzschild distance s, the above expression is always positive. However for low values of a, we can have a central region for which $v^2 \leq 0$. Let us illustrate this with a plot for several values of a. The following is a plot of v^2 for the values $a = (0.3, 0.9, 1.5, 3)$, in suitable units, with corresponding colors (red, green, blue, black).

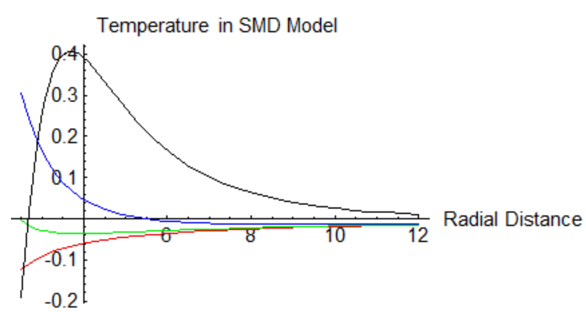


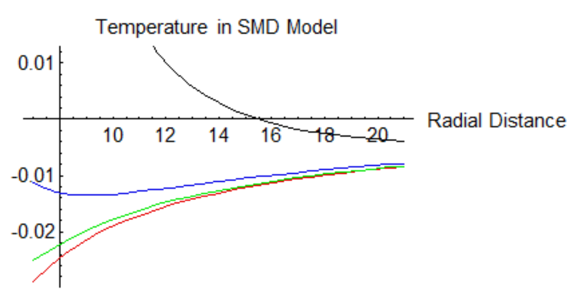
The above plot shows that for the low value $a = 0.3$ (red color) there is a region for which $v^2 < 0$. This is a *forbidden region* and its existence makes the associated value of the parameter a unacceptable. Actually, the minimum acceptable value of a should be such that the minimum value of v^2 be non-negative.

Let us examine now plots of the thermal parameter ω for the same values $a = (0.3, 0.9, 1.5, 3)$, in suitable units, with corresponding colors (red, green, blue, black).

These are plot of the thermal parameter ω in the central region and in a higher radial ranges:







The above plots show that the value $a = 0.3$ (the red curve) exhibits singularities in the thermal parameter plot, while the higher values $a = (0.9, 1.5, 3)$ produce normal curves in the central region, with positive maximum temperature values. They also show that the temperature (or pressure) changes sign at some point in the external layers in a star (the coronal and windy region). It should be noted that this change of sign occurs earlier for lower values of the parameter a.

It is clear from the above analysis that the singular Schwarzschild limit can be avoided by keeping the value of the parameter a above a minimal value. If the parameter a is regarded as a *downward evolution parameter for a collapsing star*, with associated decrease of size, then combining this scenario with the realization that the phenomenon of stellar winds being demonstrated in our SMD model, with *associated loss of stellar mass*, the end result of the collapse may not reach the socalled black hole stage, but rather *a fully evaporated star*.

5 Discussion

The work presented in this report demonstrates that it it possible to have a metrical description of a spherically symmetric stellar body with a thermal profile. The surface values for density, pressure, and temperature, can all be fitted with a single length parameter. This description involves two intimately related component functions in the line element, a temporal function $A(r)$ and the radial function $B(r)$. We have presented such functions, associated with a simple mass distribution (SMD) models. However, there is nothing special about the functions introduced except that they are simple and viable. Other forms of functions are quite possible, that we shall treat in other articles, but when fitted to a particular star like the sun, these functions would involve different values for the extension parameter a, as well as different values for the thermodynamic parameter in the central region of the star.

That we can describe the thermal profile of a star by metrical models is an important development which deserves to be pursued further. The metrical model presented could be extended much further by including *hydrodynamic* profiles like radial and rotational flows of matter. They can also be extended by including *electromagnetic* properties with appropriate charge and current distributions. The ultimate aim is to be able to describe such a complicated object like the sun with all its surface features like magnetic spots, coronal temperatures, and solar winds, and constructing complete thermodynamic, hydrodynamic, and electromagnetic profiles, down to the active core. Also, the underlying metrical models could also be useful in dealing with celestial objects like large fluid planets as well as stellar clusters, active galactic nuclei, and perhaps complete galaxies.

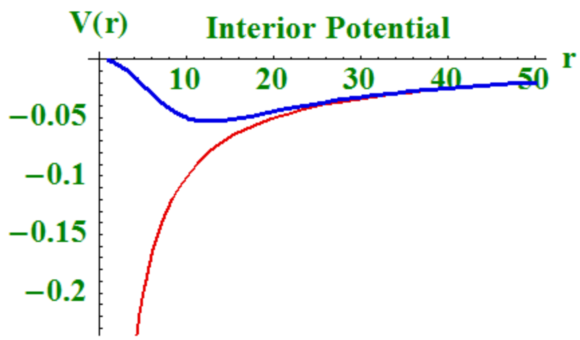
It would be interesting to compare the model of extended spherical body, presented in this paper, with Schwarzschild's model of an incompressible (liquid) sphere[3], [4] and its continuous counterpart[5].

The gravitational potential of the underlying metric, in our present model, is given by the spherical fucntion

$$V(r) = -\frac{GMr^2}{a^3 + r^3} \tag{34}$$

and the following is a schematic plot, with a convenient scale corresponding to $a = 10s$

(red color for the Newtonian potential, blue color for our potential):



In this regard, it would be interesting to investigate the motion of a particle in the gravitational potential of the underlying metric. Notice that whereas the potential for large distances ($r \gg a$) is the Newtonian $V(r) = -GM/r$, the very interior part of the potential exhibits a *repulsive* core. Does that mean that *the relativistic interior potential of any extended spherical system has a repulsive core*? Of course, and we shall return to the general question of the interior gravitational solution, and the relativistic particle motion inside an extended spherical distribution of mass in another article.[6]

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[4] N.S. Baaklini, "Schwarzschild's Metrical Model of a Liquid Sphere", *N.S.B. Letters*, **NSBL-RT-017**

[5] N.S. Baaklini, "A Continuous Counterpart to Schwarzschild's Liquid Sphere Model", *N.S.B. Letters*, **NSBL-RT-018**

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