Composite Dimensionality:
The Origin of Force and Motion

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Abstract:

Composite dimensionality is a new way of looking at the universe through a detailed analysis of the concept of motion. Motion has, from the time of Zeno of Elea twenty-four centuries ago, represented a great unknown. The universe needs nine dimensions to fully account for all forms of one-dimensional motion. Physics, viewed in nine dimensions, becomes almost classical again – phenomena detailed in both quantum mechanics and relativity theory are shown to be distortions of fundamental parameters brought on by the way nature needs to fit the information flowing in the nine basic dimensions into the three spatiotemporal dimensions we perceive (it's not a perfect process). The idea of extra dimensions is not a new concept. However, the methodology of dimensional compactification previous ideas posit as a means of dealing with the 'extra' dimensions is not needed. Composite dimensionality reveals the way in which the universe handles the nine basic dimensions, as well as to answer the question of what, exactly, are the three dimensions we exist within, as well as how the three spatial dimensions relate to the nine basic dimensions they are the product of.

Quantum mechanics is one of the most successful theories we have derived in terms of answering questions concerning the nature of the universe. However, there are few if any researchers who currently think of quantum mechanics as a complete theory. In some areas, such as the nature of single particle action, and particle pathways, the theory falls flat. Perhaps there is some part of the quantum story that we do not yet understand? But if this were the case, then how can this incomplete framework be used to make accurate predictions about anything at all? Perhaps it is not that quantum mechanics is incomplete, but our way of thinking about the universe itself that is lacking.

This is a roundabout way of saying perhaps we should examine more closely what we mean when we talk about the path a particle can take or how a particle can move. The concept of motion in general is an area of thought that has occupied the human mind for a very long time. So, I feel it is fair to ask the simple question: Do we know what motion really is? Perhaps not…

Twenty-four centuries ago, the Greek philosopher Zeno of Elea came up with several thought experiments showing that motion is not possible. Simply put, to get from where you are to where you want to be, you must get halfway there first. Before you can get to the halfway point you need to get again, halfway there, and so on unto infinity. Zeno felt that because there were an infinite number of halfway points existing between any two places, that motion is not possible because it would take an infinite amount of time to cross an infinite number of finite-in-extent halfway points. I have greatly simplified Zeno's four motion paradoxes, where he argued using logic that nothing moves. Of course, we know he is wrong, and historically it isn't known with certainty whether Zeno himself really believed motion was impossible or whether he propounded his paradoxes simply as a defense of his teacher Parmenides' ideas [1]. All one must do is to look at something moving to know that motion really occurs. It seems self-evident, doesn't it? Things change position all the time. To sum up: by observation and definition, 'movement' happens when some object changes its position with respect to time, and 'motion' is the action or process that makes movement possible. This is what our eyes tell us, and 'seeing is believing', right? Our eyes would not tell us lies, would they?

It is a common belief today that Zeno's classical paradoxes of motion have been done away with by the invention of calculus.[2] Using calculus it is possible to show that an infinite series can converge to a specific, finite, quantity. But wait, not so fast. True, calculus is an invaluable tool that allows for a numerical solution to textbook examples of motion problems, but it does not speak to the underlying cause of motion. Zeno's peculiar paradoxes speak to the heart of motion, its causative agent, and the engine behind the phenomenon - the abstruse 'process' behind the previously mentioned definition for motion. Calculus just doesn't deal with this aspect of motion; it only provides a numerical answer to a given motion problem. Using calculus to provide a quantitative answer to a motion problem serves as a bandage which conveniently covers up our conceptual ignorance as to how motion occurs. We still do not know what the engine of motion is and Zeno's paradoxes are still very much alive in the 21st century.

The problems we have with motion are reflected in quantum theory. Being a statistical theory, it is not really set up to answer questions concerning individual particles – how they may act in a particular situation, or how they undergo motion. It gives accurate results when dealing with large aggregates of particles but even then, the theory does not say anything concerning particle motion. It seems the problems first elucidated by Zeno over 24 centuries ago are still reflected in current-day thinking. When seen in this light the fact that quantum mechanics can be as correct as it is, seems something of a miracle.

What of relativity theory? This is an entire branch of physics that seems built around motion – about how quickly an object undergoing positional change with respect to an observer alters fundamental quantities of measurement. Relativity deals brilliantly with how measured phenomena such as length, time, and mass change to reflect an object's motion, but again, has nothing to say concerning the engine of motion at a fundamental level, namely, what it is.

It is my assertion that if we understood motion better than we do currently, the problems inherent in quantum theory, or relativistic theory, would clarify, revealing a much more tone, muscular theory that is also easier to understand and use.

A Universe Where Nothing Moves

Motion is an illusion.

Zeno came to the above conclusion 24 centuries ago. Can any part of his premise be correct? Is there any way we could be living in a universe where motion is not possible? Despite the information of our senses and the fact that each one of us personally experiences what we think of as motion, it is possible he was right. Because we can easily detect when movement has occurred, when some object has changed position, the only way this could be true is if motion could be divorced in some way from movement. Motion and movement are so tied together that the words seem contextually equal, and in fact many use those words interchangeably. But they're not the same thing. As I said earlier, an object's change in position is called movement, and motion is the process that causes movement. But what, exactly, is that process? In short, what exactly is motion? If the subject of the sentence is movement, then motion is the verb. So, what is the process behind this particular verb? Now, at some level it is an applied unbalanced force that sets up the conditions necessary for motion to occur, and an applied force involves the input of energy to a system. So, at this basic level the flow of energy into (or out of) a system is what causes something to move, to change position. This isn't a very satisfying explanation because when we say 'flow of energy', we are talking about the motion (of energy) to define movement in a system. This is saying motion defines itself, and is not very useful.

Of course, not having a good definition for motion doesn't mean movement cannot occur. You really need to ask precisely what you are observing when you use your eyes (or a recording apparatus) to observe movement. For example, when you observe a snail crawling slowly over the surface of a leaf, what can you really say about what you are observing? In this example, you can say that the snail is changing position, or, simply, moving. You know this because you can easily compare the results of ongoing measurements you have taken of the snail's position over a certain time interval to know there has been a change in position with respect to time. Or you could simply watch the snail, using only your eyes and keen memory to keep track of things. But, no matter how you do it, what you see or measure is movement, not the mechanism that creates the movement, which is motion. This is a subtle and important point. You are viewing the end result of a process you cannot perceive, namely motion, and calling it movement.

Some may object to this result, saying it is too restrictive. They may say that they are seeing motion occurring because they discern the snail changing position fluidly in real time as it is happening. This aspect of motion is, as Zeno would say, an illusion. To understand why, just ask yourself what motion would look like if you could turn off memory. Without the ability to remember (or record) past positions of the snail, how could the observer's brain create any kind of path at all, let alone attempt to predict where the snail may likely end up in the future? In this regard, what most people would commonly call motion is organic in nature, an artifact created in their own heads and not something seen.

The classical equations of motion work so well because they describe the results of something undergoing positional change in terms of speed or acceleration which gets expressed as a numerical quantity. This is as true for solutions expressed by calculus as it would be for any other type of mathematical methodology. Motion equations therefor do not describe the process of motion, only the results of motion, which is movement. Motion equations are really movement equations.

This conceptual fuzziness extends more deeply than we realize. When we describe motion we invent another term, force, to provide a mechanism for motion to occur through. But it all falls down upon closer inspection because we really do not know what force is either.

On a purely superficial yet functional level, force is just a numerical quantity representing the multiplicative product of mass and acceleration: $F = m \times a$ expresses the relationship between force, mass, and acceleration that forms the heart of classical physics. This equation gives no conceptual clue about *why* a given force will cause a particular mass to accelerate; only that it does. Like its bigger cousin motion, our definitions leave the juicy inner workings of force a pregnant mystery. Considering everything we either don't know or are ignoring, it really is remarkable that classical physics works as well as it does.

Quantum physics and relativity theory both incorporate within them the quintessence of classical physics, and hence suffer from much the same type of motion related problems. Within the framework of quantum mechanics, the path a particle takes is not even considered, and in relativity, results describe how motion near the speed of light (which is a sort of ultimate movement) distorts how we view certain phenomena such as length, time, mass, etc. Like classical physics, it is also remarkable how well both these recently developed scientific frameworks describe certain aspects of reality. The problem is that by leaving out the unknown factors; namely how motion occurs, and how force is related to motion dynamics, the three theories we have literally based our entire physics framework on, since at least the time of Newton, risk being placed in jeopardy when intractable problems arise that cannot be decoded.

The physicist Lee Smolin, among others, has recently identified a major phenomenological problem with all three mainline physical theories which currently form the backbone of physical science. Namely, none of them are truly fundamental theories which would allow them to describe the universe as a whole [3]. They are effective theories, in that each works within its own domain, but either fails or becomes undefined when used outside its limited arena. Is it any wonder physicists have met with so much difficulty attempting to create a grand unified theory? The problem is they are trying to cobble together a truly fundamental theory using incomplete components. You will never bake a cake by leaving out random ingredients – oh, it may look like a cake at a distance, but the imperfections will become apparent when you try to cut it or taste it.

To bake up a true unified theory of mass, energy, and action, it makes sense to begin with a fundamental theory, and that is something we do not currently possess. So, what would a fundamental theory look like? It is possible to get an overview of such a theory by looking at some of the concepts it would need to deal with:

1) It explains Zeno's paradoxes of motion by providing a conceptual mechanism that motion can occur through. This is more than simply providing 'motion equations' that one can use to predict where a moving object will be at a particular time. In other words, a fundamental theory of motion could be used across-the-board from quantum systems to classical systems to relativistic systems without major modification.

2) It provides a conceptual framework that explains the origins and mechanism of 'force'. What it is and how it acts on mass to cause positional translation.

3) It conceptually and plainly explains why energy is a conserved quantity without invalidating processes we currently utilize.

So, where do we begin our journey?

If we recognize the problem as told by Zeno, that motion is not possible in our three-dimensional universe, then we must ask: how many dimensions does it take to account for motion? To answer this question, we need to classically determine what forms of motion can occur in a single spatial dimension, and then multiply the result by three to account for all possible directions in three-dimensional space.

First, obviously, is no motion at all. This would be the simplest case: unchanging position. Next, there is motion at a constant speed. Position that varies in a constant, unchanging way with respect to time – this form of motion is also the solution to the first derivative of position with respect to time. There is also the case of accelerated motion, or the rate of change of speed with respect to time, which is also the solution to the second derivative of position with respect to time.

These forms of motion, although related to each other, can be treated as separate cases of motion, each needing its own dimension to be expressed in. Are there any other forms of one-dimensional motion? Well, the rate of change of acceleration with respect to time is called jerk, the third derivative of position with respect to time, but it can be expressed as an acceleration of an acceleration. It all boils down to three conditions: no motion, constant motion, or accelerated motion. So, it seems, there are only three forms of motion that can occur in a single spatial dimension. Since we are living in a three-dimensional universe, and there are three forms motion can take per each of those spatial dimensions, motion can be thought of as a nine-dimensional phenomenon. This explains why we cannot see it – we can only perceive the three dimensions of space. Our brains, being created from three-dimensional matter, cannot perceive anything beyond those three dimensions. Looked at in this way, it is now easier to see why Zeno got so hung up on the concept of motion. As he successfully argued, motion is simply not possible (in three dimensions). Intuitively, he somehow knew the only phenomena possible given three spatial dimensions is position, which would lead to an unchanging, eternally frozen in place universe, which can also be described as a universe where time doesn't exist or flow. When he watched something move, he was observing the results of a nine-dimensional phenomenon, motion, producing an effect, movement, on three-dimensional matter, and it all looked like magic to him.

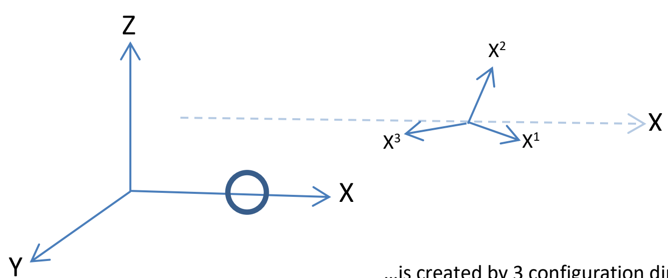
The Idea of Composite Dimensionality

This means the problem of motion reflects a more basic problem in dimensionality. It seems our concept of spatial dimensions needs to be looked at more closely. Although there are many definitions for dimensionality, in physical science a dimension is a measurable quantity. For example, the x-spatial dimension is defined as movement in a certain plane, while the y-spatial and z-spatial dimensions are defined in the same way, where all three spatial extents exist at right angles to each other. This begs the question, namely, how can a nine-dimensional phenomenon have any effect in this three-dimensional space we live in? Well, if the nine dimensions motion occurs in were totally disconnected from our three-dimensional world, as is posited in some interpretations of string theory, the answer would be that it couldn't. Historically, in theories where the number of dimensions exceeds the basic three spatiotemporal dimensions of common experience, from Kaluza – Klein theory [4], to string theory [5], the six dimensions that are beyond our perception are treated as almost superfluous. We can be grateful this is not the case else our universe would exist frozen in a single point of time for eternity, and we wouldn't be here to ask the questions! Obviously, the three-dimensional space we live in and the nine-dimensional 'configuration space' where motion occurs, are intimately connected in such a way that we can see the results of action occurring in that higher dimensional place within our own universe. Also, the way in which this higher order configuration space relates to our three-dimensional reality will dictate how basic structure in our universe is achieved and how order is maintained.

A major point of difference between composite dimensionality and most interpretations of string theory is in how the 'extra' dimensions are handled. In the string theory hypothesis for example, while the existence of nine spatial dimensions is necessary, six of those dimensions are somehow collapsed in upon themselves, through a process called compactification. The six compactified dimensions, while still existing, do not interact much with the three remaining dimensions of which our universe is created due to their extremely small size and consequently high interaction energy. Some versions of string theory utilize the extra dimensions as a possible way to explain why gravity is such a weak force, but the reasoning for such an assertion currently has no scientific evidence to back it up. The compactification process itself also represents an unknown process. Also unknown is why and how our familiar three dimensions escaped this process, and not some other arbitrary number of dimensions such as four dimensions or seven dimensions. Compactification represents an ad hoc addition to string theory. Composite dimensionality does away with the compactification process entirely. In composite dimensionality, all dimensions of the universe are placed on equal footing. All nine configuration dimensions act equally to create our familiar three-dimensional universe in an active process that continues to this very day.

Dimensional Hierarchy

At its most basic level a single spatial dimension is not a fundamental structure, but is instead a composite entity made up of three parts. If we posit a higher order domain, which is a configuration space, as existing with nine configuration dimensions, then it is possible to generate our 3-dimensional universe, where each spatial dimension we exist within is in fact a composite structure created by the orthogonal projection of three of these fundamental configuration dimensions. The way fundamental dimensions self-order to form spatial dimensions is a topic I will discuss shortly.



A single spatial dimension… …is created by 3 configuration dimensions in 9-dimensional space.

So, how does movement occur? In 3 dimensions, an unbalanced force is propagated, either by direct contact between objects, or via an electromagnetic or gravitational field, which then causes the state of motion of the afore mentioned system of objects to be altered in accordance with the classical laws of motion. A more modern interpretation of movement is that it is caused by the exchange of virtual particles which act as mediators that carry force between objects. In fact, this interpretation is not enough because mediator particles must move between objects in three dimensions to exchange force information and one is left in a catch-22 situation, using motion to define motion.

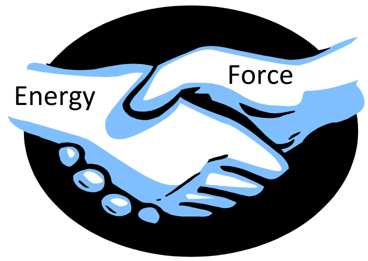
Seen in 9-dimensional configuration space, the action is more intricate. Think of the 3-dimensional space an object exists within as being the result of nine configuration dimensions coming together to form a 3-dimensional spatiotemporal domain of action – every point in space is described thusly. Mass, or rather the energy contained in mass, distorts these configuration dimensions so that they are no longer orthogonal with respect to each other. Particles may in fact be exchanged between these moving objects, but the exchange occurs in nine dimensions, not three. This is why mediator particles are virtual - they do their work in nine-dimensional configuration space. It is the distortion existing between the nine configuration dimensions, mediated by virtual particles that cause motion in configuration space, not direct contact between objects in three dimensions. In turn, it is motion in nine-dimensional configuration space that results in movement occurring in our familiar three space-time dimensions.

For an example, consider a marble rolling on a hard surface which then strikes another, stationary marble, and sets it in motion. Ignoring the contact between the marbles and table for the moment, what is really happening here? Each marble is a three-dimensional object in the center of a 9-dimensional configuration domain. The domain representing the moving marble begins to interact with the domain representing the stationary marble, which begins to distort as dimensional information begins to be shared between the two nine-dimensional configuration domains. When we observe this process translated down to three spatial dimensions, we observe the moving marble encountering the stationary marble which causes both to change their state of motion, all according to familiar classical laws. It is important to realize that there is no force in three dimensions being exchanged between these two objects, that all the action takes place in nine-dimensional configuration space. What we observe is merely the result of this dimensional interplay projected into our three-dimensional domain. The mass represented by the marble moves because its configuration domain is distorted and it gets carried along for the ride, much as a bit of flotsam on the surface of the ocean moves because it gets carried along by the tide. The tide in this case stands for the nine-dimensional distortion set up in configuration space and the movement we see is the net result of that distortion as seen in three-dimensional space.

In reality, motion is an indirect thing. In the above example, it doesn't involve the marble masses so much as it does the configuration domain each marble exists within and the way virtual particles interact in nine dimensions. Of course, the amount of distortion the configuration dimensions experience is due to the three-dimensional mass-energy equivalency each marble represents, so it is a shared process. Mass-energy creates the distortions, and it is the distortions that govern the motion of the masses, as well as other physical variables. Also, marbles don't roll in isolation. The surfaces they are resting on each represent their own configuration domain with the resultant continuous two-way exchange of dimensional information defining the state of motion for both marbles. The net result in three dimensions is a well-behaved dynamical system which obeys classical motion laws.

Energy – Force Feedback Loop

Energy in 3-dimensional space causes stress in 9-dimensional configuration space …

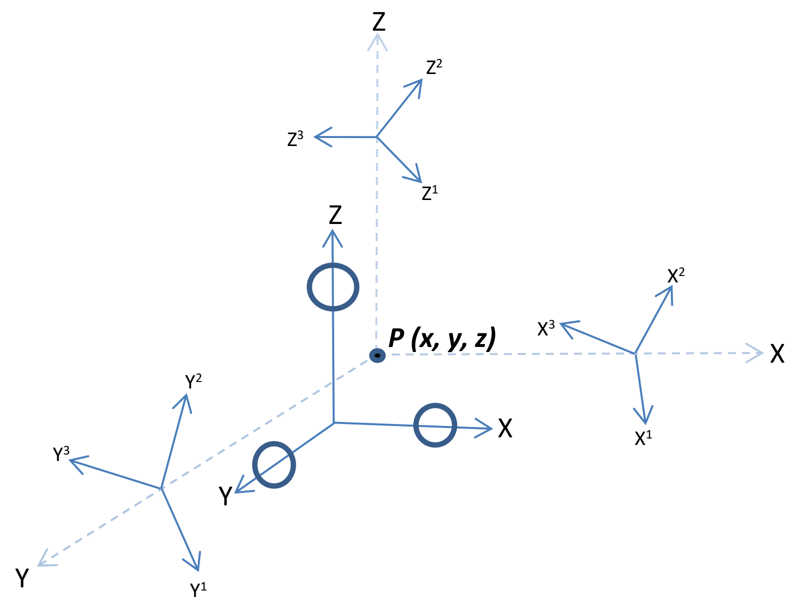


…stress in 9-dimensional configuration space gives rise to force which propagates back into 3-dimensional space and gives energy its particular characteristic.

This seems to be a complicated way to describe physical action, but it is necessary in order to preserve energy conservation, which I will go into in more detail next.

How Motion Relates to Energy Conservation

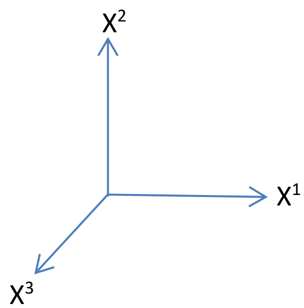
In the earlier section, I explained how each spatial dimension is a composite structure made up of three configuration dimensions. In more detail, this means that each point in the 3-manifold that defines our universe is created by the confluence of a total of nine configuration dimensions. If there were a point in the universe devoid of matter and energy such that no stress was placed on these configuration dimensions, then each of the three configuration dimensions that make up each spatial dimension would exist in perfect orthogonality with respect to any of the other two configuration dimensions:



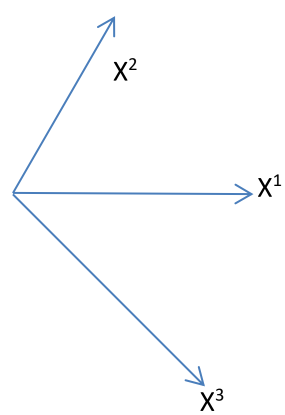
An undisturbed point ***P*** in three-dimensional space is composed of three groups of three orthogonally related configuration dimensions.

This dimensional orthogonality no longer holds when mass occupies a point in space. The presence of a mass, or more accurately, mass-energy, serves to create a tension between the three configuration dimensions which causes them to deform away from perpendicularity. The greater the mass-energy in a region, the greater this deformation will be.

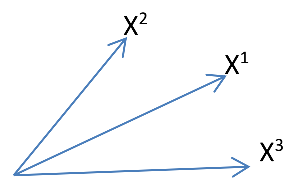
In configuration space:



In the absence of mass-energy, the composite dimensions making up a spatial dimension are perpendicular to each other.



Mass-energy causes the configuration dimensions to distort away from perpendicularity.



The greater the mass-energy in a region of space, the more distortion there will be.

It is these deformations that translate into the so-called fundamental forces of nature. It is important to realize that in this view, energy and force are two separate things. Mass (or mass – energy) in a region of space creates the stresses in configuration space that in turn generate the fundamental forces. These fundamental forces in turn shape the energy contained in mass, giving matter its physical characteristic. Also, fundamental forces are derived by the flow of quantum information. In this regard, energy defines the potential for quantum information to become real, and by real, I mean to have a measurable effect in 3-dimensional space. An explanation of motion begins when two particles, represented by these nine configuration dimensions get close enough to interact, creating more tension that gets relieved when the two particles move away from each other. Remember, all this action is happening in nine configuration dimensions, not three composite spatial dimensions.

Energy Conservation Within Composite Dimensionality

Energy is a conserved quantity. For this to be true in an absolute sense, energy must be temporally transparent. This provides a direct way to explain energy conservation. For energy to be transparent to time, it means that its magnitude can never change, that it exists as a frozen-in-place fossil, a relic whose magnitude was fixed by processes that existed only during the first instants of the universe's existence. The absolute magnitude of energy in a volume of space can never change.

Also, if time doesn't affect the magnitude of energy, then energy cannot flow from point to point in the universe. It is wrong to think of motion as the physical, fluid, movement of an object through a point in space, because if that were true then the total energy existing in the mass of an object would be able to add to the energy already existing within a point in space as the mass passes through it. This describes a time evolution which is a disallowed state because at the very least it would imply that the energy of motion would not be conserved locally.

The total amount of energy contained in a single point in space is truly an outstandingly large number, because of three variables: kinetic energy, potential energy, and vacuum state energy. Of these three quantities, vacuum state energy is by far the largest factor. At any point in space, the total energy contained in that point, is, kinetic energy (ke) + potential energy (pe) + vacuum state energy (ve) all of which can be conveniently set to unity, so, ke + pe + ve = 1. For total energy measurements, the relative proportions of these three factors can change with respect to each other, but they must always equal 1 for any point in space.

This point bears repeating. Within any given volume of space, the ratio of energy forms can change with time, but not the total amount of energy. If the total amount of energy could change with time, energy conservation would be violated. The major consequence of temporal independence means that our notion of energy 'flowing' from region to region is not correct. Indeed, if energy could flow, then it would not be conserved and our universe would look completely different. The ratio of the three energy components that exist within a region of space is in direct proportion to, and a consequence of, the amount of distortion each configuration dimension is experiencing.

Getting back to the main subject of this discussion, motion, the question that needs to be asked at this point is this one: If energy cannot flow, then how is even the appearance of motion possible? This speaks to the heart of Zeno's paradoxes of motion, and why Zeno must have thought that motion was an illusion.

To answer this question in a conceptual way, it would be helpful to consider what could be the smallest unit of volume possible in our physical world. Although the exact number is not important, due to quantum physical considerations and involving Planck's unit of length as being approximately 1.6×10^{-35} meter, we need to realize this unit is indeed minuscule. There is currently no hope of ever detecting or working with volume units so tiny that they make subatomic particles seem huge by comparison. Conceptually, this volume can simply be thought of as a region containing energy. Because this three-dimensional region is created by the confluence of nine higher order configuration dimensions, what happens in configuration space can and does have an effect within each quantum volume of 3-dimensional space. Remember from the previous section that the forces caused by matter stressing the dimensions of configuration space can shape energy contained within a volume of 3-dimensional space, giving it its physical characteristic. One of these characteristics is the displacement caused by motion in those nine dimensions. Particles cannot move fluidly in our three dimensions to reflect this displacement, but rather, 9-dimensional motion projects down to three dimensions as a displacement first appearing in one volume region, then another volume region, as the displacement propagates in configuration space.

Seen from our perspective, if we truly could witness pure motion on the scale of individual quantum volumes, motion would not be continuous, but would appear discreet, and mass would first appear frozen in one volume, then disappear to reappear in a neighboring volume in a slightly different configuration to reflect time evolution in nine dimensions, without appearing to exist between volumes. It is not the mass that is moving across the boundary of the quantum volume because if it did so it would imply energy was flowing and that is ruled out because of conservation considerations. It is the quantum information of what is happening in 9-dimensional configuration space that is seen to 'jump' from boundary condition to boundary condition, manifesting as physical force that causes the energy trapped within those quantum volumes to assume the shape of mass. Because quantum information that defines a particle's physical properties is defined as imaginary in three dimensions, the boundary condition limiting the propagation of energy does not apply to the flow of quantum information. Thus, a particle, or any other physical system can maintain its cohesion while undergoing time-evolution. In addition, the more energetic the process, the more quantum volumes the mass could skip over, which would appear to our eyes as an increase in speed. In this view, kinetic energy is directly related to this 'skip sequence'. The faster something moves, the more quantum volumes get skipped over, and we say it has a larger kinetic energy. Energy, to bring up an analogy, is the raw three-dimensional material which universal forces use as putty to shape what we call our physical reality. Those forces in turn, originate in a 9-dimensional world we cannot perceive due to the limitations of the 3-dimensional energy we are made from.

How Motion Appears in Three Dimensions:

The quantum connection

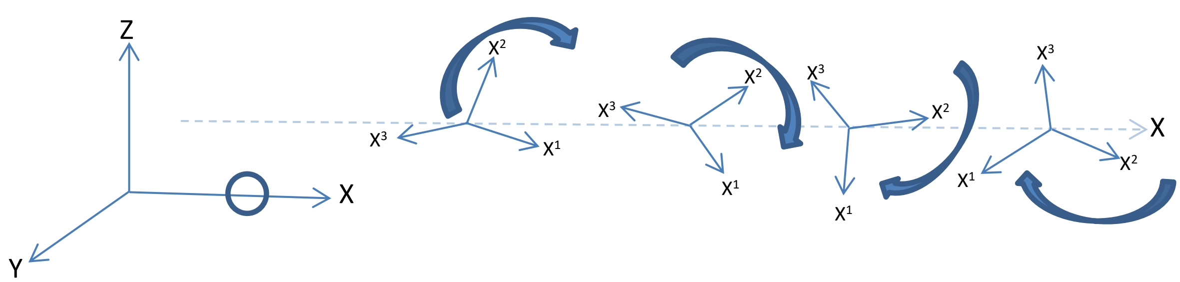
Motion depicted for a snail where speed = 2 inches per minute:

Motion depicted for a snail where speed = 4 inches per minute:

Motion in the above fun little example depends not only on the skip sequence, but is also a function of the duration the object remains bound within the quantum volume. This is so because time must also be represented for moving objects. The concept of time has always had an eneffable fuzziness about it. This is because of our lack of understanding of what the phenomena we call time actually is. In composite dimensionality, time is simply the manifestation of duration in a 3-manifold due to change occuring in a 9-manifold. Remember, when an object is locked within a quantum volume it is 'frozen' for however long it is there. This is because the energy that it is made from is time-transparent. For a given duration, no time evolution can occur. Change can only happen when the object skips to the next quantum volume. As both special and general relativity show all too well, time is also a phenomenon that is related to motion, as relative motion and gravity can alter percieved duration. Temporal ordering, the 'direction' time is observed to flow in, is also derived from the way motion occurs in nine dimensional configuration space. Again, what we observe as the flow of time, or time's arrow, is the direct outcome of a dynamical process happening outside our perceptual realm. We see the end results of this ordering of duration and call it 'time' when in fact the flow of time is as much an illusion as motion is and for the same reason.

Also, the above example attempts to represent pure motion by using the fiction of a particle (or a mollusk) which can be contained in a single quantum volume. In fact, due to the miniscule size of a single quantum volume of space there exists no particle small enough to show the discreet nature of pure motion. The opposite would be true – subatomic particles no matter how tiny would interact with many of these miniscule volumes of space, causing their overal motion to be avereged out and appearing as a smooth unbroken track in space. The universe conspires to give us the illusion of pure three dimensional motion where none exists.

Earlier I mentioned that if we understood motion better it could lead to a better understanding of quantum phenomena. Until this point, I have attempted to stay clear of quantum nomenclature as much as I could, though I must admit, with imperfect results. This is because all physical law, as well as all physical characteristics in our universe have their origin in the 9-manifold, and can be represented in our 3-manifold as imaginary quantum operators. The power of three composite dimensions is the power to make imaginary and virtual processes 'real'. An example could be the spin of a subatomic particle. Currently spin angular momentum has no conceptual real-world analogy. It is just a characteristic of subatomic particles that, while having measureable outcomes, has no easy description. In composite dimensionality spin angular momentum arises when the configuration dimensions that create a given spatial axis are themselves rotating. Particle spin is how the angular momentum of three configuration dimensions presents itself in a single composite spatial dimension for a subatomic particle along its axis of measurement.



Particle spin is caused when angular momentum along a set of dimensions in configuration space is projected into a single spatial composite dimension for a subatomic particle.

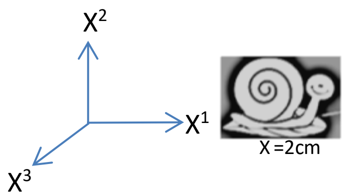
Spin angular momentum represents the rotation of 2 configuration dimensions, much as classical spin represents the rotation of 2 spatial dimensions. Precession in both cases is possible when all three dimensions, both configuration and spatial are involved. In this way, quantum processes are mirrored, albeit imperfectly, in our three-dimensional world.

Composite Dimensionality and the Limits of Physical Reality: The relativistic connection

It should be apparent by now that if continuous motion in nine dimensions can only be expressed as discreet steps in three dimensions, then 3-dimensional motion-related phenomena must in some ways be distorted, compared to the 'pure' motion happening in nine dimensions, and that the total distortion is proportional to the amount of movement an object undergoes. This distortion will also exist in any physical parameter that is being measured along the direction of motion. A way of defining the amount of distortion for a physical quantity is to first note that the three configuration dimensions that create a single spatial dimension can be conveniently represented as vectors, and the angle each vector makes with respect to the other two is defined by the relative motion as measured between the accelerated object and the observer.

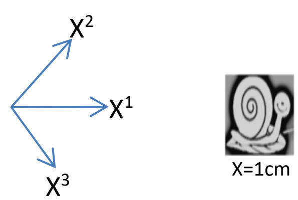
For an example relating length contraction along the x-axis of motion:

When V = 0:



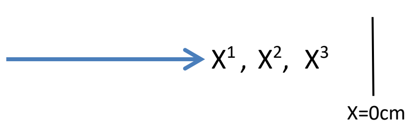
In the case where the motion is zero, there is no distortion in any of the physical parameters of the snail with respect to the observer.

When 0<V<C:



For the case where relative motion is beginning to approach the speed of light, the snail begins to appear foreshortened along the direction of motion with respect to the observer.

When V=C:



For the limiting case where the mass-energy distortion is so great all three configuration dimensions are driven into each other, the snail's length along the direction of motion is zero with respect to an observer. Because of this the speed of light is effectively an ultimate physical barrier which can never be breached because it would require an infinite amount of energy to do so.

It is interesting to note that in the above example, it would take an infinite amount of energy to drive the three configuration axes into each other to become a single dimension. This occurs when the energy of motion, or relativistic kinetic energy, becomes infinite at the speed of light. This is a pedagogic way of saying that all true motion happens beyond the light barrier and that motion itself is a superluminal phenomenon.

The above scenario can also be considered a testable hypothesis – in the last example where the poor snail has achieved the speed of light in the direction of motion, the mass of the snail attains imaginary status in only the x-direction: the snail will give up its mass- energy only in the direction of motion. From our perspective, one third of the mass-energy comprising the unlucky mollusk will be converted into radiation, spectacularly. However, this would only be true for a situation where all 3 x-configuration dimensions achieved light-speed simultaneously. For a non-symmetrical situation, where only one, or two, configuration dimensions achieved imaginary status, the resulting energy release would be one-third, or two-thirds total X-mass, which could be measured. Also, in a notable deviation from conventional theory, in a situation where some but not all configuration dimensions achieved the speed of light, the total relativistic energy in the direction of motion may not be infinite. This might be possible if the snail were undergoing rapid spin along the direction of motion, for example.

It is notable that within the theory of composite dimensionality, both relativistic effects and quantum phenomena have a common origin. Specifically, they are phenomenological distortions brought upon physical reality because of nature's attempt to fit a nine-dimensional reality within a three-dimensional physical framework. The process works, with some loss being unavoidable and showing up as the distortion effects of relativity and quantum mechanics. Philosophically, the idea of composite dimensionality brings back the ideal of classical physics, only with more dimensions. Having nine dimensions offers physics enough degrees of freedom to become a classical endeavor once again.

Dimensional Reality as an Emergent Phenomenon

Emergent phenomena come about when new and sometimes surprising properties arise within a system as it grows in complexity, properties that are not shared by the individual components that make up the system. Emergent structures are patterns that come about when large numbers of individual items interact with each other and their environment. Some examples of emergent structures in nature are the wind ripples formed on loose sand in a desert, or the fractal – ice patterns freezing water sometimes makes on your kitchen window in the winter. Now that we are looking, we are finding more emergent structures in nature. Composite dimensionality is an idea that seems to beg the question: Is the entire universe an emergent phenomenon? If so, then it is the most basic of emergent phenomena: the fundamental structure of the universe consists of nine tenuously connected configuration dimensions. The way our three-dimensional universe comes about results from the way in which these nine fundamental entities come together to create our reality. Reality is at best a second order effect arising not from fundamental principles, but instead comes from an emergent structure, the three dimensions of space.

So how could three-dimensional reality spontaneously arise from the nine separate configuration dimensions? The first thing to keep in mind is that the nine configuration dimensions are equal. There is no preferred component. A strong clue that this must be true is that we observe no fundamental difference in how objects move in any direction. There exists no preferred direction. The same amount of applied force produces the same motion no matter which direction the motion occurs in. If there were a favored composite dimension, then certain forms of motion would be easier to undergo than others.

Also following from the equality of all configuration dimensions, they all process energy the same way. X_1 interacts with energy by distorting in the same manner as Y_1, or Z_1, or any of the other configuration dimensions for that matter. If this were not true, then energy would not be conserved uniformly across all three space-time dimensions.

The reason configuration dimensions arrange themselves orthogonally with respect to each other stems from the fact that this arrangement represents geometrically the minimum energy configuration whilst at the same time ensuring X_1, X_2, X_3, Y_1, Y_2, Y_3, Z_1, Z_2, Z_3 are all equally treated. So, it seems that 90 degrees is the magic number that also represents the lowest energy state: non-interacting configuration dimensions are perpendicular with respect to each other. Deviation only occurs through mass-energy interactions with external dimensional domains and is represented by force-energy interactions. This fact defines why there are three composite spatial dimensions, as well as why one-dimensional motion can only have three forms. If this weren't the case, even large systems of particles would be able to enjoy the maximum degrees of motional freedom nine dimensions would allow. Everything would be a quantum particle.

To sum up: Although there are nine individual configuration dimensions, the only way they can equally arrange themselves into stable structure is within a 3 by 3 matrix – three composite spatial dimensions each composed of three internally orthogonal configuration dimensions. The emergent structure we perceive as three-dimensional reality is because this stability is in turn replicated by each of the three macroscopic composite spatial dimensions that we call X, Y, and Z.

If three-dimensional reality is truly an emergent pattern, then like any other emergent pattern it ought to be possible to directly observe the structures that make it up, which are the configuration dimensions themselves. For example, in the case of the fractal-ice pattern that forms on your window in the winter, it is possible to observe, at least in principle, the ice crystals that create it. So, how do we observe configuration dimensions? Since these structures exist in nine-dimensional configuration space, we can only observe them by noticing the effects they have on mass and energy in our three-dimensional world. I have already discussed some examples earlier that offer a good example of what I am talking about. Also, it is helpful to remember that a dimension is simply a way something can move – a degree of freedom.

To further clarify the point, let us return to the example for a single quantum particle, say, the electron. A single electron behaves as a quantum particle, and is seemingly capable of performing strange behaviors, such as quantum jumping, and tunneling, to name a few. Composite dimensionality is the filter through which to see what is really happening - not to the electron, but to the space the electron is in. A single electron is not acting in three composite dimensions, but in nine very tenuously connected configuration dimensions. Therefor it has many more degrees of freedom through which to act. The electron is getting blamed for how it acts as a 'quantum particle' when in fact it is just how any particle would appear when it is living in a nine-dimensional world and we can only perceive its motion through the distorting lens of three macroscopic spatiotemporal composite dimensions.

When the electron is isolated from any outside interaction, it is free to express itself in nine-dimensional space. It has maximal degrees of dimensional freedom at this point. If another electron - or any other particle- is brought into this isolated system such that the two particles interact, the behavior of both the particles will be less ‘quantum’, and more (but not very) ‘classical’. This is because, as each particle’s configuration domain begins to exchange information with the other, ways of motion become disallowed to preserve energy conservation efficiency within the system. This effectively restricts the degrees of freedom each particle can express motion in. As the number of particles increases, the number of dimensional degrees of freedom gradually reduces to a lowest common factor, and the particle system goes from a more quantum behavior to a more, and eventually almost totally, classical behavior. Therefore, the chaos of quantum action is limited mainly to the micro world. Quantum action is naturally self-limiting because on the level of the macro world, there are simply too many interacting particles forcing each other to express motion only in conservative, classically dynamic ways.

Another way to say the same thing is that classical motion is a default position which happens when a domain of information sharing particles gets too complex to express their natural, nine dimensional behaviors without violating energy conservation principles.

An interesting outcome of thinking of particle behavior in terms of composite dimensionality is to understand that the more isolated a particle is from its environment the more quantum-mechanical its behavior will be. Can the same thing be said for large aggregations of particles? Can there be any way to isolate a macroscopically large sample of particles in such a way to bring about quantum mechanical behavior? There is nothing in the hypothesis to prevent this from happening. Under special circumstances even normal macroscopic objects composed of large numbers of atoms can act in 'weird' ways because of this fact. Super cooled material such as liquid helium, for example, can behave as if it were a quantum object - if it is sufficiently isolated from the external environment. It is because the normal three space dimensions 'decouple' into their fundamental components to an extent. The common way to make this happen is to lower the temperature towards absolute zero - but it is not the only way. Other forms of isolation can lead to ever more startling behavioral modes for matter to express itself in. High temperature superconductivity and the magnetic properties of neodymium – iron – boron super magnets come firmly to mind. Doubtless, there will be other, even more startling quantum behaviors that will be discovered in the future as new and novel ways of isolating systems of particles are discovered.

Further Foundational Applications

The value of a new theory is in what it can tell us about the world we live in. So, we will now look at what new information Composite Dimensionality can tell us about the universe we live in.

The specific topic I want to look at for this discussion is why energy follows an inverse-square relationship. From direct measurement it can be shown that when a conserved quantity – electromagnetic energy such as light for example, is being radiated equally in all directions from a point source, its intensity will be inversely proportional to the square of the distance from the point source to the measuring point. This $\frac{1}{r^2}$ relationship specifically arises from the fact that the area of a sphere, $4\pi r^2$, is proportional to the square of the radius, for a sphere in three-dimensional space.

Before we look at how Composite Dimensionality treats this topic it will first be necessary to introduce some terms. As you recall, each spatial dimension is a composite structure made up of three fundamental dimensions. These fundamental structures are what allow different forms of one-dimensional motion to occur. They are not in and of themselves spatial dimensions but rather degrees of freedom within a single dimension, so it makes better sense to call them something else. I have chosen to call these fundamental structures ***compositers***. The word compositer being an amalgam of the word composite along with the suffix **TER** which is Latin for 'times three'. So, using this new term we can say that a single spatial dimension is composed of three compositers. Our three-dimensional universe is composed of nine compositers total.

I now want to show how Composite Dimensionality can be used to derive the inverse square law for our three-dimensional universe, using non-geometric means. This derivation will show how the nature and characteristic of energy within a universe-domain follow directly from the number of compositers comprising a single spatial dimension along with the total number of existing spatial dimensions. This process can be generalized for any number of compositers and dimensions in any n- dimensional domain.

Our Universe – the Domain that Matters

<u>*At first glance*</u>

The energy relationship for our universe is to multiply the total number of compositers in a single spatial dimension by the total number of dimensions there are to give to total number of degrees of freedom in the universe. As previously stated, our universe is composed of a total of 9 compositers i.e. that there are 9 degrees of linear motional freedom available in the universe (3x +3y+3z). This reduces to the square of the total number of compositers in a single spatial dimension. Therefor the exponent '2' represents the energy expression in the relationship $1/r^2$. This simplified method assumes there are equal numbers of compositers in each spatial dimension.

The General Equation

To derive a mathematical expression that will yield the exponent of the power law, start with the generalized relationship:

$\boldsymbol{X^y = Z}$ where X = the number of compositers per dimension, Z=the total number of compositers in all dimensions, and y being the variable of the force law that equates them. Thus, y=$\frac{\log z}{\log x}$ and the generalized form of the force law becomes:

General equation: $\frac{1}{r^{\left[\frac{\log z}{\log x}\right]}}$

This expression is the <u>general equation</u> that describes the relationship between dimensionality and energy propagation for any domain. However, it is not the final arbiter for defining a conservative domain.

A useful way of thinking about whether a particular universe is conservative is that the general equation shows how a quantity, such as an electromagnetic or gravitational field, for example, wants to propagate in a domain. The question now becomes can it actually propagate in this manner or are there other factors to consider? To answer this the geometry of permitted shapes also needs to be looked at. The following 3 examples demonstrate this idea.

Three examples for a 3-dimensional domain

Surface area of a sphere in 3 dimensions: $A = 4\pi r^2$

<u>Case 1: Our universe - 3 compositers per dimension</u>

3+3+3=9 total compositers, and 3 squared is 9. More rigorously, using the <u>*General equation*</u> for the case of a 3-dimensional domain where x=3, z=9, y=$\frac{\log 9}{\log 3}$ = 2, the energy conservation rule for our 3-dimensional universal domain is $\frac{1}{r^{\left[\frac{\log z}{\log x}\right]}}$, where $\frac{1}{r^{\left[\frac{\log 9}{\log 3}\right]}}$ = $\frac{1}{r^2}$ The exponent for r matches in both the general equation and the geometrical expression thus our universe is conservative.

Does this mean that any 3-dimensional domain will always be conservative? What happens if we add or take away compositers within a dimension? The equation for the surface area of a sphere remains unchanged, but what about the energy expression derived from the General Equation?

Case 2: More degrees of freedom – 3-dimensional domain with 4 compositers per each dimension

What if there were a 4th degree of linear motional freedom for each dimension i.e., 4 compositers per dimension? The total number of dimensions remains unchanged, so the surface area of a sphere also remains unchanged, meaning the expression $S = 4\pi r^2$ still holds true.

At a glance we see 4 compositers define each of the three spatial dimensions: 4x3=12 total compositers, but 4 squared is 16, and 12 ≠ 16. So, right off, we see this is not a conservative domain.

To determine the numerical value of the exponent y in the relationship $\frac{1}{r^y}$, plug the numbers into the General Equation above to derive the exponent of r:

y=$\frac{\log z}{\log x}$ if x=4, z= (4+4+4) =12, then y=$\frac{\log 12}{\log 4}$ = 1.79

thus, the equation that determines energy flow in a 3-dimensional domain with a single extra linear degree of motional freedom is $\frac{1}{r^{1.79}}$, while the geometrical expression is still $\frac{1}{r^2}$.

This shows that a 3-dimensional universe with 4 modes of linear motion within a single spatiotemporal dimension would lose energy too slowly with respect to distance. This is another example of a 3-dimensional non-conserved universe.

Case 3: Less degrees of freedom - 2 compositers

What about a 3-dimensional domain that offers only 2 modes of linear motion i.e., 2 compositers per dimension?

Again, 2x3=6 total compositers, and 2 squared is 4, and 6≠4. This is also not a conservative domain.

y=$\frac{\log z}{\log x}$ if x=2, z= (2+2+2) =6, y=$\frac{\log 6}{\log 2}$ =2.584

thus, the equation that determines energy flow in a 3-dimensional universe with a one less compositer per spatial dimension is $\frac{1}{r^{2.58}}$

This shows that a 3-dimensional universe with 2 modes of linear motion within a single spatiotemporal dimensional would too quickly lose energy with respect to distance. This 3-dimensional domain is also not a conserved domain.

Each of the domains in the preceding 3 cases were 3-dimensional spaces using the same geometrical equations for spherical surface area, but even so, only the universe-domain in case 1, where there are 3 compositers, or degrees of linear freedom per dimension, is able to experience energy conservation as we currently observe it. This is the universe we find ourselves inhabiting.

It is possible using Composite Dimensionality to show for any n-dimensional domain, if there can exist a stable solution where energy flow is conserved, and the equals sign reigns supreme.

The previous examples can be applied as a powerful general rule to define the stability of any n-dimensional domain:

Generalized Rule:

As long as the solution to the General Equation matches the geometrical expression governing the increase in surface area for that domain, it will be a conservative domain.

If a domain has too many degrees of freedom per dimension, there will not be enough energy per each compositer to support conservative motion.

If a domain has too few degrees of freedom per dimension, there will be too much energy per each compositer to support conservative motion.

To reiterate, this same analysis can be done for any arbitrary combination of compositers and dimensions. For example, a 4-dimensional domain (or any other n-dimensional domain):

In the case of a 4-dimensional hypersphere, the area increases by the equation, $\boldsymbol{A = 2\pi^2\, r^3}$

In general, for any hypersphere of dimension N, r will be of the form r^{N-1}, so for any 4-D space to be considered conservative, the output from the General Equation must = 3.

Case 4 - 4-dimensional domain with 4 compositers per dimension:

Total number of compositers per dimension = 4, and, total number of compositers = 4+4+4+4 = 16. Plug these numbers into the general equation $y=\frac{\log z}{\log x}$. So, $y=\frac{\log 16}{\log 4}$ = 2. i.e., $4^2 = 16$.

The geometry of a 4=d space demands the exponent be 3.

This shows that an inverse square law for energy propagation does not lead to energy being conserved in a 4-d space with 4 compositers per dimension.

Case 5 – 4-dimensional domain with 3 compositers per dimension:

Total number of compositers per dimension = 3, and, total number of compositers = 3+3+3+3 = 12. Plug these numbers into the general equation $y=\frac{\log z}{\log x}$. Utilizing the general equation:

$\frac{1}{r^{\left[\frac{\log z}{\log x}\right]}}$, where $\frac{1}{r^{\left[\frac{\log 12}{\log 3}\right]}} = \frac{1}{r^{2.26}}$

Again, this is a non-conserved domain.

Case 6 - 4-dimensional domain with 5 compositers per dimension:

Total number of compositers per dimension = 5, and, total number of compositers = 5+5+5+5 = 20. Plug these numbers into the general equation $y=\frac{\log z}{\log x}$. So, $y=\frac{\log 20}{\log 5}$ =1.86. Utilizing the general equation:

$\frac{1}{r^{\left[\frac{\log z}{\log x}\right]}}$, where $\frac{1}{r^{\left[\frac{\log 20}{\log 5}\right]}} = \frac{1}{r^{1.86}}$ again, a non-conserved domain.

<u>Case 7 – 4-dimensional domain with 2 compositers per dimension:</u>

Total number of compositers per dimension = 2, and, total number of compositers = 2+2+2+2 = 8. Plug these numbers into the general equation y=$\frac{\log z}{\log x}$. So, y=$\frac{\log 8}{\log 2}$ = 3. Utilizing the general equation:

$\frac{1}{r^{\left[\frac{\log z}{\log x}\right]}}$, where $\frac{1}{r^{\left[\frac{\log 8}{\log 2}\right]}} = \frac{1}{r^3}$

This example shows a match between the geometry and the general equation for a 4-dimensional domain, indicating the if each of the 4 dimensions has 2 degrees of freedom, this universe will indeed be a conserved domain following an inverse cube law.

As you can see, an analysis of energy flow using composite dimensionality, as opposed to historical techniques, yields a more complete picture of dynamical energy processes. The main reason historical analysis yielded misleading results was because researchers assumed the dimensions in a 4-dimensional universe (or any other universe) were the same kind of dimensions, with the same motional degrees of freedom, as can be found in our own universe. Indeed, if a 4-dimensional universe only had the same degrees of freedom as are found in our 3-dimensional universe, 3 compositers per dimension, it would not be a conservative place. As I have demonstrated, to be conservative a 4-dimensional universe would need to follow an inverse cube law as dictated by the particular geometry operating within such a domain, as well as being able to construct such a combination of compositers that the output of the General Equation also yields a cubic exponent. Indeed, the same general rule we derived for our universe carries to **any** n-dimensional universe. Thus, the general rule can be restated as an axiom:

Axiom 1:

<u>As long as the solution to the General Equation matches the geometrical expression governing the increase in surface area for that domain, it will be a conservative domain irrespective of how many spatial dimensions it may have.</u>

And, as well, it is now possible to derive the particular force law any theoretical universe would operate under, knowing only the total number of compositers and how they are arranged-whether conservative or not. Of course, though this preceding discourse is certainly interesting, it does not establish the reality of such alien domains, whether conservative or not, only how they might behave should they exist.

Another interesting fact is the historical link between energy conservation and $\frac{1}{r^2}$ is not necessarily always correct. In any universe, energy conservation simply means that total potential is always conserved – that energy is neither created nor destroyed in any process. Composite dimensionality shows that how energy flows (i.e., its force law) in any domain involves the interplay of the dimensions that make up a domain and the compositers that make up its dimensions. For this interplay to generate conservative forces, the geometry of said domain must also be considered. Energy conservation is a property of compositers working with geometry. An interesting point to make is that although I have begun only a preliminary investigation searching for salient examples of N-Dimensional domains where energy flows in a conserved manner, I have found only two examples of energy-stable domains. Our own habitat of 3 dimensions and 3 compositers per dimension, operating under the inverse square law, and the four-dimensional example having 2 compositers per dimension, operating under an inverse cube law.

The equivalence principle and energy conservation

In light of the above discussion, another interesting point is what Composite Dimensionality tells us about the equivalence principle, and why inertial and gravitational masses are equivalent. As I will show in this section, unequal inertial and gravitational masses would imply global energy conservation violation. Axiom 1 tells us that the analytical results of the General Equation must match the geometrical expression governing the increase in surface area for any domain. In our domain, the analytical results govern how energy flows, how it fills space, and it varies as the inverse square of distance. This describes the energy side of the energy-mass relationship of matter – this is inertial mass. Inertial mass is how an object resists a change in its state of motion which can be dealt with using dimensional analysis, while gravitational mass is wholly defined by the force of gravity. Gravity is a measure of how much acceleration is caused by the same objects ability to warp space, which is a problem in geometry. Of course, the force of gravity is also an inverse square force, but this time the exponent is given by geometric reasoning. Gravitational solutions are derived via geometric analysis (and physical experimentation) and inertial solutions are derived through algebraic reasoning via the General Equation. Both methods derive the same answer. The equivalence principle is only valid because we live in a universe where energy is a conserved quantity.

It is possible using the concepts presented within the theory of Composite Dimensionality to obtain a basic understanding of the relationship between dimensionality, energy, and geometry for any number of theoretical alien domains, and by extension, to begin to synthesize potential physics frameworks functioning within such domains.

It was William Shakespeare who said in his poem, As You Like It, “All the world's a stage, / And all the men and women merely players; / They have their exits and their entrances / And one man in his time plays many parts,” (II.VII.1-4). In a very real way, our world is much like a stage which is set up to show the results of things, not how those results were achieved. Taking all this into consideration, when you view a common garden snail crawling leisurely on the leaf of a plant, know that what you perceive as the snail’s movement is in fact the result of a process that takes place outside of your perception, faster than the speed of light, in a nine dimensional world you can have no direct knowledge of due to the fact that the energy you and I are created from is a three dimensional phenomena, which is itself an emergent phenomena brought about by factors that are normally invisible to our perception.   It is, however, a grand effort we undertake to try to understand reality in all its many, perhaps infinite, forms...

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