A very interesting formula of a subset of Poulet numbers involving consecutive powers of a power of two

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Abstract. I studied Poulet numbers for some time but I'm still amazed by the wealth of the patterns that this set of numbers offers; it's like everything that the prime numbers, in their stubbornly to let themselves understood and disciplined, refuse us, these exceptions of the Fermat's "little theorem" allow us. This paper states a conjecture about a new subset of Poulet numbers that I discovered by chance.

Conjecture:

There is an infinity of Poulet numbers P of the form P = ((2^n)^k)*((2^n)^(k + 1) + 2^n + 1) + 1, where k is a non-null positive integer, for any n non-null positive integer.

Examples:

For n = 1 the formula becomes P = 2^k*(2^(k + 1) + 3) + 1 and we obtained:

: 2^4*(2^5 + 3) + 1 = 561 = 3*11*17, a Poulet number;
: 2^7*(2^8 + 3) + 1 = 33153 = 3*43*257, a Poulet number.

For n = 2 the formula becomes P = 4^k*(4^(k + 1) + 5) + 1 and we obtained:

: 4^2*(4^3 + 5) + 1 = 1105 = 5*13*17, a Poulet number;
: 4^3*(4^4 + 5) + 1 = 16705 = 5*13*257, a Poulet number.

For n = 4 the formula becomes P = 16^k*(16^(k + 1) + 17) + 1 and we obtained:

: 16^1*(16^2 + 17) + 1 = 4369 = 17*257, a Poulet number;
: 16^2*(16^3 + 17) + 1 = 1052929 = 17*241*257, a Poulet number;
: 16^3*(16^4 + 17) + 1 = 268505089 = 17*241*65537, a Poulet number.

For n = 6 the formula becomes P = 64^k*(64^(k + 1) + 65) + 1 and we obtained:

: 64^1*(64^2 + 65) + 1 = 266305 = 5*13*17*241, a Poulet number.

For n = 7 the formula becomes P = 128^k*(128^(k + 1) + 129) + 1 and we obtained:

: 128^1*(128^2 + 129) + 1 = 2113665 = 3*5*29*43*113, a Poulet number.

For n = 8 the formula becomes P = 256^k*(256^(k + 1) + 257) + 1 and we obtained:

: 256^1*(256^2 + 257) + 1 = 16843009 = 257*65537, a Poulet number.