On Bell's Inequality

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(Dated: April 27, 2014)

We show that when spin eigenfunctions are not fully orthonormal, Bell's inequality does allow local hidden variables. In the limit where spin eigenfunctions are Dirac orthonormal, we recover a significant extremal case. The new calculation gives a possible accounting for $\alpha_{\rm MCM} - \alpha_{\rm QED}$.

As it has been understood, Bell's inequality rules out the new variable proposed in the MCM. No analytic form has been found for the eigenfunctions of the spin operator but it is assumed they are orthonormal. In this short paper we examine the case when spin eigenfunctions are not orthonormal [1]. Derivation of Bell's inequality often starts with a statement of the average value of the product of the spins when the detectors are aligned along spatial unit vectors $\vec{a}$ and $\vec{b}$ and θ is the angle between them [2].

$$P(\vec{a},\vec{b}) = -\vec{a}\cdot\vec{b} = -\cos(\theta) \tag{1}$$

This is derived by taking the expectation value of the product of two spins in a singlet state. Moving directly to the end of that calcluation we find the following.

$$\begin{aligned} P(\vec{a},\vec{b}) \;&=\; \frac{\sin(\theta)}{\sqrt{2}}\langle 0\,0|1\;-1\rangle - \qquad (2)\\ &-\;\cos(\theta)\langle 0\,0|0\,0\rangle + \frac{\sin(\theta)}{\sqrt{2}}\langle 0\,0|1\,1\rangle \end{aligned}$$

When spin states are othogonal, equation (2) reduces to equation (1). When they are not orthogonal, the $\sin(\theta)$ terms do not go to zero. Let the magnetic quantum number distinguish $\delta_\pm$ [1].

$$P(\vec{a},\vec{b}) = \delta_- - \vec{a}\cdot\vec{b} + \delta_+ \tag{3}$$

Bell's inequality is derived from the difference between $P(\vec{a},\vec{b})$ and $P(\vec{a},\vec{c})$. Using the normal prescription [2] that $-\vec{a}\cdot\vec{b} = A(\vec{a})A(\vec{b})$ and moving to the hidden variable formalism, we may write the following.

$$\begin{aligned} P(\vec{a},\vec{b}) &\;- P(\vec{a},\vec{c}) = \qquad (4)\\ =&\;\int [1 - A(\vec{b},\lambda)A(\vec{c},\lambda)]A(\vec{a},\lambda)A(\vec{b},\lambda)\rho(\lambda)\,d\lambda\\ +&\;\int (\delta_-^{ab} - \delta_-^{ac})\rho(\lambda)\,d\lambda + \int (\delta_+^{ab} - \delta_+^{ac})\rho(\lambda)\,d\lambda \end{aligned}$$

The system in question decays to two particles so it is not possible to directly test the theory's prediction for three different detector alignments $\{\vec{a},\vec{b},\vec{c}\}$. The experimenter would have to perform a test in one apparatus configuration $\{\vec{a},\vec{b}\}$, then reconfigure the table for $\{\vec{a},\vec{c}\}$ and take more data at some later time. In the process of reconfiguring, the observer moves to a different level of $\aleph$ so $\delta^{ab} \neq \delta^{ac}$ [1]. The delta resultant from the earlier measurement is infinitely smaller than the later one and can safely be ignored. When the deltas are integrated as the Dirac delta, we find the extremal case in which local hidden variables are always allowed.

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 3 + P(\vec{b},\vec{c}) \tag{5}$$

There is an asymmetry in equation (4) that will ruin equation (5) when the surviving deltas are negative. Hence, we require that $-\delta_\pm = \delta_\mp$. Since the deltas are the source of space and time [1], and $\{-+++\}$ spacetime is indistinguishable from $\{+---\}$ spacetime, it seems this requirement can be accomodated.

Now consider the case when $\delta_\pm$ are integrated according to the prescription in reference [1].

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 2\pi + (\Phi\pi)^3 + 1 + P(\vec{b},\vec{c}) \tag{6}$$

The definition $\alpha = 2\pi + (\Phi\pi)^3 + 1 + P(\vec{b},\vec{c})$ both allows and tightly constrains a varying fine structure constant. This also presents a possible accounting for the small discrepancy between the predicted value $\alpha_{\rm MCM}$ and the empirically determined one $\alpha_{\rm QED}$. Taking $\alpha_{\rm MCM}$ as $2\pi + (\Phi\pi)^3 + 1$ we arrive at the following.

$$\alpha_{\rm MCM} - \alpha_{\rm QED} = P(\vec{b},\vec{c}) \approx 1.59 \tag{7}$$

Obviously 1.59 cannot arise in the product of two unit vectors so we may assume a form $\vec{b}\cdot\vec{c} = \hat{e}_1 \cdot \Phi\hat{e}_2$ giving $\theta \approx 11.5°$. While that angle doesn't ring any Bell's, this formulation is in good agreement with the idea that the fine structure constant is a feature of the geometry between adjacent moments where reality in the later moment is scaled by Φ [1, 3].

[1] J. Tooker, viXra:1312.0168 (2013)
[2] D.J. Griffiths, *Introduction to Quantum Mechanics*, 2 Ed, Ch 12 (2004)
[3] J. Tooker, viXra:1209.0010 (2012)