HAMILTON'S PRINCIPLE AND THE SCHWARZSCHILD METRIC

DOUGLAS L. WELLER

Abstract. The Schwarzschild metric is rearranged to manifest inherent limitations based on the conservation of energy. These limitations indicate that a collapsing surface will not compact below a critical radius to form a black hole.

1. Introduction

A significant difference between physics and abstract math is the requirement that physics is anchored in physical reality, both in its starting principles and its predictions.

For discussions of gravity, the anchor to reality has preeminently been Albert Einstein's theory of general relativity and particularly his field equations. These equations are built on a very few basic assumptions about reality. Most notable of these is the fundamental concept that the laws of nature, particularly the conservation of momentum and energy, are valid irrespective of the motion of the reference system from which they are measured[1].

This paper explores the conservation of energy as expressed in Schwarzschild metric. The Schwarzschild metric has been chosen as an explanatory vehicle because it is concrete, it has very few terms and as a solution to the field equations it must express the conservation of energy in the manner circumscribed by the field equations.

2. Gravity and the Conservation of Energy

How is energy conserved in a gravitational field? In Einstein's defining works on gravity, he explicitly or implicitly invoked what he called the Hamilton Principle to explain conservation of energy[1, 2, 3]. According to Einstein's version of the Hamilton Principle, any change in energy that results from changing position in a gravity field is balanced by an equal and opposite change in the energy of matter (or in the energy of light if light is being observed). As he explicitly sets out in his 1916 paper, *Hamilton's Principle and the General Theory of Relativity*[4],

$$\frac{\delta}{\delta x_v}(F_\sigma^v + t_\sigma^v) = 0, \tag{1}$$

where F_σ^v represents the components of the energy of matter and t_σ^v represents the components of the energy of the gravitation field[4, p. 172]. Einstein included the electromagnetic field in his definition of matter[4, p. 167].

Date: Dec. 16, 2013.

The energy equivalence of matter apart from a gravitation field was derived by Einstein a few years earlier [5] and is given by the well known

$$E = mc^2. \tag{2}$$

It may be understood from (1) and (2) that for a particle of mass m (hereinafter particle m), its total energy equivalence mc^2 remains constant; however, when particle m enters a gravitational field, its energy equivalence is apportioned into components representing the energy of matter and representing the energy of the gravitation field.

The remainder of this paper explores how this is expressed in the Schwarzschild metric and the implications of this for explaining and predicting physical phenomena and in particular, black holes.

3. Verifying Conservation of Energy in the Schwarzschild metric

Shortly after Einstein's published his field equations, a German military officer, Karl Schwarzschild published a solution of the field equations for a gravitational field outside a spherical non-rotating mass[6]. In this section, the terms of the Schwarzschild metric are rearranged to make it easy and intuitive to verify that the Schwarzschild metric complies with (1).

3.1. **Rearranging the Schwarzschild metric.** The Schwarzschild metric describes a gravitational field outside a spherical non-rotating mass. For a compact mass M with a Schwarzschild radius R, the Schwarzschild metric is often expressed using reference space coordinates (r, θ, ϕ), coordinate time t and local time τ as

$$c^2 d\tau^2 = c^2(1 - \frac{R}{r})dt^2 - \frac{dr^2}{(1 - R/r)} - r^2 d\theta^2 - (rsin\theta)^2 d\phi^2. \tag{3}$$

In order to verify that (3) complies with (1), the terms of (3) can be rearranged as follows.

Begin by multiplying both sides of (3) by $\left(\frac{1}{dt}\right)^2$ yielding

$$c^2\left(\frac{d\tau}{dt}\right)^2 = c^2\left(1 - \frac{R}{r}\right)\left(\frac{dt}{dt}\right)^2 - \frac{1}{1 - R/r}\left(\frac{dr}{dt}\right)^2 - r^2\left(\frac{d\theta}{dt}\right)^2 - (rsin\theta)^2\left(\frac{d\phi}{dt}\right)^2. \tag{4}$$

The terms of (4) can be rearranged as

$$c^2 = c^2\left(\frac{d\tau}{dt}\right)^2 + c^2\frac{R}{r} + \frac{1}{1 - R/r}\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 + (rsin\theta)^2\left(\frac{d\phi}{dt}\right)^2. \tag{5}$$

To simplify interpretation, a velocity v through the three dimensions of curved space as measured using time coordinate t can be defined as

$$v = \sqrt{\frac{1}{1 - R/r}\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 + (rsin\theta)^2\left(\frac{d\phi}{dt}\right)^2}. \tag{6}$$

Using the definition in (6), (5) reduces to

$$c^2 = c^2 \left(\frac{d\tau}{dt}\right)^2 + c^2 \frac{R}{r} + v^2. \tag{7}$$

To further simplify comparison, a particle m is allowed to enter the gravitation field defined by Schwarzschild metric. Based on (7) the energy equivalence of particle m is apportioned as

$$mc^2 = mc^2 \left(\frac{d\tau}{dt}\right)^2 + mc^2 \frac{R}{r} + mv^2. \tag{8}$$

The Schwarzschild metric as expressed in (8) is mathematically equivalent to (3). The arrangement shown in (8) simply makes clear how the terms in the Schwarzschild metric represent concrete physical phenomena. Each of these terms is now individually considered.

3.2. **Energy equivalence.** The energy equivalence, mc^2 represents the energy equivalence of particle m. The gravity field around mass M does not change the total energy of particle m, but only apportions it into different energy components, affecting the behavior of particle m in the gravity field around mass M.

3.3. **The energy of motion component.** The energy of motion component mv^2 represents the effect of motion through space. In general relativity all motion is with respect to a reference point. The coordinates indicate the reference point from which measurements are made. In (8), the Schwarzschild metric is expressed using reference space coordinates (r, θ, ϕ). The origin of the reference space coordinates (r, θ, ϕ) is at $(0, 0, 0)$, the center of mass M. This indicates that the motion of particle m is measured with respect to the center of the mass M.

The energy equivalence mc^2 of particle m is the upper limit on the value of mv^2. That is, mv^2 cannot exceed mc^2 without violating (8) and thus the conservation of energy. This, therefore, makes the speed of light c the upper limit on the magnitude of v. This is the upper boundary for speed predicted by Einstein's work in special relativity.

3.4. **The energy of gravity component.** The energy of gravity component $mc^2\frac{R}{r}$ represents the effect of gravity. In this gravity component, the coordinate r represents radial distance of particle m from the center of mass M. The energy of gravity component $mc^2\frac{R}{r}$ increases as radial distance r decreases.

The energy equivalence mc^2 of particle m is the upper limit on the value of $mc^2\frac{R}{r}$. That is, $mc^2\frac{R}{r}$ cannot exceed mc^2 without violating (8) and thus the conservation of energy. This, therefore, makes the Schwarzschild radius R the minimum value for radial distance r.

While the wide acceptance of special relativity makes the upper boundary of v in accordance with our intuition, this is not necessarily the case for the minimum value of r. So let's pause to consider the source of this limitation.

When Karl Schwarzschild solved Einstein's field equations, he needed to represent the energy of gravity in the gravity field. He did this using a constant [6]. As a result, Isaac Newton's gravitational constant G appears in the Schwarzschild metric. While it is not shown directly in (3), it is there because it is used in the calculation of the Schwarzschild radius R. Specifically,

$$R = \frac{2GM}{c^2}. \tag{9}$$

The Schwarzschild radius R is the critical radius for forming a black hole because it is the radial location where Newtonian escape velocity v_g is equal to the speed of light c. That is,

$$v_g = c\sqrt{\frac{R}{r}}. \tag{10}$$

The Schwarzschild metric assumes that Isaac Newton's gravitational constant G does not vary, but remains constant while r varies from $r = \infty$ to $r = 0$. *This assumption plus the requirement that energy is conserved inevitably results in the boundary of minimal radial distance* $r = R$ in the Schwarzschild metric.

If Karl Schwarzschild's assumption is incorrect and, for example, at very small values of r Newton's gravitational constant is not an accurate representation of the physical properties of gravity, this would change the value for r where $v_g = c$, and thus the critical radius for forming a black hole. However, any radial location where v_g is greater than c will be a violation of general relativity [7].

3.5. **The energy of matter component.** The energy of matter component $mc^2\left(\frac{d\tau}{dt}\right)^2$ shows how motion and gravity affect the passage of time. When at rest and unaffected by gravity (i.e. $v = 0$ and $r = \infty$), then $\frac{d\tau}{dt} = 1$ and $mc^2\left(\frac{d\tau}{dt}\right)^2 = mc^2$. When the energy of the gravitation field increases by an increase in v or $\frac{R}{r}$, $\frac{d\tau}{dt}$ decreases with a corresponding decrease in $mc^2\left(\frac{d\tau}{dt}\right)^2$.

$\frac{d\tau}{dt}$ represents time dilation, i.e.,the difference in the clocks that measure time in the two coordinate systems represented in the Schwarzschild metric. The clock in each coordinate system is the frequency of light for that coordinate system [2]. So the term $mc^2\left(\frac{d\tau}{dt}\right)^2$ instructs us that as frequency, and thus the speed of light, slows in a coordinate system, so also, in exact correspondence, does the energy of matter component.

4. Enforcement of Limits in the Schwarzschild metric

The limit in (8) that $mv^2 + mc^2\frac{R}{r}$ cannot be greater than mc^2 is enforced by the time dilation inherent in $mc^2\left(\frac{d\tau}{dt}\right)^2$. This is well understood for velocity. No matter how much particle m is accelerated, it will never reach its top theoretical speed of c because time dilation in combination with length contraction (which can also be calculated using the Schwarzschild metric), will keep velocity c always out of reach of particle m.

This is also true for any attempt of particle m to cross the Schwarzschild radius R. No matter how much particle m is accelerated by gravity, it will never reach the Schwarzschild radius R because time dilation in combination with length contraction, will keep Schwarzschild radius R always out of reach of particle m. This is demonstrated in the next section where the journey of a free falling particle is considered.

5. Journey of a Free falling Particle

The journey of a free falling particle is significant for establishing the possibility of black holes and explaining their operation. Absent the existence of a black hole, a particle's journey to mass M will end in a collision with the surface of mass M or some intermediary. This section ignores the possibility of a collision and considers the hypothetical case of mass M being compacted below the Schwarzschild radius R. This will allow examination as to whether formation of black holes is possible, as the free falling particle can be used to represent the last particle on the surface of a collapsing mass to cross R in order to form a black hole. If a free falling particle cannot cross R, then black holes will not form from collapsing stars.

Without performing a single calculation, it can be seen very clearly why within the Schwarzschild metric a free falling particle will never cross R. For example, from (7) it can be seen that there are no valid values for the Schwarzschild metric where $r < R$. According to (8), a free falling particle m that crosses R violates the conservation of energy and thus violates a fundamental tenet of general relativity, as described by (1).

As described above, the enforcement mechanism that prevents values where $r < R$ is the same as the one that prevents $v > c$. Time dilation (and length contraction) will make it impossible for R to be crossed. No matter how much particle m is accelerated, it cannot break the barrier that defends both $r = R$ and $v = c$.

Specifically, as particle m approaches the Schwarzschild radius R, the effects of the energy of the gravity field will slow the approach of particle m. The gravitational energy component $(mc^2 \frac{R}{r})$ shown in (8) will consume energy at the expense of the energy component of movement (mv^2) and the energy of component of matter $mc^2 \left(\frac{d\tau}{dt}\right)^2$. If particle m could, without obstruction, reach the Schwarzschild radius R, there would be no energy left to pass it. At the Schwarzschild radius R, $mv^2 = 0$ and $mc^2 \left(\frac{d\tau}{dt}\right)^2 = 0$. With no ability to go forward, particle m will remain suspended in place until some ending event occurs. The ending event could be evaporation of particle m, evaporation of mass M, or perhaps some apocalyptic event. Some ending event must occur if matter is not infinite. For example, the estimated half life of a proton is on the order of 10^{32} years and a black hole, if it were to exist, is estimated to have a finite lifetime of less than 10^{100} years, a mere blink of the eye on the way to infinity [8] .

5.1. **Calculation.** Calculations using the Schwarzschild verify the journey scenario given above. For a particle falling radially

$$d\theta = d\phi = 0, \tag{11}$$

and the Schwarzschild metric in (3) reduces to

$$c^2 d\tau^2 = c^2(1 - \frac{R}{r})dt^2 - \frac{dr^2}{(1 - R/r)}. \tag{12}$$

For every radial location r_i reached from a starting location r_s, the coordinate time t_i to reach radial location r_i can be calculated using an integral

$$t_i = \int_{r_s}^{r_i} dt = \int_{r_s}^{r_i} f_1(r)dr, \tag{13}$$

where $f_1(r)$is a function of r derived from (12) [9, p. 667].

When the radial location r_i is set equal to a critical radius r_c, the integrand $f_1(r)$ for the integral in (13) is undefined and the integral does not converge. This indicates particle m will remain outside the critical radius until it experiences an ending event occurring at a time t_e and at a radial location r_e. The critical radius r_c will be at or very near R, when measured from the perspective of a distant observer.

6. Making the Physically Impossible Look Mathematically Plausible

The discussion above shows that according to the Schwarzschild metric a free falling particle m cannot cross R and by logical implication black holes cannot form. How is it some assert the Schwarzschild metric allows for a free falling particle to cross R? In this section, the rationale for asserting particle m crosses R–and thereby indicating black holes can form–is considered. The well known textbook *Gravitation* [9] often referred to as MTW is used as the source of information for the rationale. The rationale is broken down into logical steps that are briefly recited and critiqued below. The critique demonstrates that the rationale for the formation of black holes relies on logical and mathematical fallacies [10].

6.1. **The apparent paradox of two different final destinations.** An apparent paradox is put forth. Specifically, it is asserted that calculations from the Schwarzschild metric show that using a different time coordinate results in a different final destination for a free falling particle.

Specifically, the journey toward the Schwarzschild metric in the prior section was timed by a distant observer. What happens when the journey is timed by the particle? When measured using the local time τ of particle m, the elapsed time τ_i required to reach a radial location r_i can be calculated using the integral

$$\tau_i = \int_{r_s}^{r_i} d\tau = \int_{r_s}^{r_i} f_2(r)dr, \tag{14}$$

where $f_2(r)$is a function of r derived from (12) [9, p. 663].

When the radial location r_i is set equal to the critical radius r_c, the integrand $f_2(r)$ is undefined; however, the integral in (14) converges. This indicates that it is mathematically

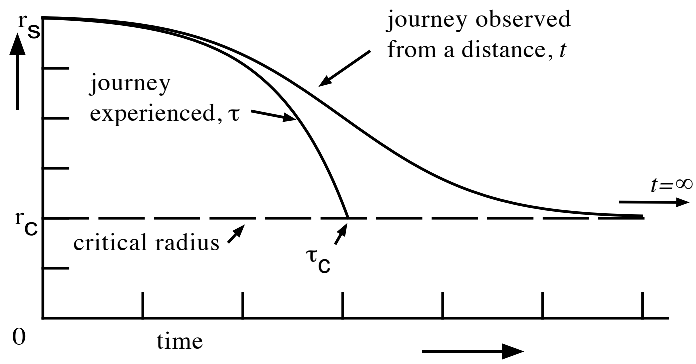

FIGURE 1. Apparent paradox. Calculations from the Schwarzschild metric seem to show different ending points for a journey to the critical radius.

possible to calculate a finite value for local time τ to reach r_c even though it is not possible to calculate a finite time using time coordinate t.

These results have been summarized as in Fig. 1 [9, p.667]. Fig. 1 gives the illusion that particle m can reach r_c in finite local time even though it cannot reach r_c in finite time when measured by a distant observer.

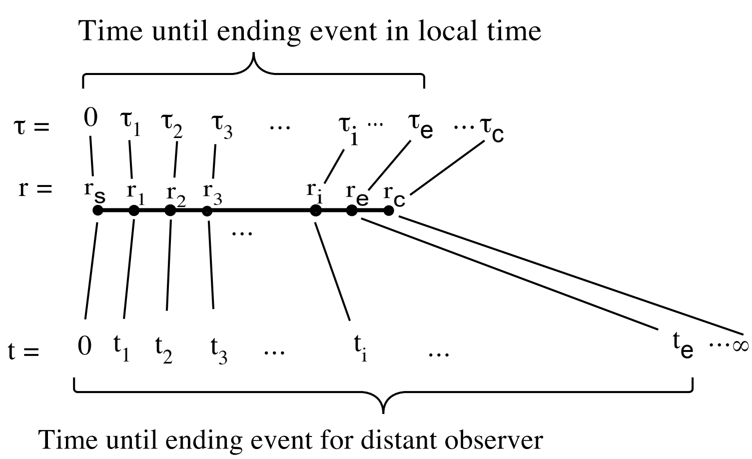

FIGURE 2. Solution to Paradox. The physical end of the journey is the same regardless of the coordinate used to measure time.

6.1.1. *Critiquing the apparent paradox.* The solution to the apparent paradox in Fig. 1 is to recognize that Fig. 1 does not show an ending event. Once the ending event is included, there is no difference in the logical sequence of the journey, regardless of the coordinate used to measure time.

To understand the location of ending event in time and space, consider the journey as graphed in Fig. 2. Fig. 2 shows the same data displayed in Fig. 1, but in a different format. Particle m starts at radial location r_s and progresses through radial locations, represented in Fig. 2 by r_1, r_2, r_3,...r_i,...and r_e. The elapsed time to make the journey to each radial location differs depending upon the location from which time is measured. When elapsed time is measured from the location of the distant observer, reaching r_i requires elapsed time t_i, reaching r_e requires elapsed time t_e, and r_c cannot be reached in finite time. Likewise, when elapsed time is measured from the perspective of particle m, reaching r_i requires elapsed time τ_i, reaching r_e requires elapsed time τ_e, and reaching r_c would require elapsed time τ_c.

While values for τ_c can be calculated, τ_c can never physically be reached because critical radius, r_c cannot be reached. The ending event (e.g., evaporation of M at t_e where $t_e < 10^{100}$ years) that occurs at r_e, t_e and τ_e occurs before particle m reaches r_c. The ending event occurs at r_e regardless of the coordinate used to measure time.

Fig. 3 shows Fig. 1 modified to include the ending event at r_e, t_e, and τ_e.

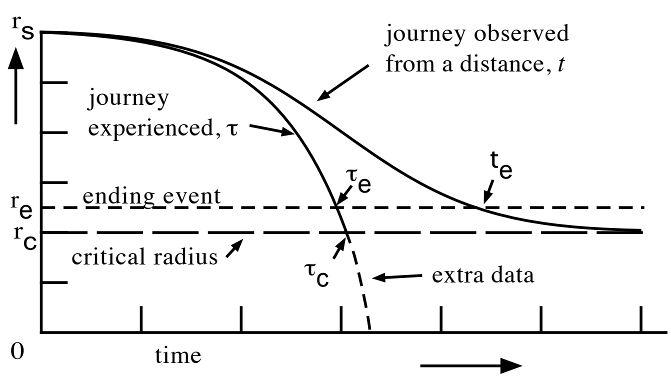

FIGURE 3. Paradox solved. The end of the journey is now shown.

6.2. **Generating data not valid within the Schwarzschild metric.** While the Schwarzschild metric can be used to generate values for elapsed time for free falling particle m to reach R, no data can be generated, at least using ordinary rules of mathematics, for any value of time τ after R is crossed. The reason for this is clear from (7). There are no valid values in the Schwarzschild metric where $r < R$. This is reflected in the integrals calculated in (13) and (14). In both these integrals the integrand at r_c is undefined making it impossible to generate values for values of r less then r_c.

Because ordinary rules of math cannot be used to generate these values, ad hoc rules of math are used. Particularly, a novel "cycloid principle" [9, See pp. 663-664] is used to generate this extra data. This extra data is represented in Fig. 3 by dashed lines.

6.2.1. *Critiquing data generation.* The extra data generated for locations where $r < R$ fall outside the boundaries of the Schwarzschild metric. This can be clearly seen in (7) and (8). As discussed above, the boundaries of the Schwarzschild metric set out in (8) are the result of the conservation of energy. The extra data, therefore, is invalid as violating the conservation of energy that is foundational to general relativity.

Further, merely showing how the extra data can be mathematically generated does not overcome the logical sequencing problem introduced by adding the extra data to Fig. 3. The extra data suggests r_c can be reached and crossed in local time τ_c. However, this is physically impossible because as shown in Fig. 2 and Fig. 3, the ending event that occurs in local time τ_e will occur before the critical radius r_c can be reached by particle m.

6.3. **Declaring coordinates to be pathological.** Once it is accepted that in local time τ particle m can traverse R, the logical inconsistency of not being able to reach R in coordinate time t needs to be addressed. This is addressed by declaring a pathology in the coordinates results in a singularity appearing at $r = R$ [9, pp. 820-823] .

6.3.1. *Critiquing coordinate pathologies.* The ad hoc assertion that coordinates are pathological ignores that the physical boundaries of (8) are imposed by the conservation of energy fundamental to the theory of general relativity. As discussed above, the singularities in the Schwarzschild metric protect these boundaries and are not the result of mathematically defective coordinates.

Further, declaring coordinates to be pathological strikes at another fundamental root of general relativity. According to general relativity, all coordinates (reference frames) will observe the same reality. As Einstein [1, p.117] made clear when setting out the basis for the theory of general relativity: "... all imaginable systems of coordinates, on principle, [are] equally suitable for the description of nature." The singularities at $r = R$ and $v = c$ exist not because of any pathology in the selected coordinates, but because there are physical limitations that result from the conservation of energy.

6.4. **Using special coordinates to show the particle crosses the Schwarzschild radius.** Having declared some coordinates to be "pathological", it is necessary to replace them with coordinates that give the desired results. Coordinates that are based on the reference frame of the free falling particle (e.g., the Novikov coordinates) and coordinates that are based on the reference frame of a radially traveling photon, (e.g., ingoing Eddington-Finkelstein coordinates and the Kruskal-Szekeres coordinates) have been used to "show" R can be reached and crossed [9, pp. 826-835].

6.4.1. *Critique of the use of special coordinates.* Regardless of the coordinates used in the Schwarzschild metric, R cannot be reached and crossed by particle m without violating the Schwarzschild metric and the conservation of energy. In general relativity, all coordinates will observe the same reality. Special coordinates such as Novikov coordinates, Eddington-Finkelstein coordinates and Kruskal-Szekeres only give the appearance that particle m reaches and crosses R by using the logical fallacy of begging the question, where the thing to be proved true is assumed to be true in a premise.

Specifically, the thing to be proved is that a free falling particle can reach and cross the critical radius. The premise is that the reference frame of special coordinates can reach and cross the critical radius. If it is assumed that a reference frame reaches and crosses R, then it can also be "shown" by measurements from that reference frame that particle m is able to reach and cross the critical radius.

But the premise is wrong. No reference frame can reach and cross R. Coordinates based on the free falling particle and coordinates using the reference frame of a radially traveling particle are considered separately in the following subsections. It is shown in these subsections that the reference frames for the special coordinates cannot cross R and so cannot be used to boot strap particle m across R.

6.4.2. *Coordinates based on the reference frame of the particle.* The Novikov coordinates utilize a reference frame that, while not identical to the reference frame of particle m, nevertheless uses the same time coordinate τ [9, p. 826]. This means the mapping of t to τ shown in Fig. 2 is valid for the Novikov coordinates as well as for particle m. The mapping shows that the Novikov coordinates cannot reach R in finite coordinate time t. The ending event that occurs at r_e, t_e, and τ_e will occur at τ_e as measured from the Novikov coordinates. This ending event (e.g., the disintegration of mass M), will occur before the Novikov time coordinate τ reaches τ_c. Since the particle cannot reach and cross r_c, before the Novikov time coordinate τ reaches τ_e, the Novikov coordinates show that particle m will not reach and cross τ_c.

6.4.3. *Coordinates based on the reference frame of a photon.* Ingoing Eddington-Finkelstein coordinates use a freely falling photon as their foundation [9, p. 828]. A radially traveling photon will not reach R in finite coordinate time t. Specifically, for a radially traveling photon, $d\theta = d\phi = 0$. Because local time for a photon does not progress, $d\tau = 0$. Setting $d\theta = d\phi = d\tau = 0$ in (3) yields

$$0 = c^2(1 - \frac{R}{r})dt^2 - \frac{dr^2}{(1 - R/r)}. \tag{15}$$

The integral in (13) can be used to calculate elapsed coordinate time t for the photon based on radial distance. Integrand $f_1(r)$ is obtained by rearranging the terms in (15), i.e.,

$$f_1(r) = \frac{dt}{dr} = \frac{1}{c(1 - R/r)}. \tag{16}$$

When the photon reaches $r = R$, the integrand in (16) is undefined and the integral in (13) does not converge. Therefore the radially traveling photon will not reach R in finite coordinate time t. Some ending event (e.g., the disintegration of mass M) will ultimately end the journey of the photon.

Further, because the integrand in (16) is undefined when the photon reaches $r = R$, no data exists for the journey when $r < R$. This data must be generated, for example, using the same or similar techniques used to generate such data for the free falling particle [9, p. 828].

Since neither a free falling particle nor a photon can reach R in finite time, these reference frames cannot be used to bootstrap free falling particle m across the critical radius. The journey of particle m from the perspective of a photon is discussed in the following subsection.

6.4.4. *Journey of the particle from the reference frame of a photon.* When viewed from the reference frame of a radially falling photon (e.g., ingoing Eddington-Finkelsten coordinates) the journey of the particle does not vary vis-a-vis the account given by the distant observer above.

Specifically, suppose coordinate t is redefined to measure time as experienced by the photon. From the perspective of a photon there is no advance of time, so $dt = 0$. Since free falling particle m travels radially, $d\theta = d\phi = 0$. Setting $dt = d\theta = d\phi = 0$ in (3) yields

$$c^2 d\tau^2 = \frac{dr^2}{(1 - R/r)}. \tag{17}$$

The integral in (14) can be used to calculate elapsed local time τ for the particle based on radial distance. Integrand $f_2(r)$ is obtained by rearranging the terms in (17), i.e.,

$$f_2(r) = \frac{d\tau}{dr} = \frac{1}{c\sqrt{(1 - R/r)}}. \tag{18}$$

When the particle reaches $r = R$, the integrand in (18) is undefined and the integral in (14) converges. The ending event at r_e and τ_e will occur before particle m reaches R. As expected in calculations made in accordance with general relativity, the same logical sequence occurs regardless of the reference frame from which observations are made.

6.5. **Surface of Last Influence.** A surface of last influence occurs when it is assumed a collapsing surface of a star will compact below R. A hypothetical is given in which a family of external observers shines a flashlight at the reflective surface of a collapsing surface [9, p. 873]. Beams of light that impact the collapsing surface at locations where $r < R$ are unable to return to the external observers. Beams of light that impact the collapsing surface just before R is reached will take a very long time to return to the external observers creating the illusion that the collapsing surface remains above R.

6.5.1. *Critiquing Surface of Last Influence.* The existence of a surface of last influence is based on the assumption that a collapsing surface of a star will compact below R. The surface of last influence, therefore, offers no evidence that a surface will compact below R, but only explains what might be expected to occur *if* a surface could collapse below R. But, as discussed above, the conservation of energy will prevent a particle from crossing

R and thus by implication will prevent the collapsing surface of a star from compacting below R.

Because the collapsing surface will not compact below R in finite time and light travels faster than matter, photons from the flashlight of an external observer will always overtake the collapsing surface and reflect back the information that the collapsing surface is still outside R. If the stationary external observer continues to shine a flashlight at the collapsing surface until the ending event at r_e, t_e, and τ_e is reached, the light will continue to impact the collapsing surface and reflect back to the external observer. This will verify to the stationary external observer that the collapsing surface continuously remains outside R until the ending event.

7. Where from Here

Once it is recognized that the Schwarzschild metric and Einstein's theory of general relativity do not predict or permit formation of black holes, this opens up a horizon of opportunities for investigation. For example, for those that want to advocate for the existence of black holes, theories other than general relativity can be explored for possible support.

Further, recognizing the possibility that black holes do not exist expands the tools to explain observed phenomena that are puzzling in the current paradigm. For example, as indicated by (8) and (1), the energy of matter is lessened by gravity. This suggests that under high gravity fields matter might more easily make the transition to radiation. This is subject to verification, if not via gravity,then by matter accelerated close to the speed of light. If experimentally verified this may prove a very fruitful field of study to provide explanations of cosmological observations. For example, the tremendous release of energy that would accompany such transitions may serve as a power source to help explain jet emissions seen from highly compacted matter, previously regarded as black holes.

References

[1] Einstein A., The foundation of the general theory of relativity. *The Principle of Relativity*, Dover Publications, Inc., New York, 1923. pp. 111-164.

[2] Einstein A., On the Influence of gravitation on the propagation of light. *The Principle of Relativity*, Dover Publications, Inc., New York, 1923. pp. 99-108.

[3] Einstein A., Is there a gravitational effect which is analogous to electrodynamic induction? *The Collected papers of Albert Einstein, Volume 4: The Swiss Years: Writings, 1912-1914*, Princeton University Press, Princeton, NJ, 1996. pp. 126-129.

[4] Einstein A., Hamilton's principle and the general theory of relativity. *The Principle of Relativity*, Dover Publications, Inc., New York, 1923. pp. 167-173.

[5] Einstein A., Does the inertia of a body depend upon its energy-content? *The Principle of Relativity*, Dover Publications, Inc., New York, 1923. pp. 69-71.

[6] Schwarzschild K., On the gravitational field of a mass point according to Einstein's theory, translated by S. Antoci, A Loinger. 1999, arXiv:Physics/9905030.

[7] Einstein A., On a stationary system with spherical symmetry consisting of many gravitating masses. *Annals of Mathematics*, 1939, 40 4.

[8] Hawking S. W., Black Hole Explosions? *Nature*, 1974, 248 (5443): 30.

[9] Misner C., Thorne K., Wheeler J., Gravitation. W.H. Freeman & Co, New York, 1973.

[10] Weller D., Five fallacies used to link black holes to Einstein's relativistic space-time. *Progress in Physics*, 2011, v. 1, 91-95.

[11] Vachaspati T., Stojkovic D., Krauss L., Generalized second law of thermodynamics in black-hole physics, *Physics Review D*, 2007, 76 024005.

[12] Weller D., How black holes violate the conservation of energy, *Progress in Physics*, 2011, v. 1, 87-90.

[13] Einstein A., On the electrodynamics of moving bodies *The Principle of Relativity*, Dover Publications, Inc., New York, 1923. pp. 37-65.

[14] Weller D., Gravity and the conservation of energy, *Progress in Physics*, 2011, v. 4, 37-39.

[15] Schwarzschild K., On the gravitational field of a sphere of incompressible fluid according to Einstein's theory. Sitzun- gsber. Preuss. Akad. Wiss., Phys. Math. Kl., 1916, 424 (see also in arXiv: physics/9912033)