A SIMPLE AND INTUITIVE PROOF OF FERMAT'S LAST THEOREM

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Abstract

In this paper it is shown a proof of the so-called "Fermat's Last Theorem" by means of application of three general principles: the converse of Pythagoras' Theorem, Dimensional Analysis and the connection algebra-geometry. These simple concepts were within the reach of Fermat himself, what allows us to infer that he could have used them for the "marvelous proof" that he claimed to have.

Fermat's Last Theorem

The so-called "Fermat's Last Theorem" (FLT) states that no three positive integers a, b, c can satisfy the equation $a^n+b^n = c^n$ for any integer $n>2$. The equation is diophantine [1] – named for Diophantus, a 3rd-century mathematician– of degree n, i.e. only solutions of integers are considered. It is a generalization of Pythagoras' Theorem when $n>2$. When $n=2$ there are infinite solutions: the so-called "Pythagorean triples" as (3,4,5), (5,12,13), (13,84,85), etc. The adjetive "last" applied to the theorem is not because it was the last work of Fermat in chronological sense, but because it has remained unsolved for more than 350 years.

Pierre de Fermat –lawer by profession and mathematician by vocation [2]– wrote in 1637 a note in latin in the margin of a copy of the 1621 edition of Arithmetica by Diophantus of Alexandria (translated from greek into latin by Claude Gaspar Bachet). The note was discovered posthumously, but the original was lost. A copy appeared in a book published in 1670 by his son Clement-Samuel. The note of Fermat was the following [3]:

Cubum autem in duos cubos, aut quadratoquadratum in duos quadratoquadratos, et generaliter nullam in infinitum ultra quadratum potestatem in duos eiusdem nominis fas est dividere cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet.	It is impossible to separate a cube into two cubes, or a fourth power into two fourth powers, or in general any power higher than the second into two like powers. I have discovered a truly marvelous proof of this, which this margin is too narrow to contain.

Since the theorem was enunciated it had baffled mathematicians for over 350 years, and became one of the most famous unsolved mathematical problems [4]. Many have been the attempts to prove it by countless profesional and amateur mathematicians around the world. It is considered the hardest mathematical problem of the world, and has the dubious honor of being the theorem

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with the greatest number of false proofs published. Really it is not a theorem, but a conjecture, i.e., it is something that is believed to be true but has not been proved yet.

Finally the theorem was proved in 1995 by Andrew Wiles [5], 358 years after the enunciation by Fermat. The proof –that consist of 109 pages– is complex, not intuitive, difficult to understand –even to professional mathematicians– because it is based in advanced and sophisticated mathematics, and it is indirect (the FLT is a corolary of a general theorem). That is why it has raised the possibility of proving the theorem in a simple way using elementary mathematics, accessible to anyone with basic mathematical knowledge. If achieved, it would agree to Fermat, that he had indeed discovered a simple and straightforward proof.

According to Hilbert, "A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man who you meet on the street" and "A proof has to be achieved, not by calculation, but rather by 'pure ideas' where possible". According to Minkowski, "Problems should be solved through a minimum of blind computations and through a maximum of forethought". It is also famous the Einstein's phrase: "You do not really understand something unless you can explan it to your grandmother".

Therefore, the strategy to try to simplify the proof and make it more intuitive is to use general concepts or principles. From that higher perspective everything is much easier.

The general principles that we are going to use in the proof of FLT are: the converse of Pythagoreas' Theorem, Dimensional Analysis and the connection algebra-geometry.

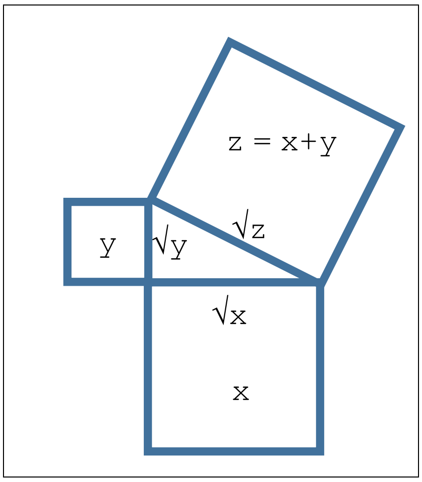
The Converse of Pythagoras' Theorem

The Pythagoras' Theorem states a property that all the right triangles fulfill: the sum of the squares of the legs is equal to the square of the hypotenuse. Algebraically this is expressed as $x^2+y^2 = z^2$, where x, y are the legs and z the hypotenuse.

The converse of Pythagoras' Theorem can be interpreted in two ways:

- Algebraic interpretation.
  If we have the expresión $x+y = z$, then it exists a right triangle of legs $x^{1/2}$, $y^{1/2}$ and hypotenuse $z^{1/2}$ (see figure).

  That is to say, that in the addition –the most fundamental operation of mathematics– the Pythagoras' Theorem is implicit. Hence its enormous importance as a universal connector between algebra and geometry:

  – To pass from geometry to arithmetics we have to square the numbers that represents the sides of a right triangle.
  – To pass from arithmetics to geometry we have to transform the numbers in

square roots.

This theorem is universal for every expression of type $x+y = z$, and connects algebra to geometry.

- Geometric interpretation.
  It correspons to the converse of Pythagoras' Theorem included in the proposition I.48 of Euclid's Elements –proposition I.47 is the Pythagoras' Theorem–, that states "If in a triangle the square on one the sides equals the sum of the squares on the remaining two sides of the triangle, then the angle contained by the remaing two sides of the triangle is right".

  This theorem is exclusively of geometric type (connects geometry to geometry).

Here we are going to use the algebraic interpretation of the converse of the Pythagoras' Theorem.

Dimensional Analysis

Dimensional Analysis [6] [7] is the study of the physical dimensions involved in the equations that model a physical phenomenon. Although certain ideas of Dimensional Analysis were implicitly present in the works of Galileo, Kepler and Newton, it is considered that the Dimensional Analysis was born formally with Fourier in his book "Analytical Theory of Heat" (1822).

The two key concepts of Dimensional Analysis are:

1. Physical magnitude.
   In mathematics numbers are considered "pure", without attributes. In physics, however, numbers are considered as magnitudes, consisting of a pure number (or quantity) and an unit. There are primary magnitudes (length, time and mass) and secondary (or compound) magnitudes (speed, force, energy, etc.).

   A physical magnitude is an expression of the form "quantity*unit", which expresses the number of times a unit is repeated. The unit can be simple (for the primary magnitudes, e.g. 5*m) or compound (for secondary magnitudes, e.g. 3*m/seg). One physical magnitude can be expressed by different units and, therefore, by different quantities, for example, 5*m ≡ 500*cm. There is an analogy between magnitude and vector, where the unit plays the role of vectorial base.

   Quantities and units are handled using the algebra's laws. For example, if an object travels 10 meters in 5 seconds, its speed is (10*m)/(5*seg) = (10/5)*m/seg = 2*m/seg.

2. Physical Dimension.
   Each primary physical magnitude has an associated dimension, which is symbolized by a letter (L: length, T: time, M: mass). The compound (or secondary) magnitudes have an associated dimensional expression, which is a monomial, i.e. a product of powers of

the primary dimensions. For example, using the Maxwell notation [x] to refer to the dimensions of a physical magnitude x:

Speed: $[v] = LT^{-1}$
Force: $[f] = [m][a] = MLT^{-2}$
Energy: $[e] = L^2MT^{-2}$

In general, the dimensional expresión of a physical magnitude is $L^{n1} \cdot T^{n2} \cdot M^{n3}$, where n1, n2, n3 are integer numbers (positive, zero or negative).

Constants and angles are dimensionless and have dimensión 1, and also the arguments and the results of the trigonometric functions, the logarithm function and the exponencial function.

Dimensional Analysis meets the rules of algebra, except for the addition and substraction: $M+M = M$, $L+L = L$, $T+T = T$, $M-M = M$, $L-L = L$, $T-T = T$.

Principles of Dimensional Analysis

Dimensional Analysis is based on the two following principles:

1. Principle of dimensional homogeneity (or consistency).
   In any equation that relates variables of physical magnitudes, the dimensional expressions on each side of the equation must be the same. It is called the "law of conservation of the dimensions". For example, the equation $s = v_0t + at^2/2$ has dimensional homogeneity, as can be easily verified, taking into account:

   $[s]=L \quad [v_0]=LT^{-1} \quad [t]=T \quad [½]=1 \quad [a]=LT^{-2}$

2. Principle of mathematical homogeneity or similarity.
   The principle of similarity states that "All laws of physics are invariant under changes of measures in one system of units". This principle is very important when it comes to model physical phenomena using prototypes or small scale models.

Dimensional Analysis and geometry

Although Dimensional Analysis was born to model physical phenomena it is also applicable to geometry:

- There is only one primary magnitude: length, which dimension is L. The secondary magnitudes have L^n as dimensional expression, where n is a positive integer. For example:

  [Circumference length] = [2π·radius] = L
  [Circle surface] = [π·radius²] = L^2
  [Square surface] = [side²] = L^2

[Cube volume] = [side^3] = L^3

The geometric constants π (ratio between the length of the circumference to its diameter) and ϕ (the golden ratio) are dimensionless. They are universal constants in the sense that they are scale independent.

The angles are measured in radians because the radian (ratio of two lengths) is dimensionless and simplifies the formulas, such as the development in series of $\sin(x)$, $\cos(x)$ and their derivatives.

- The geometric magnitudes have the form "number·u^n", where u is a unit of length and n the dimension of the geometric object. For example,

  Square surface = (side·u)^2 = side^2·u^2
  Cube volume = (side·u)^3 = side^3·u^3

  By specifying the unit a higher semantic content of expressions is given, beyond the pure algebraic formalism.

- The homogeneity principle refers to the consistency of the formulas of geometric objects. For example,

  [Triangle surface] = [base·height/2] = [base]·[height]·[1/2] = $L \cdot L \cdot 1 = L^2$

- The principle of similarity is the true essence of geometry. For example, π is the ratio between the length of the circumference to its diameter, regardless of physical size or scale.

Advantages of Dimensional Analysis

Dimensional Analysis is a conceptual, simple, powerful, generic and qualitative tool. It is widely used in pure and applied science for:

- Help in modeling, in a simple way, physical phenomena. It allows to gain insight expressing the relationships between the dimensions of the magnitudes involved in these phenomena. The first step of the modeling process is the identification of the variables involved. From purely dimensional considerations of the variables it often can be established directly the equation that relates these variables.

- Checkup of physical models by detecting posible errors in the equations in order that they make sense.

- Resolution of problems at qualitative level, which direct or quantitative solution presents big difficulties of mathematical type. After obtaining a qualitative solution, it is easier to achieve a detailed and quantitative solution.

At geometric level, Dimensional Analysis is specially useful for:

- Obtain formulas of geometric entities. Sometimes the obtained qualitative formula matches the quantitative one. Other times the qualitative formula needs an adjustement to obtain the quantitative one. For example, the length of a circumference must be proportional to its radius r. Therefore, its length must be $k \cdot r$. The adjustment consists in obtaining the value of k (in this case, 2π). Another example is the surface of a rectangle of sides a and b. Its surface S is proportional to a and b: $S = a \cdot b$. In this case, the formula does not need any adjustement.

- Make proofs of geometric theorems, since they are notably simplified. An example is the following theorem. A triangle of sides a^n, b^n, c^n cannot be a Pythagorean triple, with a, b, c, n positive integeres and $n>1$. The quantitative proof is very laborious, but the qualitative proof is inmediate: by purely dimensional reasons, n must be 1.

  The proof of LFT itself is simplified (as we shall see), to the point of being almost immediate.

The Connection Algebra-Geometry

The two modes of conciousness

As it is known, there are two modes of consciousness:

- The intuitive, deep, conceptual, synthetic, creative, general, global, imaginative, qualitative, parallel, continuous, etc. It is usually associated to the right hemisphere of the brain. We will call it "HD consciousness" for short.

- The rational, superficial, formal, analytical, particular, quantitative, sequential, discrete, etc. It is usually associated to the left hemisphere of the brain. We will call it "HI consciousness" for short.

The whole consciousness arise when both modes of consciousness are connected. This connection is in such a way that every particular thing is a manifestation of something general. In this sense, HD consciousness is higher than HI consciousness. Any particular thing cannot never be isolated. It must be linked to something general or universal. This connection is precisely the semantics of the particular, what gives it a meaning.

Algebra vs. Geometry

In mathematics, these two modes of consciousness are reflected in the duality algebra-geometry, where algebra corresponds to HI consciousness and geometry corresponds to HD consciousness. What we can call "mathematical consciousness" arises when algebra and geometry are connected.

Pythagoras' Theorem (in their direct and converse versions) plays a fundamental role in the connection between algebra and geometry, i.e. in the union of the two modes of consciosness.

Pythagoras' Theorem is the paradigm of the unión of algebra and geometry. Pythagoras' Theorem is a theorem of consciousness.

Since HD consiousness is higher than HI consciousness, and since geometry is HD consciousness and algebra is HI consciosness, geometry is on a higher level than algebra, so algebra should be a particularization or manifestation of geometry. In this sense:

- An equation like $ax+by+c = 0$ is the manifestation, representation or formalization of a straight line. And an equation like $x^2+y^2 = z^2$ is the manifestation, representation or formalization of a circumference.

- Pythagoras' Theorem was formalized in a descending way: from geometry to algebra. The converse of Pythagoras' Theorem implies to raise the semantic level: from algebra to geometry.

- LFT is an algebraic theorem, but is has to be the manifestation of a geometric property.

- Since geometry is more general and intuitive than algebra, reasoning on geometric figures improves the understanding and facilitates the proofs and the discovery of properties. The paradigm of this approach is precisely the Pythagoras' Theorem.

Numbers vs. segments

The duality algebra-geometry is reflected in the duality numbers-segments. The not exponential variables of the algebraic expressions can be interpreted as pure numbers or as segments of straight lines, where numbers corresponds to HI consciousness and segments to HD consciousness. Interpreting variables as segments has many advantages:

- Segments connect to geometry, with the higher, with the HD consciousness. This implies to consider the highest semantic level possible. On the contrary, numbers are HI consciousness, lower consciousness.

- Generalize the equations and make them independent of coordinate and unit systems.

- It is the most natural and intuitive interpretation, which is of descending type: from HD consciousness to HI consciousness, from general to particular, from qualitative to quantitative.

  Ancient greeks worked with segments and areas instead of numbers. The square of a number was not interpreted as a number multiplied by itself, but like a geometric square. Pythagoras' Theorem was expressed verbally referring only to geometry, as equality of areas. They did not express it as an equation in the symbolic modern sense. That was made later, with the development of algebra.

- They avoid to deal with irrationals, which are inexpressible numbers. For example, $\sqrt{2}$ is numerically inexpressible. But as a diagonal of a square of unitarian side it is conceived and described without difficulty.

- Segments are magnitudes (quantity*unit) and their dimensión is `L`. Numbers have no dimensions (their dimensión is `1`).

- Because they are in a higher level, segments can manifest in infinite ways, depending of the unit used. A number is a manifestation of a segment of a straight line according to a certain unit. When the unit is changed, its manifestation is different. That is to say, a segment of straight line has infinite posible (numeric) manifestations. Numbers are fixed. Segments lengths are variable, depending of the used unit.

Pythagoras vs. Plato

Pythagoras and his school (the Pythagoreans) gave more importance to arithmetics than geometry. They put emphasis in numbers, which were considered the essence of all things and the foundation of our understanding of the world. "Numbers govern the world". The Pythagoreans believed that all the mathematical problems could be solved by whole numbers and thereof. Because of that, to discover that the diagonal of a square is inconmensurable with respect to his side was a trauma, a consequence of the Pythagorean theorem itself.

Plato believed that geometry belonged to the kingdom of Ideas or Forms, the perfect, true and immutable reality. And that these Ideas manifested (at a superficial level) in multiple and singular concrete forms of an imperfect nature.

Plato considered that geometry was a discipline higher than arithmetics. In front of Plato's Academy it had been inscribed "Let no one enter who is ignorant of geometry". In Timeo he says that "Geometry is the key to unlocking the secrets of the universe". And also mantained that geometry was connecting with the divine. And he is attributed the phrase "God always use geometric procedures". To Plato numbers do not govern the world but geometry does. Plato highlighted the catastrophic character of irrationals.

Analytic geometry

Descartes and Fermat were contemporary and creators of the union of algebra and geometry with the so-called "analytical geometry", a very appropiate name that expresses that descending process from the synthetic and generic (geometry) to the analytical and specific (algebra). What we call today "Cartesian system" is two perpendicular axis representing points (by pairs of ordered numbers) and geometric places of points (by equations).

- Descartes described the analytical geometry in an appendix of "Discourse on the Method", published in 1637 (the same year that the famous Fermat's note). Descartes was interested on the algebraic formalization of geometric figures (as the conics), in a "descending" process: from geometry to algebra.

  To Descartes, analytic geometry was an step towards the universal science, towards the fundamental truths that connect all sciences. Descartes wanted to apply the mathematical method (based on reason) to philosophy.

- Fermat described the analytical geometry in a manuscript entitled "Introduction to

Plane and Solid Loci" –where "Loci" is translated as "geometric place"–, written in 1636 (a year before Descartes). Fermat diffused it among his network of postal contacts, so it is believed that Descartes had access to Fermat's manuscript and inspired him. Finally the manuscript was published pothumously en 1679. Because of that, today we say "cartesian geometry" instead of "fermatian geometry". Fermat –unlike Descartes– was interested in the geometric properties of the algebraic equations (e.g. Diophantic equations), in an "ascending" process: from algebra to geometry.

For Descartes and Fermat, the variables of an equation were representing linear segments, something more than numbers.

Pythagoras' Theorem was fundamental for the invention of analytical geometry. It is present in two fundamental issues: One is the circumference equation: $x^2+y^2 = r^2$. The other one is the calculation of the distance between two points, given their coordinates. In turn, Analytical Geometry was a fundamental tool for the development of calculus by Newton and Leibniz.

Vector and multivector algebras

They are algebras operating with vectors and multivectors.

A vector is a generalization of a segment. It is a segment of straight line having length (its module) and an orientation consisting of a direction in the space and a sense. Vectors are used to represent physical magnitudes such as velocity, force and electric field. which depend of the coordinate system used. The other physical magnitudes are scalars such as temperature, mass, density, etc., which are independents of the coordinate system. Scalar magnitudes are represented by a number (with its corresponding unit). Vectors in general can be represented by n numbers, which are the projections on the ortogonal axes of a n-dimensional space. Vectors can be considered as "multidimensional numbers".

A multivector (or n-vector), in turn, generalizes the concept of vector. It is the generalization of the concept of segment in an space of n dimensions, i.e., a multidimensional oriented segment. A 0-vector is an scalar, an 1-vector is a traditional vector, a 2-vector (or bivector) is an oriented segment of plane, a 3-vector (or trivector) is an oriented segment of volume, etc.

Geometric algebra

Nowadays, geometric algebra (or Clifford algebra) [8] is considered the culmination of the synthesis between algebra and geometry, up to such a point that some authors consider it "the mathematics of consciousness". Although it is an algebra, its great inspiring principles are of geometric type.

Geometric algebra provides multivectors (n-vectors) and defines the operations of addition and product of n-vectors. The product is called "geometric product". Any n-vector has its opposite and its inverse one.

Geometric algebra does not fulfill the principle of dimensional homogeneity because (among other things) allows the addition of vectors and scalars. This is not considered more unusual than the complex numbers, which are the sum of a real component and an imaginary one.

Due to its generic character, geometric algebra has applications in a large number of fields: number theory, topology, differential geometry, theoretical physics (classic and modern), computer graphics, robotics, etc. It is a tool that allows to solve many mathematical problems in a more simple and direct way. Besides, it generalizes the numbers (real, complex, quaternions, hypercomplex, etc.).

According to David Hestenes –one of the major impellers of geometric algebra– geometric algebra is a unified lenguaje for mathematics and physics.

Conclusions

- Algebra and geometry are manifestations (at mathematical level) of the two modes of consciousness. Geometry is higher than algebra, and segments are at a higher level than numbers.

- Algebra and geometry need one another to achieve "mathematical consciosness". Algebra needs geometry to be interpreted, in order that symbols acquire semantics. And geometry needs algebra in order to express itself. "Geometry without algebra is dumb. Algebra without geometry is blind" (David Hestenes).

- Not exponential variables of an algebraic expression must be firstly interpreted as segments and secondly as numbers.

- Geometry is the great inspiring principle of mathematics. Geometric concepts generalize, unify and simplify mathematics. Algebraic problems are simplified when approached from a geometric point of view.

- Geometry is the foundation of modern physics. Many equations of physics have a simple geometric interpretation, providing more clear and understandable conceptual models. From Plato's ideas to modern physics (quantum and relativistic), geometry describes the ultimate reality of the universe.

Proof of the Theorem

The proof consists of three steps, where in each step one of the mencioned principles is applied.

1. Application of the principle of connection algebra-geometry.
   This principle is reflected in the connection numbers-segments. The variables a, b, c of Fermat's equation can be interpreted:

   a) As numbers.
      If we interpret a, b, c as pure numbers, given a, b, n positive integers, it is always possible calculate $c = (a^n+b^n)^{1/2}$. The question is whether or not it is possible c to be integer. This is a pure algebraic interpretation. From this point of view it is very difficult to proof the theorem.

b) As segments.
In Pythagoras' Theorem, the variables a, b, c of the expression $a^2+b^2 = c^2$ must be interpreted as segments in general (the sides of the right triangle) and as numeric variables in particular. And since Fermat's equation $a^n+b^n = c^n$ is a generalization of the Pythagoras' Theorem, it is also natural to interpret the variables a, b, c the same way. Both views (segments and numbers) must be compatible, that is, must be consistent with one another. From this point of view, everything is simplified.

2. Application of the converse of Pythagoras' Theorem (at algebraic level).
According to this theorem, if Fermat's equation $a^n+b^n = c^n$ is fulfilled, then $A=a^{n/2}$, $B=b^{n/2}$ and $C=c^{n/2}$ have to be the sides of a right triangle T, where A and B are the legs and C is the hypotenuse. The corresponding deployed squares are $A^2=a^n$, $B^2=b^n$ and $C^2=c^n$, and where the Pythagorean equation $A^2+B^2 = C^2$ should be fulfilled. This involves raising the semantic level: from algebra to geometry.

3. Application of Dimensional Analysis.
Once installed in geometry, we interpret the variables a, b, c as lengths of linear segments:

$$[a] = [b] = [c] = L$$

The expressions a^n, b^n, c^n have dimension L^n:

$$[a^n] = [b^n] = [c^n] = L^n$$

The values A, B, C are lengths (the sides of the right triangle):

$$[A] = [B] = [C] = L$$

The expressions A^2, B^2, C^2 are surfaces (the deployed squares):

$$[A^2] = [B^2] = [C^2] = L^2$$

The expressions $A^2=a^n$, $B^2=b^n$, $C^2=c^n$ are relations between geometric magnitudes. These expressions do not have dimensional homogeneity. In order to achieve dimensional consistency it must necessarily be $n=2$ and, consequently, $A=a$, $B=b$ and $C=c$.

This reasoning is valid regardless of a, b, c are integer o real numbers. But since a, b, c are positive integers, and $a^2+b^2 = c^2$ is fulfilled, then (a, b, c) must be a Pythagorean triple.

Another way of looking at it is applying Dimensional Analysis to the area of the right triangle T. Its area is $(a \cdot b)^{n/2}/2$, which has dimensión L^n and should be L^2. Therefore, must be $n=2$.

In summary, the proof is based on:

1. Applying the converse of Pythagoras' Theorem (at algebraic level) to the Fermat's equation, interpreting `a, b, c` as numeric variables, that is, ascending from algebra to geometry.
2. Once istalled in geometry, variables `a, b, c` are interpreted as segments and Dimensional Analysis is applied to infer that necessarily $n=2$.

In a nutshell, this is the proof of the FLT:

> The expression $a^n+b^n = c^n$ does not fulfill for a, b, c, n positive integers and $n>2$ because, according the converse of Pythagoras' Theorem (algebraic version), there is a right triangle of sides $a^{n/2}, b^{n/2}, c^{n/2}$, which are lengths (dimension L), whose corresponding deployed squares (a^n, b^n, c^n) are surfaces (dimension L^2) and whose surface is $(a{\cdot}b)^{n/2}/2$ (also dimensión L^2). Interpreting a, b, c as lengths (dimensión L), in order to achieve dimensional consistency it must necessarily be $n=2$. And since a, b, c are positive integers, and it holds $a^2+b^2=c^2$, then (a, b, c) must be a Pythagorean triple.

Reflections on the FLT

FLT, a paradoxical theorem

FLT can be considered a paradoxical problem because in it contradictory opposites apparently converge: complexity and simplicity. Firstly, because the theorem is very easy to enunciate and seemingly very difficult to prove. Secondly, because the Wiles's proof is enormously complex, requiring sophisticated mathematical tools. Nevertheless it can also be proved easily, using only elementary mathematical concepts.

It is the paradox of simplicity-complexity that corresponds to two different ways of seeing the same problem. It is what Max Tegmark names "the perspectives of the frog and of the bird" [9].

For the frog, everything is very difficult and complex because it moves on the surface, horizontally, in the particular thing, in the detail. For the bird, from a higher perspective, everything is much easier because it sees evident relations and connections, which it is not possible to perceive from the frog perspective. It is the old proverb "can't see the forest for the trees".

Actually, the frog perspective corresponds to the mode of consciousness of the left hemisphere (analytical), and the bird perspective corresponds to the mode of consciousness of the right hemisphere (synthetic).

The strategy to use always should be that of the bird, advancing (or descending) from the universal or general to the particular, always deriving particular truths from general or universal principles.

Wiles used the frog perspective, looking for particular horizontal relationships. The proof presented here is that of the bird, since it is based on general principles and then descending to the analytical level. Therefore, the considered "most difficult mathematical problem of the world" (from the frog perspective) turns into "one of the easiest mathematical problems of the world" (from the bird perspective).

The FLT is probably the best example of these two extreme views, of the duality simplicity-complexity.

FLT, a theorem of consciousness

Pythagoras' Theorem is a theorem of consciousness because it connects algebra and geometry, but FLT is also a theorem of consciousness because in its proof several opposite concepts are united:

- The rational and the intuitive. Intuitively, almost without writing or detailing the proof, it is intuited that the theorem is truth. But the proof can also be rationalized.

- Numbers and segments. The proof makes use of the connection algebra-geometry in its most fundamental aspect: the connection between numbers and segments.

And the theorem establishes a frontier between the real or manifested (the infinite Pythagorean triples for $n=2$) and the imaginary or unmanifested (the absence of solutions for $n>2$). In short, between zero and infinite.

The possible "marvelous proof" of Fermat

There are serious doubts whether Fermat really had a "wonderful proof", given the extent and complexity of the Wiles's proof, who used elements of modern algebra that Fermat could not know. Because of this, most mathematicians think that Fermat was wrong, that he could not have a real full proof. Only a few disagree with this general opinion and think that Fermat had such a proof in a formal way or at least had known it by intuition.

But it is perfectly possible that Fermat could have a simple proof as the one included here. The general principles applied here are very simple and were within the reach of Fermat himself. This allows us to infer that he could use them for the "marvelous proof" he claimed to have.

The Generalized FLT

FLT is an attempt of "vertical" generalization of the Pythagoras' Theorem, i.e. when $n>2$, but there are two more posible generalizations:

1. The "horizontal" generalization of the Pythagoras' Theorem is when there are $m>2$ summands: $a_1^2 + ... + a_m^2 = b^2$. For example, for $m=3$, we have many quadruples: (1, 2, 2, 3), (2, 3, 6, 7), (1, 4, 8, 9), (4, 4, 7, 9), (12, 16, 21, 29), etc., a very much wider variety than the Pythagorean triples.

2. The generalization "horizontal-vertical" or complete is when there are $m>2$ summands and $n>2$: $a_1^n + \dots + a_m^n = b^n$, which is the generalized Fermat expression.

The question is: What is the minimum value of m to satisfy this expression? According to the FLT, if $n=2$, then $m=2$. For $n=3$ and $m=3$, we have many examples of expressions:

$$3^3+4^3+5^3 = 6^3 \qquad 1^3+6^3+8^3 = 9^3 \qquad 7^3+14^3+17^3 = 20^3 \qquad 3^3+36^3+37^3 = 46^3$$

The first expression is a generalization of archetypal Pythagorian expression $3^2+4^2 = 5^2$.

The generalized conjecture would be: n summands are needed at a minimum to ensure compliance with the generalized Fermat expression. And the generalized Fermat theorem would be: an entire $m<n$ does not exist such that fulfills the expression $a_1^n + \dots + a_m^n = b^n$.

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