

Inherent Energy Loss of the Thomson Scattering

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Abstract: If a low density electromagnetic wave hits an electron, it does not only perform the well-known transverse movement, but also a longitudinal movement with double frequency. This leads to an energy loss of the original electromagnetic wave without change of direction which was previously unknown and is no elastic or inelastic collision. The "lost" energy is radiated in two different frequency ranges and the relative energy loss increases with decreasing frequency of the primary wave. An experimental confirmation of this phenomenon could influence the debate about "tired light". The derivation is based solely on classical electrodynamics and therefore contains no ad hoc hypothesis. The only adjustable parameters are the duration and the envelope of the wave packet.

Keywords: envelope, wave packet, secondary wave, redshift, polarization

1 Introduction

J. J. Thomson assumed that the scattering of light by electrons is a linear process. Under the then possible measurement accuracy the wavelength remained constant. Thomson dismissed the idea that light's magnetic field was strong enough to influence electron motion. Now, a century later, the usual explanation of the Thomson scattering is still: "The electric field of the incident electromagnetic wave accelerates an unbound electron, causing it, in turn, to emit radiation at the same frequency. The magnetic field of the original waveform can be neglected." This may not be the whole truth, because the effect of the Lorentz force is ignored, even though the electron moves in the variable magnetic field of the wave. One consequence is a tiny energy loss of the primary wave, after an electron comes into its influence. In the following work, this effect is closely examined.

2 Modelling the Thomson Scattering

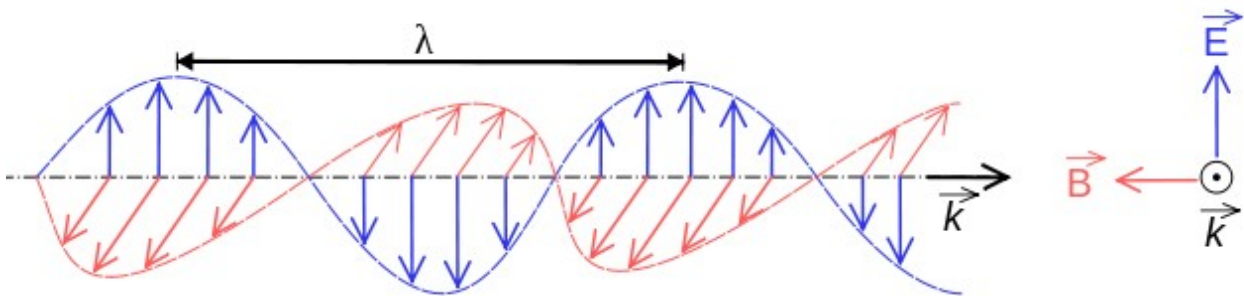
Far away from the source, the wave fronts of each electromagnetic wave are planes, which (theoretically) infinitely extend transverse to the propagation direction. The two electric field quantities E and B are described by the equations $E = E_{max} e^{i(\omega t - kz)}$ and $B = B_{max} e^{i(\omega t - kz)}$, the z -axis indicates then propagation direction. Only the real components can be measured. Since all sources emit only a finite amount of energy, they can neither generate an infinite number nor infinitely extended wave fronts. It follows that

- the number of wave fronts is limited. In z -direction, a wave packet must have a beginning and an end. In the following formulas, the factor $C(t)$ limits the total duration, its properties are defined below.
- the area of each wave front (parallel to the x - y -plane) is limited. This problem is discussed in a subsequent paper.

Hits a linear polarized wave a free, unpaired electron, this is caused to vibrate with the same frequency (the response to a circular polarized wave is also discussed in a subsequent paper).

For the mathematical description of a real wave packet the following coordinates and functions are defined:

- The wave vector \underline{k} of an electromagnetic wave is parallel to the z-axis and is oriented towards the observer. The wavefronts are parallel to the x-y-plane, the wave is linear polarized.
- The vector spaces $\vec{e}_x \times \vec{e}_y = \vec{e}_z$ and $\vec{E} \times \vec{B} = \vec{k}$ are right-handed.
- The origin ($x = y = z = 0$) of the coordinate system is the position of the electron before the arrival of the wave fronts. Then, the electron oscillates around this average coordinate and comes to rest again after the final wavefront (proof see below).
- The x-axis points to the right and because of $z = 0$, the magnetic field at the position of the electron is $B_x = -C(t) \cdot B_{max} \cdot \cos(\omega t) = \Re(-C(t) \cdot B_{max} \cdot e^{i\omega t})$
- The y-axis points upwards, the electric field at the position of the electron is $E_y = C(t) \cdot E_{max} \cdot \cos(\omega t) = \Re(C(t) \cdot E_{max} \cdot e^{i\omega t})$



The picture shows a short section of the wave train. Rightmost is a snapshot of the vectors \underline{E} and \underline{B} at time $t = 0$. The wave vector \underline{k} points to the observer.

The dimensions of the wave packet define the volume of a contiguous range in which the main part of the energy is transmitted. Subsequently, only the consequences of finite length in the z-direction are investigated. This also determines the period of time during which the electromagnetic wave influences the electron. If the energy of the wave is small enough that the maximum deflection of the electron in the y-direction is negligible compared to the expansion of the wavefront, the condition $v_y \ll c$ is satisfied and all calculations can be performed nonrelativistic.

In the far field, an electromagnetic wave is defined by the direction of propagation, surface power density S , energy density w and frequency $\omega = 2\pi f$. The values of the periodical electric and magnetic fields of the wave can be calculated with the Poynting formulae $E = \sqrt{Z_0 \cdot S}$, $B = \mu_0 \sqrt{S/Z_0}$ and $S = c \cdot w$, where $Z_0 = 377 \Omega$ is the characteristic impedance of free space. The maximum values are obtained by multiplying by a factor $\sqrt{2}$.

If unbound, quiescent electrons are in the area of these fields, they are stimulated to oscillate and radiate energy. In the following two cases are discussed:

1. The electrons are part of an extended, thin plasma, wherein the mutual distance is considerably greater than the wavelength and collisions occur rarely. The movement of the positive ions, which are also present, can be neglected because of their much bigger mass. However, they also cause a resetting force on the electrons, so that they tend to oscillate at the plasma frequency. In the interstellar medium, ω_p is very low with only a few kilohertz and is therefore neglected below.
2. The wave can excite the electron shells of densely packed atoms causing them to vibrate around their equilibrium positions. Even in transparent materials there are resonance effects

which mostly lie in the UV region and are subsequently neglected. In solids, the distances between the "electron clouds" are significantly smaller than the wavelength, resulting in interference effects, these direct the radiation in certain directions. Those effects are well known in antenna technology.

3 Model of a wave packet

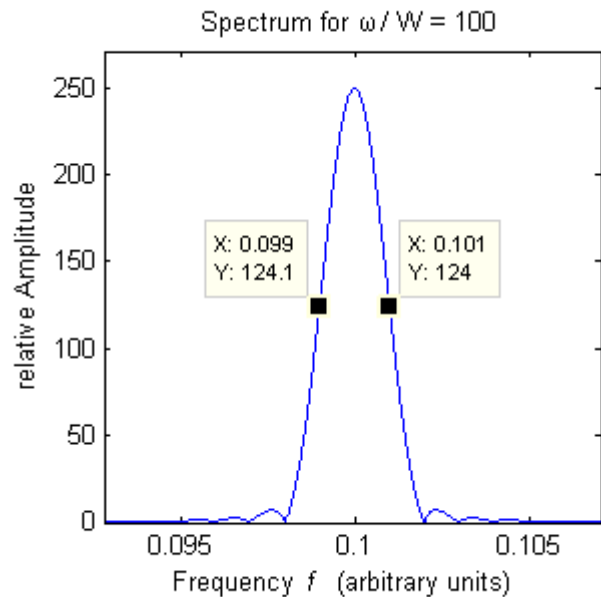
Every electromagnetic wave is limited in time, that is, it has a beginning and an end. First, an envelope is modeled, which is zero beyond a certain interval (Slowly varying envelope approximation). Some continuous functions $C(t)$, such as a Gaussian wave packet with cut slopes have discontinuities and are therefore less suitable. The frequently expressed assumption, the envelope is a decaying exponential function (damped wave), as one can hear after striking a bell, lacks any basis in the field of light. In no atomic model can one find evidence for a harmonic oscillator to produce a damped wave.

Technically, the envelope function $C(t)$ is formed by the amplitude modulation of a carrier with a frequency mixture, in the simplest case with a single modulation frequency W , where $W \ll \omega$ is presumed. Each modulation generates so-called side band frequencies in the vicinity of ω whose amplitudes typically decrease with increasing frequency separation. The side-band frequencies occupy a frequency range which is called *natural line width*.

Numerical verifications show that modifications to the shape of the envelope have little effect, they change the results of this work only marginally. Therefore, the well-known Hann-Window $C = (1 - \cos(W \cdot t))/2$ with the acceptable range $0 \leq W \cdot t = 2\pi$ is chosen as the envelope of E and B . Outside this range is $C = 0$. With this choice, the FWHM-bandwidth is $\Delta\omega \approx 2W$ and the line width is $\Delta\lambda \approx \frac{4\pi W c}{\omega^2} = \frac{\lambda^2 W}{c \pi}$. Only those

wave trains will be considered, which include at least 100 oscillations, so it is assumed

$W \ll \omega$. This assumption is likely to be met for most spectral lines. For example, the natural line width of the sodium D-line amounts to about 10 MHz and therefore the length of a wave packet is approximately 10^7 oscillation periods. The limiting case $W \rightarrow 0$ describes an non-physical wave of constant amplitude and infinite extent and is not covered here.



When a free, unbound electron enters the sphere of influence of a wave packet, it is affected during the period of time $0 \leq W \cdot t < 2\pi$. Before and after the electron is at rest. The biggest part of the energy, about 92.44 %, is transported through the central region of the envelope between the time points $W t_1 = \pi/2$ and $W t_2 = 3\pi/2$. This interval is called the *coherence length* $L \approx \frac{\pi c}{W}$ with the above definitions. The extensions are energetically negligible. If the wave moves in the dispersion-free space, the coherence length is unchanged and there is no wave packet spreading.

The $C(t)$ function describes only the dimension of the wave packet in parallel to the wave vector \underline{k} .

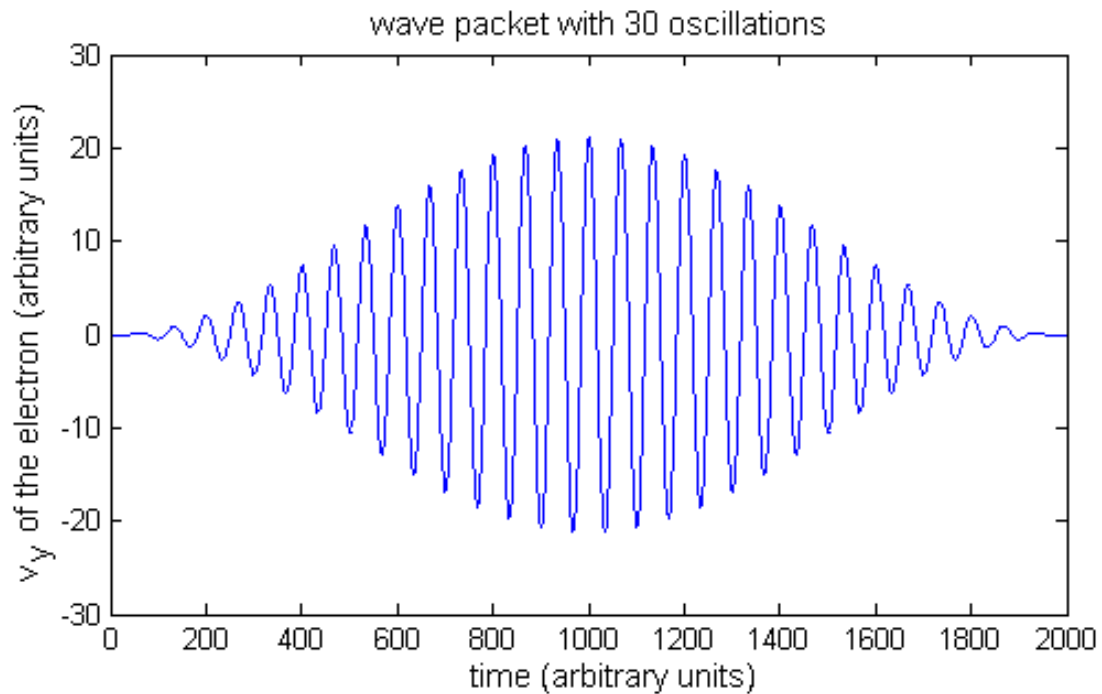
The extent transversely thereto, that is the three-dimensional structure and the volume are treated in a separate paper in relation to the Fresnel zones.

4 Movement of the electron in the y-direction

The fields of an electromagnetic wave exert a force on each charged particle. A negatively charged electron is accelerated parallel to the y-axis and the speed follows from the formula $a_y = q_e E_y / m_e$ by integration:

$$v_y = \frac{q_e E_{max}}{2 m_e \omega (\omega^2 - W^2)} \left(e^{i\omega t} (\omega W \sin(W t) + i \omega^2 \cos(W t) - i \omega^2 + i W^2) - i W^2 \right) \quad (1)$$

The integration constant is chosen such that at the start of the electromagnetic wave at the time $W \cdot t = 0$ the condition $v_y = 0$ is satisfied. The picture gives an idea of the time course of v_y .



After a further integration, the real component gives the y-coordinate of the electron

$$y = \frac{q_e E_{max}}{2 m_e \omega^2} (\cos(\omega t) (\cos(W t) - 1)) \quad (2)$$

This solution in the y-direction does not contain physical news, if the effects of the surrounding plasma and the movement of the electron in the z-direction are ignored. This is discussed further below.

5 Radiation due to y-acceleration

The electron is accelerated by the incoming wave packet (primary wave)

$$a_y = \frac{q_e E_{max}}{m_e} \frac{1 - \cos(W \cdot t)}{2} \cos(\omega t) \quad (3)$$

and emits energy. The value is obtained by calculating the power (Larmor formula) and subsequent integration over time during the duration of the wave packet $0 \leq W \cdot t < 2\pi$.

$$P_y = \frac{q_e^4 E_{max}^2}{24 \pi \epsilon_0 c^3 m_e^2} (1 - \cos(W \cdot t))^2 (\cos(\omega t))^2 \quad (4)$$

$$\int P_y dt = \frac{q_e^4 E_{max}^2}{24 \pi \epsilon_0 c^3 m_e^2} \frac{3 W t - 4 \sin(W t) + \cos(W t) \sin(W t)}{4 W} \quad (5)$$

$$A_y = \frac{q_e^4 E_{max}^2}{24 \pi \epsilon_0 c^3 m_e^2} \frac{3 \pi}{2 W} = \frac{q_e^4 \mu_0 c S_y}{24 \pi \epsilon_0 c^3 m_e^2} \frac{3 \pi}{2 W} = \frac{q_e^4 \mu_0^2 \cdot S_y}{8 m_e^2 W} \quad (6)$$

S_y is the surface power density of the primary wave. This classical approximation is not valid for big power densities.

6 The cross section of the electron

The incident wave packet with the surface power density S_y is limited to the time period

$0 \leq W \cdot t < 2\pi$. Since the envelopes of the electric and magnetic components are modelled with the function $C = \frac{1 - \cos(W \cdot t)}{2}$, one gets $S_y(t) = S_0 \frac{(1 - \cos(W \cdot t))^2}{4}$. The integration over the period of the wave packet results in the energy Q , which is distributed over the total cross-sectional area of the electron.

$$Q = \int_0^{2\pi/W} S_0 \frac{(1 - \cos(W \cdot t))^2}{4} dt = \frac{3 \pi S_y}{4 W} \quad (7)$$

A free electron can not store energy and has to re-radiate the amount it has previously absorbed from the wave packet. Thus its actual absorption cross section σ can be calculated:

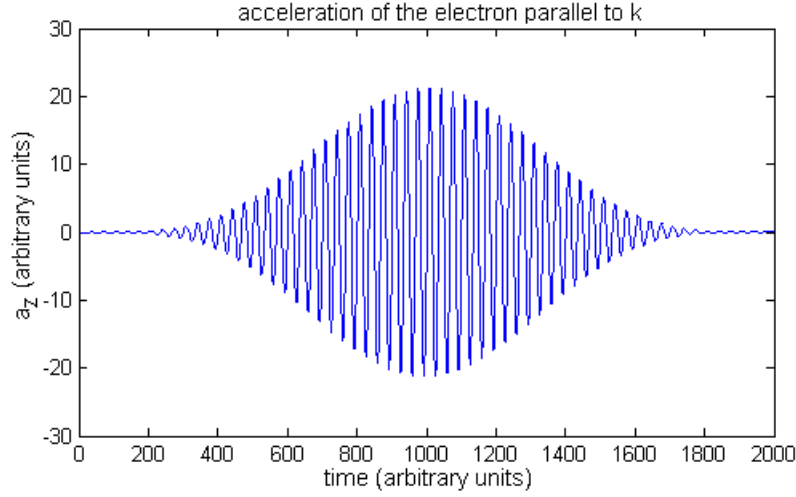
$$\sigma_{Elektron} = \frac{A_y}{Q} = \frac{q_e^4 \mu_0^2 \cdot S_y}{8 m_e^2 W} \frac{4 W}{3 \pi S_y} = \frac{q_e^4 \mu_0^2}{6 \pi m_e^2} = 6.625 \cdot 10^{-29} m^2$$

This result is identical with the classically calculated cross section of the electron.

7 Motion of the electron in the z-direction

The previously described motion of the electron is in the y-direction. The magnetic field of the wave is oriented antiparallel to the x-axis. The Lorentz force $F_z = q_e v_y \times B_x$ ties together both and provides for the fact that the electron is accelerated in the direction of the positive z-axis, so in parallel with \underline{k} . When using formula (1) results for

$$a_z = \frac{q_e^2 E_{max} B_{max} (1 - \cos(W \cdot t)) e^{i\omega t}}{-4 m_e^2 \omega (\omega^2 - W^2)} \left(e^{i\omega t} (i\omega^2 \cos(Wt) - i\omega^2 + \omega W \sin(Wt) + iW^2) - iW^2 \right) \quad (8)$$



When compared to the preceding image, it can be seen that there is a temporal compression and doubling of the frequency. The cause is not inherent "non-linearity" of the electron, but the fact that in the far field of every electromagnetic wave the electric and magnetic fields reverse polarity at the same time. The integration of equation (8) provides the speed, whose real part is

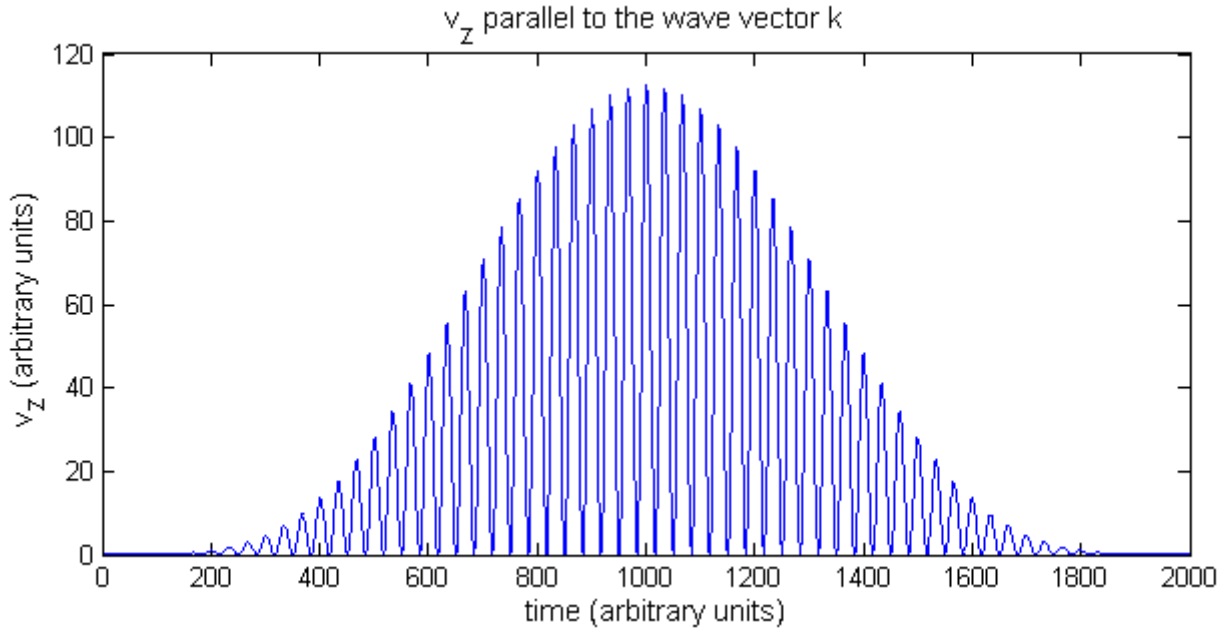
$$v_z = \frac{q_e^2 \mu_0 S_y}{m_e^2} \frac{R_1 \cdot R_2}{4 \omega^6} \quad \text{with} \quad (9)$$

$$R_1 = \cos(\omega t) \cos(Wt) \omega^2 - W^2 - \cos(\omega t) (\sin(Wt) W + \omega) \omega$$

$$R_2 = \cos(\omega t) \cos(Wt) \omega^2 - W^2 + \cos(\omega t) (\sin(Wt) W - \omega) \omega$$

The integration constant has been chosen so that at time $t = 0$ the initial condition $v_z = 0$ is satisfied. The following figure shows that the velocity v_z is never negative, and therefore the unbound electron will only move *away* from the light source. The same result is reached by elementary numerical integration (see Appendix). It is noteworthy that the initial and final velocities are equal, so that the electron is not permanently accelerated. Therefore, this displacement of the electron consumes no energy and is not a consequence of the radiation pressure.

Evidently, the waveform shown in the image below is the superposition of a cosine pulse and a wave packet with the frequency 2ω . Accordingly, the Fourier analysis of v_z provides two well-defined frequency ranges: A very broad band with a focus on W and a narrow band with the centre frequency 2ω . In both of these frequency bands energy is emitted at the expense of the primary wave.



By integration over the period $0 \leq W \cdot t < 2\pi$, the distance travelled by the electron is calculated during the passage of the wave packet

$$z = \frac{q_e^2 \mu_0 S_y \pi W^3}{m_e^2 2 \omega^6} \quad (10)$$

This displacement in the z-direction can be observed only in unbound electrons, because their mass is sufficiently small. If the primary wave hits atoms and stimulates their electron shells to vibrate, the far higher total mass of the atom must be inserted into the formula. Therefore, z is reduced by at least a factor of $\left(\frac{m_{proton}}{m_{electron}}\right)^2 = 1836^2 \approx 3.4 \cdot 10^6$ and is thus negligible.

The average velocity component of the electron in parallel with the wave vector \underline{k} and the z-axis is

$$v_z(\text{mean}) = \frac{q_e^2 \mu_0 S_y W^4}{m_e^2 4 \omega^6}$$

The wave packet loses no net energy by the displacement of the electron, because the electron subsequently is at rest. In the period $0 \leq W \cdot t < \pi$ radiation energy is temporarily converted into kinetic energy and transforms back in the period $\pi \leq W \cdot t < 2\pi$. Overall, there is no momentum transfer from the wave packet to the electron.

These two consecutive accelerated movements cause the low frequency radiation, which is discussed below. The z-velocity obviously includes two different frequency components, which are discussed separately.

8 High-frequency emission due to the z-acceleration (2ω)

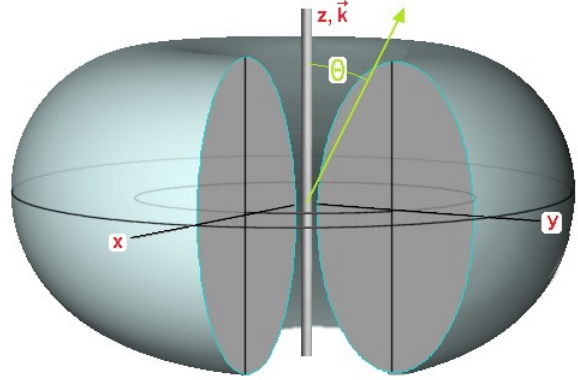
The incoming wave packet accelerates the electron by the interaction of electric and magnetic field components in the z-direction at the frequency 2ω , in parallel with the wave vector \underline{k} . First, the power is calculated with the Larmor formula using equation (8)

$$P_z = \frac{q_e |a_z|^2}{6\pi \varepsilon_0 c^3} = \frac{q_e^6 \mu_0^3 S_y^2}{24\pi c m_e^4} \frac{(\sin(\omega t))^4 (\cos(Wt) - 1)^4}{\omega^2}$$

The integration over the period $0 \leq W \cdot t < 2\pi$ results in the value of the radiated energy:

$$A_z(\text{fast}) = \frac{q_e^6 \mu_0^3 S_y^2}{24\pi c m_e^4 \omega^2} \frac{105\pi}{32W} = \frac{35 q_e^6 \mu_0^3 S_y^2}{256 c m_e^4 \omega^2 W} \quad (11)$$

It should be noted that the power P_z is generated and emitted during a very short interval. The radiation pattern of the secondary radiation of a single electron is a torus, as we know from a dipole antenna. The axis of which is the \mathbf{k} -vector of the primary wave and whose maximum radiation lies in the xy-plane ($\Theta \approx \pi/2$). The intensity of the secondary wave in the z-direction ($\Theta = 0$), that is parallel to the \mathbf{k} -vector, is zero, because no longitudinal electromagnetic waves exist. Nothing is changed when the primary wave is circularly polarized.

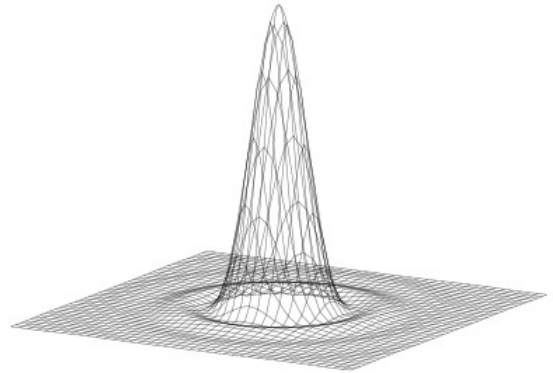


Because of the rotational symmetry of the torus, the radiation energy can *not* create a pulse component in the x-y-plane, and therefore also causes *no change* in the direction of the \mathbf{k} -vector. This could reignite the discussion of the hypothesis "tired light". Although the wave packet loses energy, there is no scattering in the physical sense, which is of course always connected with a change in direction of the \mathbf{k} -vector. The energy loss of the primary wave described above produces *no image blur* - for astronomical observations or during irradiation of compact transparent material such as lenses. If only a few electrons vibrate, the energy loss will be very small and difficult to detect. Whether it can be ignored in any case, is not the subject of this physical derivation and deserves a separate investigation.

9 Radiation pattern with a small mutual spacing of the electrons

The torus is strongly distorted when the wave packet of the primary wave stimulates not only a single electron to oscillate, but many that are in close proximity. In solids, the mutual distance of the electrons and electron shells is much smaller than the wavelength, this simplifies the description.

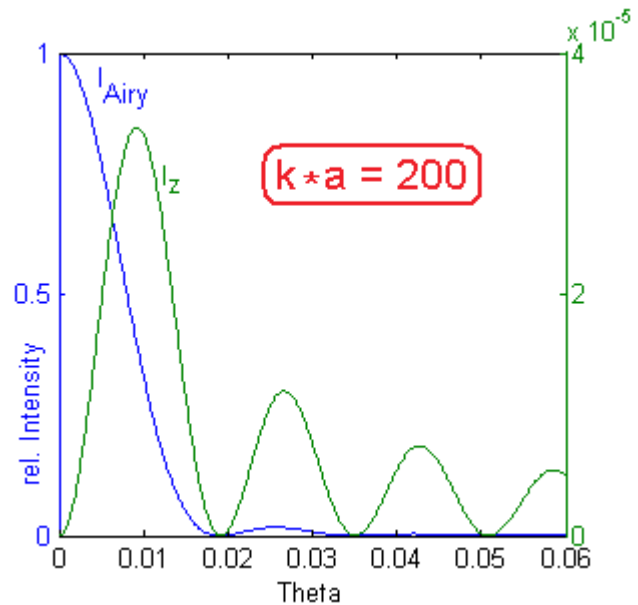
If the electrons form a circular area having a diameter $2a$ significantly exceeding the wavelength and the polarization is parallel to the surface, the radiation focuses close to the surface normal ($\Theta = 0$) and the intensity function is the known Airy disk, as shown in the image. The surface normal of the circle indicates the direction of maximum intensity. With increasing angular deviation from the normal, the intensity decreases rapidly and, in particular in the plane of the circular surface, the radiation disappears due to destructive interference.



Since the polarization of the secondary wave is parallel to the surface normal, any radiation in the direction of the normal is prohibited. Therefore, a compromise between the conflicting radiation patterns torus and Airy directivity pattern must be found.

If a glass cylinder of radius a is illuminated with light in parallel to the axis of symmetry, the plane wave fronts of the primary electromagnetic wave are parallel to the x-y-plane of the radiation. All electrons in any x-y-plane are forced into synchronous oscillations. As shown above, the velocity in conjunction with the magnetic field of the primary wave generates a very small movement of all electron clouds in the z-direction. Destructive interference ensures that in the x-y-plane ($\Theta \approx \pi / 2$) no secondary wave is emitted.

The compromise is possible, if the radiation patterns torus and Airy disc are mathematically formulated and multiplied together:



a) Electrons oscillating in the z-direction do not radiate energy in the z-direction ($\Theta = 0$) because longitudinal electromagnetic waves do not exist. For the radiated field strength, the formula is $E \sim \sin(\Theta)$.

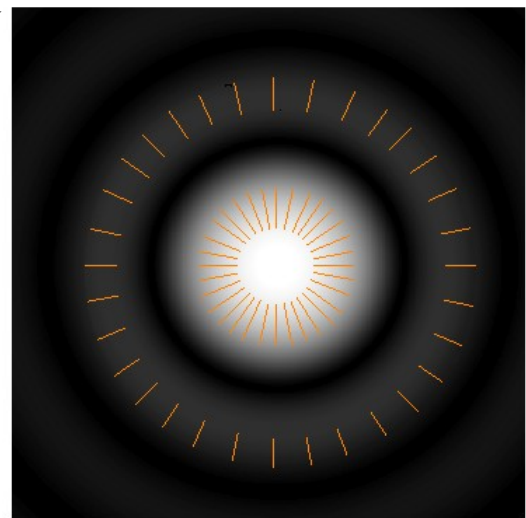
b) For constructive interference applies $E \sim \frac{2J_1(ka \sin(\Theta))}{ka \sin(\Theta)}$ as for the diffraction at a circular aperture, where J_1 is the Bessel function of the first kind and a is the radius of the glass cylinder.

The intensity of the radiation generated by the synchronous movement of the electrons in the z-direction, is the square of the product

$$I_z \sim (E_z)^2 \sim \left(\frac{2J_1(ka \sin(\Theta))}{ka} \right)^2 \quad (12)$$

As the picture shows, the radiation power of the secondary radiation disappears for $\Theta = 0$ and has a maximum for $1,842 = ka \sin(\Theta)$. After the light has passed through the glass cylinder, an analysis should show, that that the strong primary wave can be separated from the considerably weaker secondary wave due to different polarization and angle of observation.

The primary wave does not change polarization and frequency and, after a sufficient distance, the intensity profile corresponds to the well known Airy disk with a strong central peak, the angle of the first intensity minimum (innermost dark ring) can be calculated with the well-known formula $(\sin \Theta)_{Airy} = 1,22 \frac{\lambda}{2a}$. In the image, the intensity of the primary wave is indicated by the area brightness.



The highest intensity of the secondary wave (2ω) is observed on concentric circular rings. For half the opening angle of the innermost circle, the formula is $(\sin \Theta)_{2\omega} = 0,586 \frac{\lambda}{2a}$. In the picture orange lines indicate two annular areas in which the secondary radiation should be measurable. The main difference, however, is to be expected in terms of polarization. The orientation of the lines shows the position-dependent linear polarization of the secondary wave. This is independent of the polarization of the primary wave, and upon whether it is linear or circular. Illustratively stated, the orange lines are short sections of the z-axis - seen from the respective view. In this section, only the electron motion in parallel to the z-axis is discussed.

An analysis in the opposite direction of the primary wave can facilitate the detection of the secondary wave, because there the intensity of the reflected primary wave can be significantly reduced by a suitable coating of both ends of the cylinder.

10 Low-frequency radiation as a result of z-acceleration

In the z-direction, the electron is accelerated with two different time constants. The previous section dealt with the radiation of frequency 2ω , subsequent the low-frequency component is discussed, which can be observed only in unbound electrons. A suitable low-pass filtering of equation (9) yields the mean velocity of the electrons:

$$v_z(\text{slow}) = \frac{q_e^2 \mu_0 S_y}{m_e^2} \frac{(1 - \cos(Wt))^2}{8\omega^2} \quad (13)$$

$$a_z(\text{slow}) = \frac{q_e^2 \mu_0 S_y}{m_e^2} \frac{(1 - \cos(Wt)) \sin(Wt)}{4\omega} \quad (14)$$

One first calculates the power with the Larmor formula

$$P_z(\text{slow}) = \frac{(q_e \cdot a_z(\text{slow}))^2}{6\pi \epsilon_0 c^3} = \frac{q_e^6 \mu_0^3 S_y^2}{96\pi c m_e^4} \frac{((\cos(Wt) - 1) \sin(Wt))^2}{\omega^2}$$

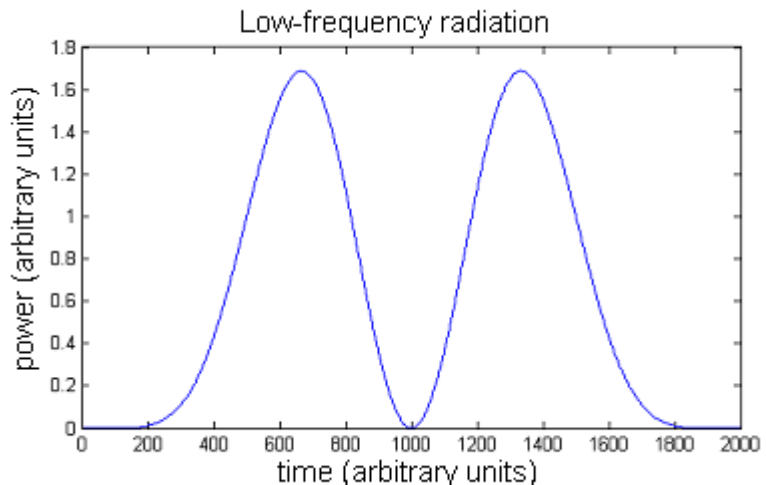
and then integrates over the period of the wave packet. The radiated energy is

$$A_z(\text{slow}) = \frac{q_e^6 \mu_0^3 S_y^2}{96\pi c m_e^4 \omega^2} \frac{5\pi}{4W} = \frac{5 q_e^6 \mu_0^3 S_y^2}{384 c m_e^4 \omega^2 W} \quad (15)$$

Since the movement of the electrons is considered parallel to the \mathbf{k} -vector, the energy is mainly emitted in the x-y-plane.

The picture shows, that the low-frequency radiation is not continuously emitted during the period $0 \leq W \cdot t < 2\pi$, it focuses around the two time points

$$t_1 \approx \frac{2\pi}{3W} \quad \text{and} \quad t_2 \approx \frac{4\pi}{3W},$$



measured from the start of the primary wave. The difference depends on the contour of the envelope.

Since the electric field strength is proportional to $a_z(\text{slow})$, the mean radiated frequency corresponds to the variable factor $f(\text{slow}) \approx (1 - \cos(Wt)) \sin(Wt)$. Because the period of oscillation differs only slightly from the duration of the wave packet, the frequency can only be imprecisely defined, the bandwidth of this secondary wave is very large and the signal is closer to that of a noise. For the polarization, the same considerations apply as for the high-frequency component.

Example: From the wavelength $\lambda_{\text{prim}} = 550 \text{ nm}$ and line width $\Delta\lambda = 2 \text{ nm}$ follows

$W = 6,23 \cdot 10^{12} \text{ Hz}$. If light passes through thin plasma, secondary radiation is a broadband spectrum in the frequency range $f(\text{slow}) \approx 1,5 \cdot 10^{12} \text{ Hz}$ corresponding $\lambda(\text{slow}) \approx 200 \mu\text{m}$. Interestingly, the cosmic background radiation is measured approximately the same range. This radiation should be detectable perpendicular to the direction of propagation of the primary wave.

11 Specific energy loss as a result of z-acceleration

It is not possible for a free electron to store energy and therefore radiates all of the energy which it receives from the primary wave. Every time a wave packet excites a free electron to oscillate, it loses a tiny fraction of its energy, and then continues running with slightly reduced frequency. This extends the wavelength of the primary wave by a tiny amount (redshift). For the three energy components, the following relationships apply:

$$A_y = \frac{q_e^4 \mu_0^2 S_y}{8 m_e^2 W} \quad (6)$$

$$\frac{A_z(\text{fast})}{A_z(\text{slow})} = \frac{21}{2} \quad A_z(\text{fast}) + A_z(\text{slow}) = \frac{115 q_e^6 \mu_0^3 S_y^2}{768 c m_e^4 \omega^2 W}$$

$$R = \frac{A_z(\text{fast}) + A_z(\text{slow})}{A_y} = \frac{115 q_e^2 \mu_0}{96 c m_e^2} \cdot \frac{S_y}{\omega^2} \quad (16)$$

Of particular interest is the energy or power ratio R , when an electron is stimulated by the primary wave packet to execute short-term oscillations it radiates secondary waves. The relative energy loss does not depend on the modulation frequency W for the duration of the wave packet. Because the energy difference is emitted with *no preferred direction*, the initial wave packet undergoes no change of direction, there is no scattering in the classic sense.

12 Concluding Remarks

The trigger for this investigation was a side note in http://en.wikipedia.org/wiki/Thomson_scattering (version October 25, 2013). This is "... *the main cause of the acceleration of the particle will be due to the electric field component of the incident wave, and the magnetic field can be neglected.*"

Really? After only two hours programming work for a simulation using the Matlab program (see Appendix) amazing features of the electron motion in the z-direction showed up. They could not be removed by program changes (for example, better Integration procedures or smaller step size). The rest of the examination was - despite the use of symbolic algebra programs - the painstaking proof that the simulation is correct. Perhaps the side note will be adjusted to reality someday.

Although the results of this work may contribute to the discussion about interesting aspects of *tired light*, this was neither starting point nor target of the investigation.

13 Equipment:

Eugene Hecht: Optics

John David Jackson: Classical Electrodynamics

The Software packets „Mathematica“ , „Reduce“ and „Matlab“

OpenOffice Writer

perseverance and the infinite patience of my wife

14 Links:

Szu-yuan Chen, Anatoly Maksimchuk & Donald Umstadter: Experimental observation of nonlinear Thomson scattering, <http://arxiv.org/pdf/physics/9810036v3.pdf>

15 Appendix

The following Matlab program is a simple simulation without any mathematical tricks. It generates some pictures to illustrate some results mentioned in this investigation. After copying in the Matlab editor window, it should be stopped after the selected rows (breakpoint) to look at the intermediate results carefully. A change from the Hann Window to the Blackman Window shows that the shape of the envelope has little effect. It would be a great pleasure if someone would discover the correct form of the envelope.

```
%Thomson scattering; Author: Herbert Weidner
qe=-1.6e-19; me=9.11e-31; c=3e8; %basic units
cc=100; %points per oscillation
lambda=1e-6; %Meter
T=lambda/c; dt=T/cc;
S=1e6; %W/m2 surface power density (Poynting)
%wave propagation z direction
Ey0=sqrt(377*S); %linear polarzation; Z0=377 Ohm, points in y-direction
Bx0=12.566e-7*S/Ey0; %mu0, points in -x-direction
%z-direction to the observer, E x B = k (right-handed)
%a free electron is moved
c=30; %number of cycles
yz=zeros(2,c*cc); vyz=zeros(2,c*cc); %az=zeros(1,c*cc);
n=1:c*cc; %ma(y)=qE
%in the far field E and B are in-phase
ay=(hann(c*cc))' .*sin(2*pi*n/cc)*qe*Ey0/me; %x-acceleration
Bx=- (hann(c*cc))' .*sin(2*pi*n/cc)*Bx0; %- because EBk is right-handed
for n=2:c*cc-1 %compute vy and y
    vyz(1,n)=vyz(1,n-1)+dt*(ay(n-1)+ay(n))/2;
    yz(1,n)=yz(1,n-1)+dt*(vyz(1,n-1)+vyz(1,n))/2;
end
az=-qe*vyz(1,:).*Bx/me; %qv(y)B(x)=ma(z)
plot(vyz(1,:)); ylabel('y-velocity (m/s)');
title('wave packet, 30 oscillations'), xlabel('time')
%----Breakpoint in the following line -----
for n=2:c*cc-1 %compute vz and z
    vyz(2,n)=vyz(2,n-1)+dt*(az(1,n-1)+az(1,n))/2;
```

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    yz(2,n)=yz(2,n-1)+dt*(vyz(2,n-1)+vyz(2,n))/2;
end
plot(vyz(2,:)); ylabel('z-velocity (m/s)');
title('velocity of the electron parallel to the k-Vektor')
xlabel('time (arbitrary units)')
n=1:size(vyz,2);
%----Breakpoint in the following line -----
plotyy(n,az,n,vyz(2,:));
ylabel('left: az, right: vz (m/s)');
title('a and v in z-direction'), xlabel('time')
%----Breakpoint in the following line -----
plotyy(n,vyz(1,:),n,vyz(2,:));
ylabel('left: vy, right: vz (m/s)');
title('velocity in y- and z-directions')
xlabel('time (arbitrary units)')
%----Breakpoint in the following line -----
plotyy(n,yz(1,:),n,yz(2,:));
ylabel('left: y, right: z in m');
title('coordinates of the electron in y- and z-direction')
xlabel('time (arbitrary units)')
%----Breakpoint in the following line -----
fprintf(1, 'vy/vz= %e\n',max(abs(vyz(2,:)))/max(vyz(1,:)))
%path of the electron in the y-z-plane
scatter(yz(1,:),yz(2,:),'.')
xlabel('deflection in y-direction (in m)')
ylabel('deflection in z-direction (in m)')
title('path of the electron in the y-z-plane')

```

Herbert Weidner, 11. December 2013