Brief Comment on "Dimensional Transmutation by Monopole Condensation in QCD"

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Abstract: Approaches to confinement which employ "monopole condensation" and "Abelian dominance" conjectures to explain chromodynamic color confinement are on the right track, but simply do not go far enough. These are complementary, not competing conjectures. They reach their logical conclusion when married together in the view that baryons are one and the same as the magnetic monopoles of Yang-Mills gauge theory and that confinement results from the Abelian aspects of these Yang-Mills monopoles.

In paper [1] which is the subject of this comment, the authors correctly identify "monopole condensation" and "Abelian dominance" as the two "outstanding" conjectures for explaining color confinement in the $SU(3)_C$ chromodynamic theory of strong nuclear interactions. The shortcoming of this paper is that it simply does not go far enough. By adopting the view that these are "competing conjectures" and not complementary ones, and by concluding that it is "the monopole condensation which is responsible for the confinement," the authors adopt a mutually-exclusive, "either / or" stance with respect to these two conjectures. In so doing, they deprive themselves and others of the key insight that confinement arises from what is an inseparable combination of *both* of these conjectures, namely, that baryons are *one and the same* as the magnetic monopoles of Yang-Mills gauge theory and that confinement results from the Abelian subset behaviors of these Yang-Mills monopole baryons.

The foregoing is a conceptual problem, but it is to some degree rooted in the mathematical difficulty that many have faced to date in separating the Abelian from non-Abelian aspects of Yang-Mills gauge theory in gauge-independent fashion. Additionally, while using such simplifications as SU(2) Chromodynamics may be helpful, this is simply one of the "variety of highly oversimplified models" which Jaffe and Witten state at page 3 of [2] provide only a "severely simplified truncation" for explaining confinement and the other QCD properties that fall under the rubric of the Yang-Mills and Mass Gap Problem. It also bears emphasis that this Problem, see page 6 of [2], requires solution proponents to "prove that for *any* compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap > 0. . ." Thus, even a convincing explanation of confinement using SU(3) rather than SU(2) would by itself be no more than a partial solution to this problem, because the requirement is to provide a more general proof that there is a mass gap for <u>any</u> compact simple gauge group, without restriction as to group G. So while a better understanding of SU(3) Chromodynamics is a very necessary part of resolving the Yang-Mills and Mass Gap Problem, it is by itself insufficient. This Problem requires a better conceptual understanding and mathematical exposition of *Yang-Mills gauge theory in general*, which does not rely on SU(2) or SU(3) or any other *specific* compact simple gauge group.

As to the particular problem of confinement, the authors in [1] are sniffing at the correct "scent" of a solution, as is anyone who conjectures in one form or another that the magnetic monopoles of Yang-Mills gauge theory or Meissner effects or some Yang-Mills version of magnetism has something very substantial to do with confinement. Again, the problem is not that these efforts are wrong-headed, but that they are right-headed but merely not audacious enough. The main shortcoming of [1] is that "monopole condensation" and "Abelian dominance" are treated as if one or the other is the right solution when in fact the right solution requires an organic combination of both.

As the author of this comment has shown in a separate preprint paper [3], the confinement problem is understood by recognizing that baryons themselves – including the observed protons and neutrons – are *one and the same* as the non-Abelian magnetic monopoles of Yang-Mills gauge theory, and that confinement results from the Abelian behaviors of these Yang-Mills monopoles, which monopoles become topologically-stable via spontaneous symmetry breaking according to the approach laid out by the author in sections 6 through 8 of [4]. To use the language of [1], the observed baryons are the "monopole condensate," while the confinement of color (no free gauge or quark fields) arises from "Abelian dominance" within these monopoles. In other words, confinement emanates from that portion of the Yang-Mills monopoles which in an Abelian gauge theory such as Maxwell's electrodynamics is responsible for the absence of *any* net magnetic field flux across any closed two-dimensional surface, and which is colloquially thought of in electrodynamics as the simple non-existence of magnetic monopoles.

Specifically, in differential forms, $P = dF = ddG = 0$ with $F = dG$ expresses the absence of magnetic monopoles in electrodynamics, and makes integral use of the geometric result $dd = 0$ rooted in the Bianchi identity $R_{\tau\sigma\mu\nu} + R_{\tau\mu\nu\sigma} + R_{\tau\nu\sigma\mu} = 0$ that "the exterior derivative of an exterior derivative is zero." Thus, using Gauss' / Stokes' theorem, the integral form of the Abelian monopole equation is:

$$\iiint P = \iiint dF = \iiint ddG = \oint\!\!\oint F = \oint\!\!\oint F^{\mu\nu} dx_\mu dx_\nu = \oint\!\!\oint dG = 0 . \tag{1}$$

As is very well known, one may extract Maxwell's magnetic charge equation, which is Gauss' law for Magnetism, in the integral form $\oint\!\!\oint \mathbf{B} \cdot d\mathbf{A} = 0$, from the space-space *ij* bivector components of $\oint\!\!\oint F^{\mu\nu} dx_\mu dx_\nu = 0$.

But in Yang-Mills gauge theory, where the field strength $F = DG = dG - iG^2$ with $D = d - iG$ being the gauge-covariant extension of the exterior derivative, the vanishing Abelian monopole equation $P = dF = ddG = 0$ is replaced by the non-vanishing:

$$\begin{aligned} P = DF = (d - iG)F = D(dG - iG^2) = (d - iG)(dG - iG^2) = ddG - idG^2 - iGdG - G^3 \\ = \mathbf{0} - i(dG^2 + GdG) - G^3 = \mathbf{0} - i(dG^2 + GDG) \end{aligned} , \tag{2}$$

which still embeds $dd = \mathbf{0}$ as a subset equation.

This means that the gauge-group independent counterpart to (1) in Yang-Mills theory is:

$$\begin{aligned}\iiint P &= \iiint DF = \oiint F - \iiint iGF = \iiint \left(ddG - i\left(dG^2 + GdG \right) - G^3 \right) = \iiint \left(-i\left(dG^2 + GdG \right) - G^3 \right) \\ &= \oiint dG - i\oiint G^2 - \iiint \left(iGdG + G^3 \right) = \mathbf{0} - i\oiint G^2 - \iiint \left(iGdG + G^3 \right) \\ &= \oiint dG - i\oiint G^2 - i\iiint GDG = \mathbf{0} - i\oiint G^2 - i\iiint GDG\end{aligned} \quad .(3)$$

Here $\iiint P = \oiint F = 0$ now becomes $\iiint P = \oiint F - i\iiint GF = -i\oiint G^2 - i\iiint GDG$. The above is the equation for the magnetic monopoles of Yang-Mills gauge theory, and it still embeds an Abelian portion $\oiint dG = 0$. But because $\oiint dG = 0$, this means that under the local gauge-like transformation $F \to F' = F - dG$, a.k.a. $F^{\mu\nu} \to F^{\mu\nu}{}' = F^{\mu\nu} - \partial^{[\nu}G^{\mu]}$, the monopole volume integral transforms invariantly as:

$$\iiint P = \oiint F \to \oiint F' = \oiint \left(F - dG \right) = \oiint F = \iiint P \quad . \qquad (4)$$

Consequently, that there is no *net* flux of any colored gauge field across any *closed* surface of the monopole volume. The "monopole condensate" is represented by and enclosed within the three-volume $\iiint P$, the "Abelian dominance" is expressed by $\oiint dG = 0$, and the symmetry of $\iiint P$ under $F \to F' = F - dG$ by virtue of $\oiint dG = 0$ is the mainspring of confinement and draws from *both* of these two "outstanding" conjectures referenced by [1]. The Abelian expression $\oiint dG = 0$ which represents an absence of monopoles in electrodynamics, yields the *symmetry principle* (4) for the confining behavior of monopoles in Yang-Mills theory generally.

It will of course be appreciated that all of the foregoing differential forms equations are relativistically-invariant, and gauge invariant, and do not depend on any lattice. And, it will be appreciated that (3) does "separate the monopole potential from the QCD potential gauge independently." This helps us to understand how these two "outstanding" conjectures are *not* mutually-exclusive, but instead coact together to cause at least the bi-colored gauge fields to be confined by the Abelian behaviors of the monopole / baryon.

But while $\oiint dG = 0$ prevents any net flux of gauge fields, (3) goes even further to inform us that the only thing which *does* flow in and out of these monopoles, is whatever is represented by the surface-integral term $\oiint G^2$, while whatever is contained within the non-integrable terms $\iiint GDG$ remains confined. As the author shows in section 11 of [3], this $\oiint G^2$ term represents a net flow of mesons with a color-neutral symmetric wavefunction of $\overline{R}R + \overline{G}G + \overline{B}B$ over closed monopole surfaces. So by implication, individual quarks remain confined because they are not color-neutral and the only entities permitted to net flow over a closed surface are $\overline{R}R+\overline{G}G+\overline{B}B$ color-neutral mesons. Of further importance is that the monopole condensate $\iiint P=\iiint P^{\sigma\mu\nu}dx_{\sigma}dx_{\mu}dx_{\nu}$ contains a third rank antisymmetric tensor $P^{\sigma\mu\nu}$ which itself is shown in [3] to have precisely the colorless antisymmetric wavefunction $R[G,B]+G[B,R]+B[R,G]$ required for a baryon.

So in sum, a complete understanding of confinement must marry both Abelian dominance and Yang-Mills monopole condensation together into the view that the observed baryons, including the proton and neutron flavors of baryon, are synonymous with Yang-Mills monopoles which inherently are both color-neutral and via their Abelian subset properties, color-confining.

References

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[3] J. Yablon, *Filling the Mass Gap: Chromodynamic Symmetries, Confinement Properties, and Short-Range Interactions of Classical and Quantum Yang-Mills Gauge Theory*, http://vixra.org/abs/1310.0057 (2013)

[4] J. Yablon, *Why Baryons Are Yang-Mills Magnetic Monopoles*, Hadronic Journal, Volume 35, Number 4, 401-468 (2012) Link: http://www.hadronicpress.com/issues/HJ/VOL35/HJ-35-4.pdf