Ten conjectures about certain types of pairs of primes arising in the study of 2-Poulet numbers

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Abstract. There are many interesting, yet not studied enough, properties of Poulet numbers. In particular, the study of the 2-Poulet numbers appears to be most seductive because in their structure are found together three of the most important concepts in number theory: those of primes, semiprimes and pseudoprimes. In this paper we make few conjectures about primes or pairs of primes, including twin primes, that could be associated to the pairs of primes represented by the two prime factors of a 2-Poulet number.

Conjecture 1:
There is an infinity of pairs of primes of the form (p, p + 24).
Note: It is not necessarily for the two primes to be consecutive, as is stipulated in de Polignac's Conjecture.

Conjecture 2:
There is, for any odd prime q, q > 3, an infinity of primes of the form q + p*(p + 24) – 1, where p and p + 24 are both primes.

Examples:
For q = 5, we have:
: 5 + 17*41 – 1 = 701, prime;
: 5 + 19*43 – 1 = 821, prime.
For q = 7, we have:
: 7 + 5*29 – 1 = 151, prime;
: 7 + 19*43 – 1 = 823, prime.

Conjecture 3:
There is, for any pair of twin primes (p, q), (p, q) ≠ (3, 5), an infinity of pairs of twin primes of the form (p + m*n – 1, q + m*n – 1), where m and n are both primes and n = m + 24.

Examples:
For (p, q) = (5, 7) we have:
: 5 + 19*43 – 1 = 821 and 7 + 19*43 – 1 = 823, primes.
For (p, q) = (11, 13) we have:
: 11 + 7*31 – 1 = 227 and 13 + 7*31 – 1 = 229, primes.

Conjecture 4:
There is an infinity of 2-Poulet numbers P, $P = p_1*p_2$, for which $p_2 - p_1 + 1 = q_1*q_2$, where q_1 and q_2 are primes and $q_2 - q_1 = 24$.

Note: In other words, we can associate to an infinity of pairs of primes (p_1, p_2), where p_1 and p_2 are the two prime factors of a 2-Poulet number, another pair of primes (q_1, q_2).

Examples:
For P = 129889 = 193*673 we have:
673 – 193 = 13*37 and 37 – 13 = 24.
For For P = 130561 = 137*953 we have:
953 – 137 = 19*43 and 43 – 19 = 24.

Conjecture 5:
There is an infinity of 2-Poulet numbers P, $P = p_1*p_2$, for which $p_2 - p_1 + 1 = q$, where q is prime or a square of prime.

Note: In other words, we can associate to an infinity of pairs of primes (p_1, p_2), where p_1 and p_2 are the two prime factors of a 2-Poulet number, a prime or a square of prime q.

Examples:
For P = 2047 = 23*89 we have: 89 – 23 + 1 = 67, prime.
For P = 2701 = 23*89 we have: 73 – 37 + 1 = 37, prime.
For P = 4033 = 37*109 we have:109 – 37 + 1 = 73, prime.
For P = 4369 = 17*257 we have:257 – 17 + 1 = 241, prime.
For P = 4681 = 31*151 we have:151 – 31 + 1 = 11^2.

Conjecture 6:
There is an infinity of 2-Poulet numbers P, $P = p_1*p_2$, for which $p_2 - p_1 - 1 = q$, where q is prime or a square of prime.

Examples:
For P = 4033 = 37*109 we have:109 – 37 - 1 = 71, prime.
For P = 8321 = 53*157 we have:157 – 53 – 1 = 103, prime.

Conjecture 7:
There is an infinity of 2-Poulet numbers P, $P = p_1*p_2$, for which $p_2 - p_1 - 1$ and $p_2 - p_1 + 1$ are both primes.

Conjecture 8:
For any pair of twin primes (q_1, q_2) there exist at least a pair of primes (p_1, p_2) such that $q_1 = p_2 - p_1 - 1$ and $q_2 = p_2 - p_1 + 1$ are both primes and $P = p_1*p_2$ is a 2-Poulet number.

Conjecture 9:
There is an infinity of pairs of primes, not necessarily consecutive, of the form (p, p + 84).

Conjecture 10:
There is an infinity of 2-Poulet numbers P, $P = p_1*p_2$, for which $p_2 - p_1 = 84$.

Examples:
For P = 3277 = 29*113 we have: 113 – 29 = 84.
For P = 5461 = 43*127 we have: 127 – 43 = 84.

Reference:
See the sequence A214305 posted by us in OEIS for a list of Fermat pseudoprimes to base two (Poulet numbers) with two prime factors (2-Poulet numbers).