The cyclic variation in the density of primes in the intervals defined by the Fibonacci sequence

J. S. Markovitch
P.O. Box 2411
West Brattleboro, VT 05303[*]

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The Riemann R-function can be used to estimate the number of primes in an interval, where its accuracy is affected by the interval to which it is applied. Here, the successive intervals defined by the Fibonacci sequence will be shown to cause more cycles of R-function over- and under-estimation of primes than any of a large landscape of related sequences (calculations were continued up to one billion). The size of this landscape suggests that a special relationship exists between the Fibonacci sequence and the distribution of primes.

I. INTRODUCTION

The author earlier explored the distribution of primes, where Riemann's R-function was shown to alternately *under*- and *over*-estimate the number of primes in the intervals defined by the Fibonacci sequence, specifically from the interval $(55, 89]$ to the interval $(317\,811, 514\,229]$ [1]. Below, the values on the right-hand-side reveal whether Riemann's R-function either under- or over-estimates the number of primes in an interval defined by consecutive Fibonacci numbers, where $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, etc., and where $R(x)$ equals `RiemannR[x]` in *Mathematica*. The function $\pi(x)$ gives the number of primes less than or equal to x.

$$\frac{R(F_4) - R(F_3)}{\pi(F_4) - \pi(F_3)} = \frac{R(3) - R(2)}{\pi(3) - \pi(2)} \approx 0.4636676 \tag{1}$$

$$\frac{R(F_5) - R(F_4)}{\pi(F_5) - \pi(F_4)} = \frac{R(5) - R(3)}{\pi(5) - \pi(3)} \approx 0.8164928 \tag{2}$$

$$\frac{R(F_6) - R(F_5)}{\pi(F_6) - \pi(F_5)} = \frac{R(8) - R(5)}{\pi(8) - \pi(5)} \approx 1.0800166 \tag{3}$$

$$\frac{R(F_7) - R(F_6)}{\pi(F_7) - \pi(F_6)} = \frac{R(13) - R(8)}{\pi(13) - \pi(8)} \approx 0.8014656 \tag{4}$$

$$\frac{R(F_8) - R(F_7)}{\pi(F_8) - \pi(F_7)} = \frac{R(21) - R(13)}{\pi(21) - \pi(13)} \approx 1.1490585 \tag{5}$$

$$\frac{R(F_9) - R(F_8)}{\pi(F_9) - \pi(F_8)} = \frac{R(34) - R(21)}{\pi(34) - \pi(21)} \approx 1.1231519 \tag{6}$$

$$\frac{R(F_{10}) - R(F_9)}{\pi(F_{10}) - \pi(F_9)} = \frac{R(55) - R(34)}{\pi(55) - \pi(34)} \approx 0.9881781 \tag{7}$$

$$\frac{R(F_{11}) - R(F_{10})}{\pi(F_{11}) - \pi(F_{10})} = \frac{R(89) - R(55)}{\pi(89) - \pi(55)} \approx 0.9129715 \tag{8}$$

$$\frac{R(F_{12}) - R(F_{11})}{\pi(F_{12}) - \pi(F_{11})} = \frac{R(144) - R(89)}{\pi(144) - \pi(89)} \approx 1.0844899 \tag{9}$$

$$\frac{R(F_{13}) - R(F_{12})}{\pi(F_{13}) - \pi(F_{12})} = \frac{R(233) - R(144)}{\pi(233) - \pi(144)} \approx 0.9522431 \tag{10}$$

$$\frac{R(F_{14}) - R(F_{13})}{\pi(F_{14}) - \pi(F_{13})} = \frac{R(377) - R(233)}{\pi(377) - \pi(233)} \approx 1.0552858 \tag{11}$$

[*] jsmarkovitch@gmail.com

$$\frac{R(F_{15}) - R(F_{14})}{\pi(F_{15}) - \pi(F_{14})} \approx 0.9877612 \tag{12}$$

$$\frac{R(F_{16}) - R(F_{15})}{\pi(F_{16}) - \pi(F_{15})} \approx 1.0044540 \tag{13}$$

$$\frac{R(F_{17}) - R(F_{16})}{\pi(F_{17}) - \pi(F_{16})} \approx 0.9859970 \tag{14}$$

$$\frac{R(F_{18}) - R(F_{17})}{\pi(F_{18}) - \pi(F_{17})} \approx 1.0205233 \tag{15}$$

$$\frac{R(F_{19}) - R(F_{18})}{\pi(F_{19}) - \pi(F_{18})} \approx 0.9836314 \tag{16}$$

$$\frac{R(F_{20}) - R(F_{19})}{\pi(F_{20}) - \pi(F_{19})} \approx 1.0039880 \tag{17}$$

$$\frac{R(F_{21}) - R(F_{20})}{\pi(F_{21}) - \pi(F_{20})} \approx 0.9993921 \tag{18}$$

$$\frac{R(F_{22}) - R(F_{21})}{\pi(F_{22}) - \pi(F_{21})} \approx 1.0004330 \tag{19}$$

$$\frac{R(F_{23}) - R(F_{22})}{\pi(F_{23}) - \pi(F_{22})} \approx 0.9982875 \tag{20}$$

$$\frac{R(F_{24}) - R(F_{23})}{\pi(F_{24}) - \pi(F_{23})} \approx 1.0032416 \tag{21}$$

$$\frac{R(F_{25}) - R(F_{24})}{\pi(F_{25}) - \pi(F_{24})} \approx 0.9987094 \tag{22}$$

$$\frac{R(F_{26}) - R(F_{25})}{\pi(F_{26}) - \pi(F_{25})} \approx 1.0003750 \tag{23}$$

$$\frac{R(F_{27}) - R(F_{26})}{\pi(F_{27}) - \pi(F_{26})} \approx 0.9997837 \tag{24}$$

$$\frac{R(F_{28}) - R(F_{27})}{\pi(F_{28}) - \pi(F_{27})} \approx 1.0004894 \tag{25}$$

$$\frac{R(F_{29}) - R(F_{28})}{\pi(F_{29}) - \pi(F_{28})} \approx 0.9995964 \tag{26}$$

$$\frac{R(F_{30}) - R(F_{29})}{\pi(F_{30}) - \pi(F_{29})} \approx 0.9992383 \tag{27}$$

$$\frac{R(F_{31}) - R(F_{30})}{\pi(F_{31}) - \pi(F_{30})} \approx 1.0005718 \tag{28}$$

$$\frac{R(F_{32}) - R(F_{31})}{\pi(F_{32}) - \pi(F_{31})} \approx 0.9993671 \tag{29}$$

$$\frac{R(F_{33}) - R(F_{32})}{\pi(F_{33}) - \pi(F_{32})} \approx 1.0006990 \tag{30}$$

$$\frac{R(F_{34}) - R(F_{33})}{\pi(F_{34}) - \pi(F_{33})} \approx 0.9999341 \tag{31}$$

$$\frac{R(F_{35}) - R(F_{34})}{\pi(F_{35}) - \pi(F_{34})} \approx 0.9998355 \tag{32}$$

$$\frac{R(F_{36}) - R(F_{35})}{\pi(F_{36}) - \pi(F_{35})} \approx 1.0000960 \tag{33}$$

$$\frac{R(F_{37}) - R(F_{36})}{\pi(F_{37}) - \pi(F_{36})} \approx 1.0000344 \tag{34}$$

$$\frac{R(F_{38}) - R(F_{37})}{\pi(F_{38}) - \pi(F_{37})} \approx 0.9999823 \tag{35}$$

$$\frac{R(F_{39}) - R(F_{38})}{\pi(F_{39}) - \pi(F_{38})} \approx 1.0000619 \tag{36}$$

$$\frac{R(F_{40}) - R(F_{39})}{\pi(F_{40}) - \pi(F_{39})} \approx 0.9999415 \tag{37}$$

$$\frac{R(F_{41}) - R(F_{40})}{\pi(F_{41}) - \pi(F_{40})} \approx 1.0000607 \tag{38}$$

$$\frac{R(F_{42}) - R(F_{41})}{\pi(F_{42}) - \pi(F_{41})} \approx 0.9999453 \tag{39}$$

$$\frac{R(F_{43}) - R(F_{42})}{\pi(F_{43}) - \pi(F_{42})} \approx 1.0000317 \tag{40}$$

$$\frac{R(F_{44}) - R(F_{43})}{\pi(F_{44}) - \pi(F_{43})} \approx 0.9999653 \tag{41}$$

$$\frac{R(F_{45}) - R(F_{44})}{\pi(F_{45}) - \pi(F_{44})} \approx 1.0000053 \tag{42}$$

$$\frac{R(F_{46}) - R(F_{45})}{\pi(F_{46}) - \pi(F_{45})} \approx 0.9999977 \tag{43}$$

$$\frac{R(F_{47}) - R(F_{46})}{\pi(F_{47}) - \pi(F_{46})} \approx 1.0000050 \tag{44}$$

$$\frac{R(F_{48}) - R(F_{47})}{\pi(F_{48}) - \pi(F_{47})} \approx 0.9999997 \tag{45}$$

$$\frac{R(F_{49}) - R(F_{48})}{\pi(F_{49}) - \pi(F_{48})} \approx 1.0000142 \tag{46}$$

$$\frac{R(F_{50}) - R(F_{49})}{\pi(F_{50}) - \pi(F_{49})} \approx 0.9999886 \tag{47}$$

$$\frac{R(F_{51}) - R(F_{50})}{\pi(F_{51}) - \pi(F_{50})} \approx 1.0000059 \tag{48}$$

$$\frac{R(F_{52}) - R(F_{51})}{\pi(F_{52}) - \pi(F_{51})} \approx 0.9999961 \tag{49}$$

$$\frac{R(F_{53}) - R(F_{52})}{\pi(F_{53}) - \pi(F_{52})} \approx 0.9999981 \tag{50}$$

$$\frac{R(F_{54}) - R(F_{53})}{\pi(F_{54}) - \pi(F_{53})} \approx 1.0000027 \tag{51}$$

Above, beginning at Eq. (8) and ending at Eq. (26), Riemann's R-function alternately under- and over-estimates the number of primes, where it is the even-numbered equations that underestimate. In addition, beginning at Eq. (34) and ending at Eq. (49) the above zigzag pattern again manifests itself, though for these equations it is the odd-numbered equations that underestimate. As noted by the author earlier with regard to different evidence [2, 3], it is possible that a sequence dense in primes tends to be followed by one less dense, which in turn is likely followed by one more dense, etc., where this tendency replicates itself at ever larger scales governed by the Fibonacci sequence.

This leads to the question of whether there is anything special about the Fibonacci sequence's ability to produce such patterns. Might other sequences, about as simple, perform as well or better? Can a brute-force computer search find alternatives to the Fibonacci sequence that achieve equally remarkable results, thereby showing that the Fibonacci sequence is not alone in its ability to produce such cyclic patterns? It is these questions that this article will address.

II. METHOD OF GENERATING AND SCORING VARIOUS SEQUENCES

In order to gauge the specialness of the Fibonacci sequence's relationship to the distribution of primes, one needs an algorithm for generating many related sequences, as well as some means of scoring how closely these alternatives match the Fibonacci sequence's above-described behavior. At this point this article will depart from the above analysis, by focusing not on the *length* of zigzag runs, but on the total number of zigs and zags. More precisely, sequences will be scored as follows:

$$\text{Score} = \#\text{ of transitions from over- to under-estimation } + \#\text{ of transitions from under- to over-estimation}$$

where over- and under-estimation are computed as earlier. Accordingly, a sequence causing a pattern of R-function (O)ver and (U)nder estimation such as "OUOU" would receive a score of three. In turn, the following patterns yield the following scores:

$$\begin{aligned} \text{OOOOO} &\rightarrow 0 \\ \text{UUUUUUUU} &\rightarrow 0 \\ \text{UOOOOO} &\rightarrow 1 \\ \text{UOOOOOUUUUU} &\rightarrow 2 \\ \text{OUOUO} &\rightarrow 4 \end{aligned}$$

In this way, the above scores equal the total number of *half-cycles of R-function misestimation.*

To generate the needed *landscape of alternate sequences* we note that for the Fibonacci sequence

$$F_n = F_{n-1} + F_{n-2}$$

where

$$F_1 = F_2 = 1 \quad ,$$

which produce $1, 1, 2, 3, 5, 8$, etc. By choosing alternatives to the initiators $(F_1, F_2) = (1, 1)$ an indefinite number of related alternatives to the Fibonacci sequence can be generated. In the tables that follow these alternative sequence initiators will be designated (I_1, I_2), where

$$I_n = I_{n-1} + I_{n-2} \quad .$$

So, whereas Eqs. (1)–(51) derive from the initiators

$$(I_1, I_2) = (2, 3) \quad ,$$

the scores in the first columns of Tables I, IV, and V derive from the initiators

$$\begin{aligned} (I_1, I_2) &= (2, 3) \\ (I_1, I_2) &= (2, 4) \\ &\vdots \\ (I_1, I_2) &= (2, 44) \quad . \end{aligned}$$

III. SUMMARY OF RESULTS

In Tables I–V the score described above is computed along the number line up to one billion (or ten million for Table V) for various sequence initiators. These scores occupy the body of each column, where the scores in $\boxed{\text{boxes}}$ are the highest scores *in each column.* Table VI is a special case.

- Table I gives scores for columns headed by the Fibonacci numbers 2 through 21. It is found that the column and row (I_1, I_2) "winning" every column is a pair of consecutive Fibonacci numbers.
- Table II gives scores for columns headed by Fibonacci numbers 34 through 6765. In order to make the row headers in the 20^{th} row consist of consecutive Fibonacci numbers, rows are numbered separately for each column. It is found that the column and row (I_1, I_2) "winning" every column is a pair of consecutive Fibonacci numbers, but there are also many scores tied for first that fan out above and below the table's 20^{th} row. Accordingly, one can only claim that a pair of consecutive Fibonacci numbers *do at least as well* as any other sequence initiators.
- Table III gives the same results as Table II, but with results presented using color; also, columns are now headed by Fibonacci numbers from 2 through 28 657; and there are now 53 rows. The continuous red bar across the table's 27^{th} row represents the highest scoring initiators in each column, specifically, the pairs of Fibonacci numbers: (2,3), (3,5), (5,8), (8,13), . . . , (28 657, 46 368). Again, the many first place ties (now in red) only allow one to claim that a pair of consecutive Fibonacci numbers do at least as well as any other sequence initiators. Note especially the many scores tied for first (in red) that fan out *above and below* the table's 27^{th} row.

- Table IV gives the same results as Table I, but with columns headed by *non*-Fibonacci numbers also filled. It is found that the columns headed by Fibonacci numbers contain maximum scores that are always higher than the maximum scores contained in the immediately adjacent columns headed by non-Fibonacci numbers.
- Table V gives the same results as Table I, but with primes computed only below ten million. It is found that there is no change in the Fibonacci sequence's favored status.
- Table VI is a special case, focusing on primes of the form $4n + 1$. It was computed the same way as Table III, but it only considers primes of the form $4n + 1$ below ten million. To facilitate comparison against the number of primes predicted by Riemann's R-function, each prime of the form $4n + 1$ was simply counted twice. In contrast to the results reported in Table III, the above conditions generally cause consecutive Fibonacci numbers to produce the *lowest* scores in each column. Hence, in a reversal of the previous scoring scheme, the lowest scores of Table VI appear in pink, with yellow, green, gold, and blue representing ever-higher scores (essentially, the same colors are used in the same order as before, but reversed and with pink replacing red). For some reason, double-counting primes of the form $4n + 1$ (while ignoring primes of the form $4n + 3$) causes the Fibonacci numbers to reverse earlier behavior and *suppress* the cycles of R-function over- and under-estimation. Curiously, if, instead, primes of the form $4n + 3$ are double-counted (while ignoring primes of the form $4n + 1$) no equivalent pattern emerges. And, finally, note the many scores tied for first (in pink) that fan out *below* the table's 27^{th} row.

It is important to keep in mind that for all of these tables, the top scorers in each column depend upon the scope of the column: so, simply adding or deleting rows in a column may alter the "column winner."

This point is made clearer by Tables VII–XVI, where each table has only a few rows. In these, the red and pink horizontal lines (again representing "winning" pairs of Fibonacci numbers) are in all instances continuous, which is partly a consequence of there being fewer competing scores in each column. Tables VII–XI copy Table III in considering all primes, but these tables have just seven rows, with calculations carried up to powers of ten as high as ten billion. Tables XII–XVI copy Table VI in considering only primes of the form $4n + 1$, but these also have just seven rows, again with calculations carried up to powers of ten as high as ten billion. Collectively, these tables demonstrate the robustness of the tendencies noted earlier.

IV. ANALYSIS

The purpose of this investigation has been to gather evidence by brute-force computer search that the Fibonacci sequence does *not* have a special relationship to primes; specifically, that its intervals are not especially prone to cycles of R-function over- and under-estimation compared to other related sequences. However, given the sheer number of alternative sequences that consistently fail to improve upon the results achieved by the Fibonacci sequence, it would appear that the Fibonacci sequence *does*, in fact, have some special relationship with the distribution of primes. Moreover, one might conjecture that if these calculations were extended without limit (rather than only up to one billion), then using a Fibonacci number F_n and its successor F_{n+1} as initiators might produce at least as many cycles of over- and under-estimation of primes as the sequences generated from F_n and *any* integer $k > F_{n+1}$.

V. CONCLUDING QUESTIONS

It remains to consider why the R-function tends to alternately over- and under-estimate the number of primes in the intervals defined by the Fibonacci sequence. Apparently, as shown in Table III, a Fibonacci interval more dense in primes tends to be followed by one less dense, and vice versa. But why? Moreover, as shown in Table VI, for primes of the form $4n + 1$ the reverse appears to be true: more dense intervals tend to be bunched, as are less dense intervals. Again,why?

TABLE I. The number of half-cycles of R-function misestimation for various sequence initiators (I_1, I_2), where I_1 and I_2 occupy, respectively, the column and row headers. Calculations were carried up to one billion. For clarity, this table has scores only in those columns having Fibonacci numbers as their column headers. Scores in boxes are the highest in their columns. It is found that, consistently, these boxed entries occupy a row and column headed by a pair of consecutive Fibonacci numbers. That is to say, consecutive Fibonacci numbers produce more cycles of R-function over- and under-estimation of primes than any other sequence initiators.

	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22
03	**34**	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
04	22	30	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
05	28	**34**	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
06	23	28	–	24	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
07	32	27	–	28	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
08	27	27	–	**33**	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
09	19	29	–	24	–	–	29	–	–	–	–	–	–	–	–	–	–	–	–	–	–
10	27	24	–	27	–	–	26	–	–	–	–	–	–	–	–	–	–	–	–	–	–
11	25	26	–	22	–	–	27	–	–	–	–	–	–	–	–	–	–	–	–	–	–
12	24	29	–	26	–	–	19	–	–	–	–	–	–	–	–	–	–	–	–	–	–
13	25	23	–	26	–	–	**32**	–	–	–	–	–	–	–	–	–	–	–	–	–	–
14	27	25	–	24	–	–	23	–	–	–	–	25	–	–	–	–	–	–	–	–	–
15	23	22	–	31	–	–	24	–	–	–	–	24	–	–	–	–	–	–	–	–	–
16	22	27	–	28	–	–	29	–	–	–	–	22	–	–	–	–	–	–	–	–	–
17	24	21	–	23	–	–	26	–	–	–	–	28	–	–	–	–	–	–	–	–	–
18	23	25	–	27	–	–	22	–	–	–	–	25	–	–	–	–	–	–	–	–	–
19	27	30	–	26	–	–	26	–	–	–	–	24	–	–	–	–	–	–	–	–	–
20	24	28	–	21	–	–	29	–	–	–	–	26	–	–	–	–	–	–	–	–	–
21	26	25	–	23	–	–	23	–	–	–	–	**31**	–	–	–	–	–	–	–	–	–
22	23	22	–	24	–	–	25	–	–	–	–	23	–	–	–	–	–	–	–	27	–
23	23	25	–	27	–	–	22	–	–	–	–	23	–	–	–	–	–	–	–	26	–
24	27	27	–	22	–	–	28	–	–	–	–	24	–	–	–	–	–	–	–	25	–
25	23	23	–	25	–	–	24	–	–	–	–	24	–	–	–	–	–	–	–	25	–
26	24	23	–	31	–	–	18	–	–	–	–	27	–	–	–	–	–	–	–	22	–
27	23	25	–	27	–	–	24	–	–	–	–	25	–	–	–	–	–	–	–	22	–
28	27	27	–	20	–	–	22	–	–	–	–	21	–	–	–	–	–	–	–	24	–
29	25	22	–	24	–	–	27	–	–	–	–	24	–	–	–	–	–	–	–	22	–
30	19	24	–	26	–	–	23	–	–	–	–	27	–	–	–	–	–	–	–	25	–
31	25	25	–	26	–	–	22	–	–	–	–	27	–	–	–	–	–	–	–	21	–
32	27	28	–	26	–	–	21	–	–	–	–	24	–	–	–	–	–	–	–	20	–
33	24	28	–	23	–	–	23	–	–	–	–	24	–	–	–	–	–	–	–	27	–
34	24	23	–	24	–	–	22	–	–	–	–	24	–	–	–	–	–	–	–	**31**	–
35	22	16	–	23	–	–	22	–	–	–	–	23	–	–	–	–	–	–	–	24	–
36	20	23	–	26	–	–	26	–	–	–	–	18	–	–	–	–	–	–	–	28	–
37	24	26	–	26	–	–	22	–	–	–	–	22	–	–	–	–	–	–	–	26	–
38	23	26	–	24	–	–	23	–	–	–	–	20	–	–	–	–	–	–	–	18	–
39	25	26	–	24	–	–	23	–	–	–	–	29	–	–	–	–	–	–	–	24	–
40	24	27	–	23	–	–	23	–	–	–	–	26	–	–	–	–	–	–	–	26	–
41	24	25	–	21	–	–	22	–	–	–	–	24	–	–	–	–	–	–	–	22	–
42	22	25	–	21	–	–	29	–	–	–	–	20	–	–	–	–	–	–	–	27	–
43	22	26	–	23	–	–	25	–	–	–	–	21	–	–	–	–	–	–	–	24	–
44	25	23	–	26	–	–	23	–	–	–	–	22	–	–	–	–	–	–	–	24	–

TABLE II. The number of half-cycles of R-function misestimation for various sequence initiators (I_1, I_2), where I_1 and I_2 occupy, respectively, the column and row headers. Columns are headed by Fibonacci numbers from 34 through 6765. In order to make the row headers in the 20$^{\text{th}}$ row consist of all Fibonacci numbers, rows are numbered separately for each column. Scores in boxes are the highest in their columns. Calculations were carried up to one billion. Again, consecutive Fibonacci numbers achieve the highest scores, as shown by the row of boxes in the 20$^{\text{th}}$ row. But there are also many tie scores, which fan out above and below the 20$^{\text{th}}$ row as the column initiators grow. Hence, one can only claim that pairs of consecutive Fibonacci numbers produce *at least as many* cycles of R-function over- and under-estimation of primes as any other sequence initiators.

	34		55		89		144		233		377		610		987		1597		2584		4181		6765
36	23	70	22	125	24	214	22	358	20	591	20	968	19	1578	18	2565	19	4162	20	6746	19	10927	18
37	22	71	25	126	21	215	17	359	18	592	20	969	21	1579	20	2566	19	4163	20	6747	19	10928	18
38	26	72	25	127	19	216	17	360	20	593	19	970	21	1580	22	2567	19	4164	20	6748	19	10929	18
39	24	73	23	128	18	217	21	361	19	594	20	971	19	1581	20	2568	19	4165	20	6749	19	10930	19
40	20	74	23	129	20	218	23	362	21	595	18	972	20	1582	20	2569	19	4166	20	6750	19	10931	19
41	22	75	21	130	19	219	21	363	18	596	20	973	17	1583	20	2570	19	4167	20	6751	19	10932	19
42	21	76	19	131	16	220	23	364	22	597	17	974	19	1584	20	2571	19	4168	20	6752	19	10933	19
43	21	77	22	132	20	221	18	365	20	598	21	975	19	1585	20	2572	21	4169	20	6753	19	10934	19
44	24	78	23	133	17	222	20	366	23	599	20	976	21	1586	20	2573	21	4170	20	6754	19	10935	19
45	25	79	23	134	21	223	23	367	20	600	17	977	20	1587	20	2574	21	4171	20	6755	19	10936	19
46	21	80	22	135	23	224	25	368	20	601	19	978	18	1588	20	2575	21	4172	20	6756	19	10937	**20**
47	23	81	19	136	19	225	24	369	17	602	20	979	21	1589	20	2576	21	4173	20	6757	19	10938	**20**
48	24	82	20	137	22	226	21	370	21	603	18	980	21	1590	22	2577	21	4174	20	6758	**21**	10939	**20**
49	22	83	22	138	21	227	19	371	22	604	22	981	21	1591	22	2578	21	4175	20	6759	**21**	10940	**20**
50	22	84	20	139	25	228	15	372	19	605	22	982	21	1592	22	2579	21	4176	20	6760	**21**	10941	**20**
51	19	85	24	140	18	229	22	373	23	606	20	983	22	1593	22	2580	21	4177	20	6761	**21**	10942	**20**
52	21	86	23	141	19	230	20	374	23	607	22	984	22	1594	22	2581	21	4178	**22**	6762	**21**	10943	18
53	21	87	20	142	19	231	20	375	23	608	24	985	22	1595	22	2582	**23**	4179	**22**	6763	19	10944	**20**
54	26	88	21	143	23	232	25	376	25	609	24	986	23	1596	**24**	2583	**23**	4180	20	6764	**21**	10945	**20**
55	**30**	89	**30**	144	**29**	233	**28**	377	**27**	610	**26**	987	**25**	1597	**24**	2584	**23**	4181	**22**	6765	**21**	10946	**20**
56	25	90	28	145	27	234	26	378	25	611	24	988	**25**	1598	**24**	2585	**23**	4182	20	6766	19	10947	**20**
57	26	91	22	146	21	235	22	379	23	612	22	989	23	1599	22	2586	**23**	4183	**22**	6767	19	10948	18
58	25	92	25	147	19	236	24	380	23	613	**26**	990	21	1600	22	2587	**23**	4184	**22**	6768	**21**	10949	**20**
59	23	93	21	148	21	237	18	381	23	614	24	991	20	1601	22	2588	21	4185	**22**	6769	**21**	10950	**20**
60	18	94	24	149	20	238	19	382	21	615	20	992	24	1602	20	2589	21	4186	**22**	6770	**21**	10951	**20**
61	17	95	22	150	19	239	20	383	21	616	22	993	23	1603	18	2590	21	4187	20	6771	19	10952	**20**
62	23	96	20	151	21	240	23	384	25	617	20	994	23	1604	22	2591	21	4188	20	6772	**21**	10953	**20**
63	23	97	19	152	24	241	23	385	19	618	22	995	19	1605	22	2592	19	4189	20	6773	**21**	10954	**20**
64	21	98	22	153	22	242	22	386	21	619	22	996	19	1606	22	2593	19	4190	20	6774	19	10955	18
65	24	99	20	154	20	243	22	387	21	620	20	997	18	1607	22	2594	19	4191	20	6775	19	10956	18
66	26	100	18	155	22	244	22	388	24	621	20	998	20	1608	22	2595	21	4192	20	6776	19	10957	**20**
67	28	101	21	156	24	245	20	389	22	622	16	999	20	1609	20	2596	**23**	4193	18	6777	19	10958	**20**
68	26	102	21	157	18	246	21	390	20	623	18	1000	21	1610	18	2597	21	4194	18	6778	19	10959	**20**
69	23	103	19	158	18	247	23	391	22	624	20	1001	21	1611	18	2598	21	4195	18	6779	19	10960	18
70	27	104	21	159	20	248	23	392	21	625	18	1002	23	1612	18	2599	21	4196	18	6780	19	10961	18
71	24	105	25	160	14	249	19	393	21	626	18	1003	21	1613	20	2600	21	4197	18	6781	19	10962	18
72	24	106	22	161	16	250	24	394	19	627	20	1004	15	1614	18	2601	21	4198	16	6782	19	10963	18
73	20	107	25	162	22	251	23	395	19	628	21	1005	19	1615	22	2602	21	4199	16	6783	19	10964	18
74	22	108	23	163	24	252	24	396	18	629	21	1006	15	1616	22	2603	21	4200	18	6784	17	10965	18

TABLE III. This table gives the same results as Table II, but with results presented using color, with columns headed by Fibonacci numbers from 2 through 28 657, and with 53 rows. As before, calculations were carried up to one billion. The continuous red bar across the table's 27^{th} row represents the highest scoring initiators in each column, specifically, the pairs of Fibonacci numbers: (2,3), (3,5), (5,8), (8,13), . . . , (28 657, 46 368), whereas the mostly solid-red region represents scores tied for first. The Fibonacci pairs win or tie in all columns, clearly showing their effectiveness in producing cycles of R-function over- and under-estimation of primes. Note the rarity of "even rankings" (second and fourth: yellow and gold, respectively) compared to "odd rankings" (first, third, and fifth: red, green, and blue, respectively); also, note the ascending and descending "staircases" of green embedded in the red region. Ever-lighter shades of gray represent ever lower scores, and the upper left-hand corner is solid gray, as no score is calculated when the row initiator is less than or equal to the column initiator.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
							21	23	17	17	19	20	16	19	18	19	18	18	18	17
							19	22	23	15	19	19	19	19	18	17	18	18	18	17
							23	20	22	19	18	20	19	19	18	19	18	18	18	17
							21	22	21	21	20	20	16	19	18	19	18	18	18	17
							17	24	20	17	16	19	17	19	20	19	18	18	18	17
							21	22	17	17	22	17	18	19	18	19	18	18	18	17
						23	22	22	18	19	20	21	18	19	18	19	18	18	18	17
						23	22	24	22	20	20	19	18	19	20	19	18	18	18	17
						22	25	21	17	18	20	21	20	19	20	19	18	18	18	17
						26	25	19	17	20	19	21	22	19	20	19	18	18	18	17
						24	23	18	21	19	20	19	20	19	20	19	19	18	18	17
						20	23	20	23	21	18	20	20	19	20	19	19	18	18	17
						22	21	19	21	18	20	17	20	19	20	19	19	18	18	17
						21	19	16	23	22	17	19	20	19	20	19	19	18	18	17
					27	21	22	20	18	20	21	19	20	21	20	19	19	19	18	17
					26	24	23	17	20	23	20	21	20	21	20	19	19	19	18	17
					25	25	23	21	23	20	17	20	20	21	20	19	19	19	18	17
					25	21	22	23	25	20	19	18	20	21	20	19	20	19	18	17
					22	23	19	19	24	17	20	21	20	21	20	19	20	19	16	17
				25	22	24	20	22	21	21	18	21	22	21	20	21	20	19	16	17
				24	24	22	22	21	19	22	22	21	22	21	20	21	20	19	18	17
				22	22	22	20	25	15	19	22	21	22	21	20	21	20	17	18	17
			29	28	25	19	24	18	22	23	20	22	22	21	20	21	20	18	18	17
			26	25	21	21	23	19	20	23	22	22	22	21	22	21	18	18	18	17
		24	27	24	20	21	20	19	20	23	24	22	22	23	22	19	20	19	18	17
	30	28	19	26	27	26	21	23	25	25	24	23	24	23	20	21	20	19	18	17
34	34	33	32	31	31	30	30	29	28	27	26	25	24	23	22	21	20	19	18	17
22	28	24	23	23	24	25	28	27	26	25	24	25	24	23	20	19	20	19	18	17
28	27	27	24	23	28	26	22	21	22	23	22	23	22	23	22	19	18	18	18	17
23	27	22	29	24	26	25	25	19	24	23	26	21	22	23	22	21	20	18	18	17
32	29	26	26	24	18	23	21	21	18	23	24	20	22	21	22	21	20	18	18	17
27	24	26	22	27	24	18	24	20	19	21	20	24	20	21	22	21	20	18	18	17
19	26	24	26	25	26	17	22	19	20	21	22	23	18	21	20	19	20	18	18	17
27	29	31	29	21	22	23	20	21	23	25	20	23	22	21	20	21	20	19	18	17
25	23	28	23	24	27	23	19	24	23	19	22	19	22	19	20	21	20	19	18	17
24	25	23	25	27	24	21	22	22	22	21	22	19	22	19	20	19	18	19	18	17
25	22	27	22	27	24	24	20	20	22	21	20	18	22	19	20	19	18	19	18	17
27	27	26	28	24	21	26	18	22	22	24	20	20	22	21	20	19	20	19	18	17
23	21	21	24	24	27	28	21	24	20	22	16	20	20	23	18	19	20	17	18	17
22	25	23	18	24	25	26	21	18	21	20	18	21	18	21	18	19	20	19	18	17
24	30	24	24	23	21	23	19	18	23	22	20	21	18	21	18	19	18	17	18	17
23	28	27	22	18	28	27	21	20	23	21	18	23	18	21	18	19	18	19	18	17
27	25	22	27	22	22	24	25	14	19	21	18	21	20	21	18	19	18	19	16	17
24	22	25	23	20	22	24	22	16	24	19	20	15	18	21	16	19	18	19	18	17
26	25	31	22	29	28	20	25	22	23	19	21	19	22	21	16	19	18	19	18	17
23	27	27	21	26	25	22	23	24	24	18	21	15	22	21	18	17	18	19	18	17
23	23	20	23	24	24	23	26	22	23	19	21	17	18	21	22	17	18	19	16	17
27	23	24	22	20	20	21	26	22	19	22	21	17	20	17	22	17	18	19	18	17
23	25	26	22	21	23	19	24	20	18	24	19	17	20	19	22	17	17	19	18	17
24	27	26	26	22	25	22	24	18	18	18	20	19	20	17	22	17	17	17	16	17
23	22	26	22	28	23	20	24	18	21	20	18	17	22	17	22	17	17	17	18	17
27	24	23	23	28	19	21	24	24	17	18	20	17	20	17	22	17	17	17	18	17
25	25	24	23	28	23	21	24	26	17	18	18	18	18	19	20	17	17	16	16	17

TABLE IV. The number of half-cycles of R-function misestimation for various sequence initiators (I_1, I_2), where I_1 and I_2 occupy, respectively, the column and row headers. Calculations were carried up to one billion. This table gives the same results as Table I, but with the columns headed by *non*-Fibonacci numbers also filled. It is found that the columns headed by Fibonacci numbers contain maximum scores (in boxes) consistently higher than the maximum scores contained in the immediately adjacent columns headed by *non*-Fibonacci numbers.

	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22
03	**34**	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
04	22	30	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
05	28	**34**	25	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
06	23	28	22	24	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
07	32	27	**29**	28	27	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
08	27	27	20	**33**	23	24	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
09	19	29	24	24	28	**31**	29	–	–	–	–	–	–	–	–	–	–	–	–	–	–
10	27	24	24	27	21	26	26	28	–	–	–	–	–	–	–	–	–	–	–	–	–
11	25	26	24	22	23	28	27	19	20	–	–	–	–	–	–	–	–	–	–	–	–
12	24	29	26	26	26	27	19	28	26	24	–	–	–	–	–	–	–	–	–	–	–
13	25	23	**29**	26	24	27	**32**	24	23	25	25	–	–	–	–	–	–	–	–	–	–
14	27	25	21	24	28	26	23	23	23	26	24	25	–	–	–	–	–	–	–	–	–
15	23	22	23	31	**29**	22	24	27	26	23	28	24	27	–	–	–	–	–	–	–	–
16	22	27	26	28	22	26	29	**31**	20	22	25	22	26	26	–	–	–	–	–	–	–
17	24	21	19	23	24	27	26	28	25	23	25	28	25	23	24	–	–	–	–	–	–
18	23	25	27	27	28	25	22	24	25	28	25	25	20	21	22	22	–	–	–	–	–
19	27	30	26	26	26	25	26	26	27	27	26	24	24	23	26	24	22	–	–	–	–
20	24	28	28	21	24	25	29	22	26	19	18	26	**28**	**30**	26	21	23	22	–	–	–
21	26	25	26	23	23	24	23	26	21	20	28	**31**	25	29	27	18	25	26	22	–	–
22	23	22	25	24	25	26	25	20	25	23	26	23	22	21	21	22	26	**29**	23	27	–
23	23	25	27	27	23	22	22	22	**30**	27	23	23	23	23	25	24	26	25	28	26	22
24	27	27	**29**	22	24	26	28	22	26	27	21	24	23	27	28	26	28	25	27	25	23
25	23	23	20	25	25	25	24	25	27	25	25	24	25	26	**30**	25	24	25	20	25	21
26	24	23	25	31	25	22	18	27	25	26	23	27	23	23	20	27	22	24	23	22	24
27	23	25	22	27	21	18	24	20	29	**29**	25	25	25	27	22	24	23	25	24	22	24
28	27	27	22	20	24	22	22	28	23	**29**	23	21	23	24	25	22	24	25	28	24	24
29	25	22	23	24	27	24	27	22	24	23	26	24	24	26	21	24	27	26	21	22	25
30	19	24	23	26	24	25	23	26	26	21	21	27	20	24	25	**28**	24	26	25	25	25
31	25	25	25	26	26	26	22	28	23	24	24	27	27	27	25	25	24	25	19	21	20
32	27	28	24	26	22	27	21	25	22	24	21	24	25	25	25	23	19	23	17	20	25
33	24	28	26	23	26	22	23	21	23	23	**29**	24	26	23	22	23	21	23	26	27	22
34	24	23	26	24	21	23	22	21	24	26	21	24	25	23	22	19	21	22	27	**31**	25
35	22	16	21	23	22	24	22	26	23	22	25	23	19	25	23	25	22	25	**29**	24	23
36	20	23	24	26	26	24	26	24	26	25	23	18	21	24	25	21	**29**	23	25	28	22
37	24	26	21	26	25	24	22	27	17	23	14	22	25	22	28	27	27	21	20	26	21
38	23	26	25	24	25	25	23	22	18	25	23	20	25	25	22	23	24	25	22	18	20
39	25	26	25	24	23	22	23	23	26	26	21	29	23	25	25	24	22	20	22	24	22
40	24	27	25	23	21	19	23	23	20	27	27	26	23	24	27	22	22	20	22	26	26
41	24	25	22	21	20	24	22	24	**30**	24	22	24	20	21	21	22	23	24	26	22	**28**
42	22	25	18	21	19	27	29	20	25	23	21	20	18	22	23	22	28	26	**29**	27	23
43	22	26	24	23	26	28	25	18	23	23	23	21	21	23	25	24	21	25	26	24	26
44	25	23	23	26	22	21	23	25	23	21	23	22	19	27	23	24	22	23	**29**	24	24

TABLE V. The number of half-cycles of R-function misestimation for various sequence initiators (I_1, I_2), where I_1 and I_2 occupy, respectively, the column and row headers. This table gives the same results as Table I, but with calculations carried only up to ten million. Despite the 100-fold reduction in the size of the interval considered, a pair of consecutive Fibonacci numbers again produces more cycles of R-function over- and under-estimation of primes than any other sequence initiators.

	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22
03	26	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
04	16	23	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
05	21	26	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
06	18	20	–	19	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
07	25	20	–	21	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
08	20	19	–	25	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–	–
09	15	22	–	16	–	–	23	–	–	–	–	–	–	–	–	–	–	–	–	–	–
10	23	19	–	20	–	–	19	–	–	–	–	–	–	–	–	–	–	–	–	–	–
11	18	19	–	17	–	–	19	–	–	–	–	–	–	–	–	–	–	–	–	–	–
12	17	23	–	19	–	–	16	–	–	–	–	–	–	–	–	–	–	–	–	–	–
13	18	18	–	21	–	–	24	–	–	–	–	–	–	–	–	–	–	–	–	–	–
14	17	19	–	16	–	–	18	–	–	–	–	17	–	–	–	–	–	–	–	–	–
15	17	18	–	23	–	–	19	–	–	–	–	17	–	–	–	–	–	–	–	–	–
16	15	21	–	21	–	–	22	–	–	–	–	17	–	–	–	–	–	–	–	–	–
17	18	16	–	15	–	–	18	–	–	–	–	20	–	–	–	–	–	–	–	–	–
18	17	18	–	20	–	–	15	–	–	–	–	20	–	–	–	–	–	–	–	–	–
19	19	23	–	18	–	–	20	–	–	–	–	17	–	–	–	–	–	–	–	–	–
20	17	20	–	14	–	–	21	–	–	–	–	20	–	–	–	–	–	–	–	–	–
21	20	18	–	20	–	–	19	–	–	–	–	23	–	–	–	–	–	–	–	–	–
22	16	16	–	16	–	–	18	–	–	–	–	17	–	–	–	–	–	–	–	21	–
23	16	20	–	21	–	–	19	–	–	–	–	18	–	–	–	–	–	–	–	20	–
24	19	21	–	15	–	–	20	–	–	–	–	17	–	–	–	–	–	–	–	18	–
25	17	19	–	17	–	–	18	–	–	–	–	18	–	–	–	–	–	–	–	17	–
26	19	15	–	23	–	–	13	–	–	–	–	20	–	–	–	–	–	–	–	19	–
27	18	21	–	21	–	–	16	–	–	–	–	17	–	–	–	–	–	–	–	17	–
28	19	21	–	15	–	–	16	–	–	–	–	15	–	–	–	–	–	–	–	16	–
29	18	16	–	17	–	–	20	–	–	–	–	17	–	–	–	–	–	–	–	18	–
30	13	20	–	20	–	–	17	–	–	–	–	19	–	–	–	–	–	–	–	18	–
31	18	17	–	19	–	–	16	–	–	–	–	19	–	–	–	–	–	–	–	17	–
32	19	21	–	18	–	–	16	–	–	–	–	17	–	–	–	–	–	–	–	15	–
33	17	21	–	17	–	–	15	–	–	–	–	16	–	–	–	–	–	–	–	20	–
34	18	17	–	17	–	–	17	–	–	–	–	18	–	–	–	–	–	–	–	23	–
35	16	13	–	15	–	–	17	–	–	–	–	16	–	–	–	–	–	–	–	17	–
36	15	19	–	19	–	–	18	–	–	–	–	14	–	–	–	–	–	–	–	21	–
37	16	18	–	19	–	–	18	–	–	–	–	16	–	–	–	–	–	–	–	19	–
38	16	18	–	18	–	–	16	–	–	–	–	14	–	–	–	–	–	–	–	14	–
39	20	18	–	18	–	–	18	–	–	–	–	21	–	–	–	–	–	–	–	17	–
40	18	20	–	16	–	–	15	–	–	–	–	19	–	–	–	–	–	–	–	18	–
41	17	17	–	16	–	–	15	–	–	–	–	17	–	–	–	–	–	–	–	14	–
42	15	18	–	14	–	–	21	–	–	–	–	15	–	–	–	–	–	–	–	20	–
43	15	18	–	17	–	–	18	–	–	–	–	15	–	–	–	–	–	–	–	17	–
44	19	18	–	18	–	–	17	–	–	–	–	15	–	–	–	–	–	–	–	18	–

TABLE VI. This table was computed the same way as Table III, but computations considered only primes of the form $4n+1$ below ten million. To allow comparison against the number of primes predicted by Riemann's R-function, each prime of the form $4n+1$ was simply counted twice. Unlike for Table III, here consecutive Fibonacci numbers tend to give the *lowest* score in each column. So, the near-continuous pink bar across the table's 27^{th} row represents the lowest scoring initiators in each column, specifically, the pairs of Fibonacci numbers: (2,3), (3,5), (5,8), (8,13), . . . , (28 657, 46 368), whereas the mostly solid-pink region represents scores tied for first. Just as pink now signifies the lowest score, yellow, green, gold, and blue now represent ever-higher scores, with the lightest shades of gray indicating the highest scores (i.e., the most cycles of over- and under-estimation by the R-function). For some reason, counting only primes of the form $4n+1$ causes the Fibonacci numbers to reverse earlier behavior and *suppress* the cycles of R-function over- and under-estimation.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
							17	13	13	10	14	11	08	11	07	09	08	08	07	06
							17	13	11	13	13	11	10	11	07	09	08	08	07	06
							18	17	11	12	14	15	08	11	07	09	08	08	07	06
							12	13	12	13	16	15	08	09	09	09	08	08	07	06
							14	17	12	13	14	08	08	09	09	09	08	08	07	06
							14	13	12	13	14	08	08	07	09	09	08	08	07	04
						20	16	19	12	13	14	08	08	07	09	09	08	08	07	06
						17	16	15	12	13	14	08	08	07	09	08	08	08	07	06
						18	20	17	12	13	12	10	10	07	09	08	08	08	07	06
						20	18	17	12	13	08	08	10	09	09	08	08	08	07	04
						17	15	15	13	15	14	08	12	07	09	08	10	08	07	06
						15	18	15	15	13	12	08	12	07	09	08	08	08	07	06
						15	18	12	17	15	08	08	10	11	09	08	08	08	07	04
						13	14	12	16	12	06	10	08	11	09	08	08	08	07	04
					15	15	16	12	14	14	08	10	10	11	09	08	08	08	07	06
					19	21	14	12	14	12	10	10	08	11	09	08	08	08	07	06
					22	17	12	12	14	14	10	13	08	11	09	08	08	08	07	06
					14	19	14	15	14	12	08	11	08	11	09	08	08	08	07	06
					16	17	12	14	12	08	12	08	10	11	09	08	08	06	05	06
				19	20	17	14	17	12	10	10	08	10	11	09	08	08	08	07	06
				22	18	15	15	16	12	08	14	08	12	11	08	08	08	08	07	06
				16	14	15	16	15	10	10	10	12	12	11	08	08	08	06	07	06
			23	21	12	12	16	10	10	11	10	12	12	09	09	09	06	08	07	06
			18	17	10	16	16	10	12	09	12	12	12	09	09	09	06	08	07	06
		14	20	13	18	19	12	10	12	13	12	12	10	09	09	07	08	08	07	06
	24	16	14	18	14	15	14	10	14	12	12	10	10	09	09	08	08	08	07	06
14	13	12	11	10	10	09	08	08	08	08	08	08	08	07	07	06	06	06	05	04
23	23	22	17	14	15	14	10	12	14	14	12	08	10	09	09	06	06	06	05	04
17	21	22	17	19	18	13	15	12	12	14	12	12	08	09	09	08	06	06	05	04
19	21	24	18	16	14	17	15	13	12	12	16	12	12	09	09	08	08	06	05	04
18	16	18	20	22	14	15	10	13	14	10	12	14	14	12	07	08	08	08	05	04
18	21	18	21	18	13	14	14	15	15	10	12	14	14	12	09	08	08	08	05	04
14	22	19	15	15	20	17	14	15	18	12	10	12	14	14	07	06	08	08	09	04
17	21	14	23	17	21	19	14	10	13	13	11	12	14	14	11	06	08	08	07	04
18	17	16	15	22	19	14	12	12	17	15	15	12	14	14	09	06	08	08	07	04
20	17	19	12	16	17	15	10	10	13	15	15	14	16	15	11	10	06	08	07	06
23	14	22	18	18	19	17	14	12	09	16	12	12	12	15	13	08	06	08	07	04
22	14	19	18	22	15	21	16	12	08	13	11	12	12	15	12	10	06	08	07	04
16	13	18	12	20	19	21	15	12	12	12	13	12	12	15	12	08	06	08	07	06
18	14	16	14	18	18	19	12	15	12	12	13	10	12	15	12	10	06	08	07	04
18	19	16	18	16	14	18	12	13	14	10	15	12	12	15	10	12	08	08	07	04
13	20	16	20	16	14	22	16	15	12	10	11	12	14	13	10	08	10	08	07	04
21	23	14	20	16	14	18	18	17	10	08	16	14	13	13	14	13	08	08	05	06
21	14	18	19	10	14	17	20	14	10	10	13	14	12	13	14	12	10	06	05	06
19	20	18	19	12	22	14	18	14	10	10	11	13	12	13	14	12	10	06	05	06
18	18	14	22	14	15	18	18	10	12	10	10	13	12	13	12	12	10	06	05	06
14	16	14	20	17	16	19	17	16	14	09	14	11	12	12	13	12	10	06	05	06
14	18	15	14	13	14	15	19	13	14	13	14	13	12	12	13	12	06	06	05	06
16	16	22	22	17	10	17	19	15	12	15	10	11	12	12	13	10	12	06	05	06
14	18	22	18	17	10	17	17	14	10	12	10	11	12	12	13	12	12	08	05	06
18	17	19	20	22	14	14	17	14	12	13	12	13	13	14	13	10	10	10	05	06
16	18	19	20	20	14	14	16	20	14	13	08	12	13	13	11	12	10	08	05	06
14	18	18	16	20	16	12	17	20	14	11	08	13	13	11	11	12	12	08	05	06

TABLE VII. Calculations were carried up to 1 000 000 for all primes. Red equals *highest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			17	16	15	13	15	14	12	13	12	10	10	09	10	09	06	06	06	05
		15	16	15	12	15	13	13	12	11	12	10	10	11	10	07	08	07	06	05
	19	17	13	18	17	16	11	13	13	13	12	11	12	11	08	09	08	07	06	05
22	22	21	20	19	19	18	18	17	16	15	14	13	12	11	10	09	08	07	06	05
12	16	13	14	14	14	17	16	15	14	13	12	13	12	11	08	07	08	07	06	05
17	16	16	17	16	17	16	13	12	10	11	10	11	10	11	10	07	06	06	06	05
14	16	14	18	15	15	16	15	12	13	11	14	09	10	11	10	09	08	06	06	05

TABLE VIII. Calculations were carried up to 10 000 000 for all primes. Red equals *highest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			19	20	17	14	18	16	16	17	16	14	14	13	14	13	10	10	10	09
		19	19	17	15	18	15	16	16	15	16	14	14	15	14	11	12	11	10	09
	23	21	16	20	20	20	15	17	17	17	16	15	16	15	12	13	12	11	10	09
26	26	25	24	23	23	22	22	21	20	19	18	17	16	15	14	13	12	11	10	09
16	20	16	18	17	17	21	20	19	18	17	16	17	16	15	12	11	12	11	10	09
21	20	20	19	18	21	19	16	15	14	15	14	15	14	15	14	11	10	10	10	09
18	19	17	22	17	19	19	18	15	16	15	18	13	14	15	14	13	12	10	10	09

TABLE IX. Calculations were carried up to 100 000 000 for all primes. Red equals *highest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			22	24	19	18	21	16	17	20	19	17	17	16	17	16	13	13	13	12
		22	23	21	16	20	17	18	17	18	19	17	17	18	17	14	15	14	13	12
	26	25	17	24	22	22	18	20	20	20	19	18	19	18	15	16	15	14	13	12
29	29	28	27	26	26	25	25	24	23	22	21	20	19	18	17	16	15	14	13	12
20	23	20	21	20	20	24	23	22	21	20	19	20	19	18	15	14	15	14	13	12
25	24	24	21	20	24	22	19	18	17	18	17	18	17	18	17	14	13	13	13	12
21	23	20	26	21	22	22	21	18	19	18	21	16	17	18	17	16	15	13	13	12

TABLE X. Calculations were carried up to 1 000 000 000 for all primes. Red equals *highest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			26	25	21	21	23	19	20	23	22	22	22	21	22	21	18	18	18	17
		24	27	24	20	21	20	19	20	23	24	22	22	23	22	19	20	19	18	17
	30	28	19	26	27	26	21	23	25	25	24	23	24	23	20	21	20	19	18	17
34	34	33	32	31	31	30	30	29	28	27	26	25	24	23	22	21	20	19	18	17
22	28	24	23	23	24	25	28	27	26	25	24	25	24	23	20	19	20	19	18	17
28	27	27	24	23	28	26	22	21	22	23	22	23	22	23	22	19	18	18	18	17
23	27	22	29	24	26	25	25	19	24	23	26	21	22	23	22	21	20	18	18	17

TABLE XI. Calculations were carried up to 10 000 000 000 for all primes. Red equals *highest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			30	27	23	25	27	20	23	26	25	25	25	24	25	26	21	23	23	22
		27	30	27	22	25	23	22	23	24	27	25	25	26	25	20	25	24	23	22
	33	32	22	31	28	27	24	26	26	28	27	26	25	26	25	26	25	24	23	20
39	39	38	37	36	36	35	35	34	33	32	31	30	29	28	27	26	25	24	23	22
26	31	27	24	26	26	30	31	30	31	30	29	30	29	28	25	22	23	22	21	22
32	28	32	27	28	31	30	25	24	25	26	27	28	27	28	27	22	21	21	21	20
24	30	25	31	27	29	29	27	24	29	26	31	26	27	28	27	26	25	21	21	20

TABLE XII. Calculations were carried up to 1 000 000 for primes of the form $4n+1$. Pink equals *lowest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			15	13	08	14	14	08	10	07	09	09	09	06	06	06	04	05	04	03
		13	16	11	16	17	10	08	09	10	09	09	07	06	06	05	05	05	04	03
	19	14	12	16	12	13	12	08	11	09	09	07	07	06	06	05	05	05	04	03
12	11	10	09	08	08	07	06	06	06	06	06	06	06	05	05	04	04	04	03	02
19	19	18	17	14	12	11	06	08	09	09	09	06	08	07	07	04	04	04	03	02
15	16	18	14	18	14	11	12	08	08	09	09	08	06	07	07	06	04	04	03	02
19	17	19	14	14	14	13	12	10	08	08	11	09	09	06	07	06	06	04	03	02

TABLE XIII. Calculations were carried up to 10 000 000 for primes of the form $4n+1$. Pink equals *lowest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			18	17	10	16	16	10	12	09	12	12	12	09	09	09	06	08	07	06
		14	20	13	18	19	12	10	12	13	12	12	10	09	09	07	08	08	07	06
	24	16	14	18	14	15	14	10	14	12	12	10	10	09	09	08	08	08	07	06
14	13	12	11	10	10	09	08	08	08	08	08	08	08	07	07	06	06	06	05	04
23	23	22	17	14	15	14	10	12	14	14	12	08	10	09	09	06	06	06	05	04
17	21	22	17	19	18	13	15	12	12	14	12	12	08	09	09	08	06	06	05	04
19	21	24	18	16	14	17	15	13	12	12	16	12	12	09	09	08	08	06	05	04

TABLE XIV. Calculations were carried up to 100 000 000 for primes of the form $4n+1$. Pink equals *lowest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			22	21	15	19	21	14	16	13	16	16	16	13	13	13	10	12	11	10
		18	24	16	21	22	16	14	16	17	16	16	14	13	13	11	12	12	11	10
	29	19	17	21	18	19	18	14	18	16	16	14	14	13	13	12	12	12	11	10
18	17	16	15	14	14	13	12	12	12	12	12	12	12	11	11	10	10	10	09	08
26	27	24	19	16	18	17	14	16	18	18	16	12	14	13	13	10	10	10	09	08
20	25	24	18	22	20	17	18	16	16	18	16	16	12	13	13	12	10	10	09	08
21	25	28	22	19	16	19	18	16	16	16	20	16	16	13	13	12	12	10	09	08

TABLE XV. Calculations were carried up to 1 000 000 000 for primes of the form $4n+1$. Pink equals *lowest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			25	25	18	22	24	18	21	17	20	20	20	17	17	17	14	16	15	14
		21	26	19	24	25	20	18	21	21	20	20	18	17	17	15	16	16	15	14
	31	22	20	24	22	23	22	18	22	20	20	18	18	17	17	16	16	16	15	14
22	21	20	19	18	18	17	16	16	16	16	16	16	16	15	15	14	14	14	13	12
29	30	28	21	20	22	22	19	20	22	22	20	16	18	17	17	14	14	14	13	12
23	29	27	22	26	24	21	23	21	21	22	20	20	16	17	17	16	14	14	13	12
23	27	32	25	22	20	23	22	21	21	20	24	20	20	17	17	16	16	14	13	12

TABLE XVI. Calculations were carried up to 10 000 000 000 for primes of the form $4n+1$. Pink equals *lowest* in column.

00002	00003	00005	00008	00013	00021	00034	00055	00089	00144	00233	00377	00610	00987	01597	02584	04181	06765	10946	17711	28657
			27	28	21	27	28	20	24	19	22	24	24	21	19	19	16	18	17	16
		25	31	22	27	27	22	22	24	23	22	24	22	19	19	17	18	18	17	16
	33	27	23	29	22	23	24	20	24	24	24	22	20	19	19	18	18	18	17	16
24	23	22	21	20	20	19	18	18	18	18	18	18	18	17	17	16	16	16	15	14
32	34	31	23	22	24	25	22	22	24	24	22	18	20	19	19	16	16	16	15	14
28	32	32	25	28	27	24	26	24	24	24	22	22	18	19	19	18	16	16	15	14
25	32	36	30	25	22	26	24	24	24	22	26	22	22	19	19	18	18	16	15	14

[1] J. S. Markovitch, “Riemann’s R-function and the distribution of primes,” (2013) http://vixra.org/abs/1307.0033.
[2] J. S. Markovitch, “On the Fibonacci sequences, the Koide formula, and the distribution of primes,” (2012) http://vixra.org/abs/1211.0022.
[3] J. S. Markovitch, “On the distribution of prime numbers in the intervals defined by the Fibonacci sequences,” (2010) http://vixra.org/abs/1008.0036.