Few interesting results regarding Poulet numbers and Egyptian fraction expansion

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Abstract. Considering r being equal to the positive rational number 1/(d1 – 1) + 1/(d2 – 1) +...+ 1/(dn – 1), where d1,..., dn are the prime factors of a Poulet number, the Egyptian fraction expansion applied to r leads to interesting results.

Note:

An Egyptian fraction is a sum of distinct unit fractions, such as 1/a + 1/b + 1/c +...+/m, where the denominators a, b, c, ..., m are positive, distinct, integers. Every positive rational number can be represented by an Egyptian fraction.

The Egyptian fraction expansion is an algorithm due to Fibonacci for computing Egyptian fractions: the number x/y, where x, y are positive, distinct, integers, is written as follows:

x/y = 1/ceiling(y/x) + ((-1)mod x)/y*ceiling(y/x), where the function ceiling(z) represents the smaller integer equal to or greater than z.

This algorithm is repetead to the second term of the summation above and so on until is obtained an egyptian fraction.

Conjecture 1:

If r is equal to the positive rational number 1/(d_1 – 1) + 1/(d_2 – 1) +...+ 1/(d_n – 1), where d_1,..., d_n are the prime factors of a Poulet number P, and m is equal to the last denominator obtained applying the Egyptian fraction expansion to r, then the number m + 1 is a prime or a power of prime for an infinity of Poulet numbers.

Examples:

: For P = 341 = 11*31, we have r = 1/10 + 1/30 = 2/15 = 1/8 + 1/120; the number m + 1 = 120 + 1 = 121 = 11^2, a square of prime.

: For P = 561 = 3*11*17, we have r = 1/2 + 1/10 + 1/16 = 53/80 = 1/2 + 1/7 + 1/51 + 1/28560; the number m + 1 = 28560 + 1 = 28561 = 13^4, a power of prime.

: For P = 645 = 3*5*43, we have r = 1/2 + 1/4 + 1/42 = 65/84 = 1/2 + 1/4 + 1/42; the number m + 1 = 42 + 1 = 43, a prime number.

: For P = 1105 = 5*13*17, we have r = 1/4 + 1/12 + 1/16 = 19/48 = 1/3 + 1/16; the number m + 1 = 16 + 1 = 17, a prime number.

: For P = 1387 = 19*73, we have r = 1/18 + 1/72 = 5/72 = 1/15 + 1/360; the number m + 1 = 360 + 1 = 361 = 19^2, a square of prime.

: For P = 1729 = 7*13*19, we have r = 1/6 + 1/12 + 1/18 = 11/36 = 1/4 + 1/18; the number m + 1 = 18 + 1 = 19, a prime number.

: For P = 1905 = 3*5*127, we have r = 1/2 + 1/4 + 1/126 = 191/252 = 1/2 + 1/4 + 1/126; the number m + 1 = 126 + 1 = 127, a prime number.

: For P = 6601 = 7*23*41, we have r = 1/6 + 1/22 + 1/40 = 313/1320 = 1/5 + 1/27 + 1/11880; the number m + 1 = 11880 + 1 = 11881 = 109^2, a square of prime.

: For P = 8911 = 7*19*67, we have r = 1/6 + 1/18 + 1/66 = 47/198 = 1/5 + 1/27 + 1/2970; the number m + 1 = 2970 + 1 = 2971, a prime number.

: For P = 52633 = 7*73*103, we have r = 1/6 + 1/72 + 1/102 = 233/1224 = 1/6 + 1/43 + 1/2289 + 1/8031644 + 1/80634123646776; the number m + 1 = 80634123646776 + 1 = 80634123646777, a prime number.

Note:
For the first ten Carmichael numbers C divisible by 7 and 19 (we don't have a comprehensive list of Poulet numbers indexed together with their prime factors) we always obtain for the number m + 1 a prime or a square of prime; we have the following values for (C, m + 1): (1729, 19), (8911, 2971), (63973, 2^2), (126217, 19^2), (188461, 433), (748657, 433), (825265, 1009), (997633, 577), (1050985, 23), (1773289, 1321).

Conjecture 2:

If r is equal to the positive rational number $1/(d_1 - 1) + 1/(d_2 - 1) +...+ 1/(d_n - 1)$, where $d_1,..., d_n$ are the prime factors of a Poulet number P, and r is represented by the irreducible fraction x/y, where x, y positive integers, then the number y + 1 is a prime or a power of prime for an infinity of Poulet numbers.

Examples:
(as it can be seen above)

: For P = 341, we have r = x/y = 2/15; the number y + 1 = 15 + 1 = 16 = 2^4, a power of prime.
: For P = 561, we have r = x/y = 53/80; the number y + 1 = 80 + 1 = 81 = 3^4, a power of prime.
: For P = 1105, we have r = x/y = 19/48; the number y + 1 = 48 + 1 = 49, a square of prime.
: For P = 1387, we have r = x/y = 5/72; the number y + 1 = 72 + 1 = 73, a prime number.
: For P = 1729, we have r = x/y = 11/36; the number y + 1 = 36 + 1 = 37, a prime number.
: For P = 6601, we have r = x/y = 313/1320; the number y + 1 = 1320 + 1 = 1321, a prime number.
: For P = 8911, we have r = x/y = 47/198; the number y + 1 = 198 + 1 = 199, a prime number.

Note:
As it can be seen above, the number y is sometimes equal to lcm((d_1 – 1), (d_2 – 1), ..., (d_n – 1)), which is, for instance, the case of the Poulet number 1387 = 19*73, where y = 72 = lcm(18,72), but this is not always true: this is, for instance, the case of Poulet number 341, where y = 15 and lcm(10,30) = 30.

Conjecture 3:

If d_1,..., d_n are the prime factors of a Poulet number P, then the number lcm((d_1 – 1), (d_2 – 1), ..., (d_n – 1)) is a prime or a power of prime for an infinity of Poulet numbers.