Solution of the DICE 2007 Integrated Assessment Model in continuous time and evaluation of the DICE 2007 Savings Ratio constraint

Working Paper

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Abstract
This research demonstrates a solution for the DICE 2007 integrated assessment model in the continuous domain through the use of a Runge-Kutta sampling technique for solving differential transcendental equations. The use of a savings ratio helper constraint was not required. It is shown that the introduction of a savings ratio constraint leads to a 12% underestimation of maximum atmospheric temperature rise. In addition, evidence of a savings ratio within economic data was unable to be confirmed using the equivalent proxy of an investment ratio and model selection techniques for mixed Gaussian probabilistic graphical models. However, evidence of a dilute intertemporal relationship between investment and an increase in production was detected. The results of this research support the use of Runge-Kutta sampling differential transcendental solvers with Chebyshev function outputs for continuous solutions in integrated assessment models without the requirement for helper constraints.

1. Background

It is often difficult to appreciate the sensitivity of complex social systems to changing constraints without simulating key interrelationships in the system. The DICE 2007 integrated assessment model was developed to understand the interrelationships between climate change, the social cost of carbon and efficient carbon abatement trajectories [1], [2], [3]. It has become a classic climate change policy simulation tool for evaluating the social and geophysical effects of global warming. An indicator of the success of models such as DICE2007 in policy formation is the embedding of results and recommendations within the national climate change policies of many countries.

Although the ordinary differential transcendental equations describing the DICE 2007 model are elegant and concise, researchers have been thwarted in seeking a direct solution by the computational complexity of transcendental functions, conditional logic, intra-period equilibrium and intertemporal optimisation over a long time-frame of up to 1400 years. This has led to DICE 2007 being formulated as a discrete model with decade time intervals.

Single year time period solutions have been developed using alternative approaches such as dynamic programming, approximate dynamic programming and optimal control [4], [5], [6], [7].

$C(T) = Y(T) - I(T)$
$S(T) = I(T)/(.001+Y(T))$
$s.fx(t)=.22$ Fix savings assumption for standardization if needed*

$where:$
$C(T)$ is Consumption trillions US dollars
$I(T)$ is Investment trillions US dollars
$S(T)$ is Gross savings rate as fraction of gross world product
$Y(T)$ is Gross world product net of abatement and damages
* The GAMS compiler does not distinguish between upper-and lowercase letters

Figure 1: Excerpts from DICE 2007 GAMS program for the Savings ratio. Source: Appendix B Basic GAMS Program for DICE-2007 Model [6]

Recent developments in the solution

of ordinary differential equations have facilitated a new approach to solving intractable systems of differential transcendental equations in continuous or near continuous time. The research detailed herein demonstrates a such a solution to the DICE 2007 model by globally optimising a complete Chebyshev polynomial as the function for the solution while at each step solving the differential transcendental equations using Runge-Kutta sampling techniques with Chebyshev function outputs [8].

This approach to obtaining a continuous solution is used to evaluate the necessity of setting savings to be a fixed proportion of production, which is a stabilising helper constraint in the DICE 2007 model and included in most developments of the model [9].

Figure 1 provides DICE 2007 savings assumptions [10]. The introduction of a savings to production ratio, or equivalently the companion investment to production ratio, has the effect of eliminating the consumption variable to significantly simplify the system of equations.

Along with investigating the effect of removing a savings ratio constraint from the DICE 2007 mode, the rationale for such a constraint needs to be considered. The relationship between capital, production and investment cycles has long been a topic of interest for economists of all persuasions. In the absence of government regulation there is no prima facie accounting relationship between investment and production.

However investment in productive capacity results from the careful commitment of resources by profit seeking decision makers. The decision to invest in productive capacity is typically based on a careful analysis of return on investment [11, pp. 367–374,653]. Pierre Samuel DuPont (1870-1954) demonstrated the benefits of systematically measuring return on investment for growth planning and forecasting to manage capital allocation and measure management efficiency across diversified business units at both E. I. du Pont de Nemours and Company and General Motors [12], [13, p. 446], [14, pp. 140–8], [15]. DuPont's Return on Net Assets (RONA) analysis is based on the observation that Return on Net Assets is separable into the controllable performance indicators of Return on Sales (or net margin on sales) multiplied by Sales to Assets (asset turnover). Asset turnover ratios are usually stable for long periods and revert to a mean within a period of one to two years following major investment [11, p. 512]. Manufacturing asset turnover is typically one times, implying one dollar of infrastructure per dollar of sales.

The Global Trade Analysis Project (GTAP) collates, harmonises and provides a consistent set of economic data across 129 regions and 57 sectors for the purpose of CGE models [16, pp. 3–5], [17]. Figure 2 provides the sources and uses associated with Consumption and Investment in 2004 and 2007. The ratios of Consumption and Investment to Production are also shown.

It may be noted that the ratio of Investment to Production is 0.11 in 2004 and 0.12 in 2007. Approximately equal amounts are contributed by Savings and Capital Depreciation. The ratios of investment to production may be contrasted to the DICE 2007 assumption of a savings ratio of 0.22, being the ratio of investment to production as shown in the DICE 2007 equations in Figure 1.

	2004		2007	
US$ million million	**Consumption Priv & Govt**	**Investment**	**Consumption Priv & Govt**	**Investment**
Sources				
Labour Factor Returns	15.2		20.0	
Labour Factor Taxes	3.7		4.6	
Capital Factor Returns	9.0		12.5	
Capital Factor Taxes	0.3		0.3	
Land Factor Returns	0.7		1.1	
Land Factor Taxes	0.0		-0.1	
Production & Direct Taxes	6.2		8.0	
Import Duties	0.3		0.3	
Export Taxes	0.1		0.2	
Sales Taxes	2.4		3.1	
Savings	-4.5	4.5	-6.9	6.9
Depreciation Capital Factor		4.5		5.9
Total Sources	33.2	9.0	43.0	12.8
Uses				
Domestic Services	23.5	5.7	30.1	8.1
Domestic Manufacturing	2.7	1.8	3.8	2.5
Imported Manufacturing	2.6	1.2	3.5	1.7
Domestic Food	1.6	0.0	2.1	0.1
Imported Food	0.6	0.0	0.7	0.0
Imported Services	0.4	0.1	0.6	0.1
Taxes	1.7	0.2	2.2	0.3
Total Uses	33.2	9.0	43.0	12.8
Production	80.1		108.2	
Ratio to Production	0.41	0.11	0.40	0.12

Figure 2: Consumption & Investment in 2004 & 2007, and ratios to Production Source: GTAP 8.1 Social Accounting Matrix

Broadly speaking the inclusion of a Savings Ratio constraint, or the equivalent Investment Ratio constraint, in an intertemporal model may be supported by dynamic considerations. This is because a change in capital (investment) translates to a change in production through the mean reverting asset turnover ratio in the period (as noted above). Therefore a Saving Ratio proportionality of investment to mean production has meaning in a discrete model such as DICE 2007 where decade periods are settled both intra-period and inter-period.

This research investigates the effect of eliminating the Savings Ratio helper constraint utilising the ability to solve the DICE 2007 continuous model using Runge-Kutta sampling techniques with Chebyshev function outputs for solving differential transcendental equations. It also investigates whether evidence for the equivalent Investment Ratio can be empirically detected within GTAP economic data by mining the data using model selection techniques for mixed Gaussian probabilistic graphical models.

2. Methodology

2.1. Continuous time solution over 1400 years

While DICE 2007 is a model of 600 years (60 periods) with no terminal value, other researchers include a Chebyshev function for terminal value representing a further 800 years [18, p. 18]. Since an extension in the time period from 600 years to 1400 years is not onerous for a continuous model, the latter is demonstrated in this research. Figure 3 summarises a continuous 2007 DICE formulation utilising parameters fitted through dynamic programming [4], [9].

Maximize:

$$utility\ per\ capita = 381000 + \frac{1}{194}\int_0^{1400} \mathrm{e}^{-0.015t}\, l_t \left(1 - \frac{l_t}{c_t}\right) dt$$

where:

c_t is global consumption at time t

l_t is global population at time t

k_t is capital stock at time t

Tat_t, Tlo_t are the global mean surface and lower ocean temperature rises at time t

Mat_t, Mup_t, Mlo_t are the masses of carbon in the atmosphere, upper & lower oceans t

Subject to:

$$\begin{aligned}
k_t' &= -c_t - 0.1\,k_t' - \frac{269.4\,\mathrm{e}^{-9.2\mathrm{e}^{-0.001t}}\, k_t^{0.3}\, l_t^{0.7}\left(0.00245981\,\mathrm{e}^{2.43333\mathrm{e}^{-0.003t}}\left(1+\mathrm{e}^{-0.005t}\right)\mu_t^{2.8} - 1\right)}{If\left[Tat_t \le 0,\ 1,\ 1+0.0028388\,Tat_t^2\right]} \\
Tat_t' &= 0.22\left(If\left[t \le 100,\ 0.0036t - 0.06\ , 0.03\right] + 5.48224\log\left(Mat_t\right) - 36.7073\right) \\
&\quad -0.0644175\,Tat_t + 0.0110022\,Tlo_t \\
Tlo_t' &= 0.0048\left(Tat_t - Tlo_t\right) \\
Mat_t' &= -3.17177\,\mathrm{e}^{2.43333\mathrm{e}^{-0.003t} - 9.2\mathrm{e}^{-0.001t}}\, k_t^{0.3}\, l_t^{0.7}\left(\mu_t - 1\right) - 0.0190837\,Mat_t \\
&\quad +0.00980087\,Mup_t + 1.1\,\mathrm{e}^{-0.01t} \\
Mup_t' &= 0.0190837\,Mat_t + 0.000336993\,Mlo_t - 0.0152039\,Mup_t \\
Mlo_t' &= 0.005403\,Mup_t - 0.000336993\,Mlo_t \\
l_t &= 6514\,\mathrm{e}^{-0.035t} + 8600\left(1 - \mathrm{e}^{-0.035t}\right) \\
Tat_t &\preccurlyeq \text{a maximum temperature rise level for all } t \\
0 &\preccurlyeq \mu_t \preccurlyeq 1 \\
c_t, k_t &\succcurlyeq 0
\end{aligned}$$

Figure 3: Continuous Formulation of the DICE 2007 Model

Inspection of the above equations reveals the consumption functions c_t and abatement/amelioration function μ_t as a set of independent functions. Prima facie these two functions need to be jointly optimised. The alternative used in DICE 2007 is to introduce a savings ratio that defines savings as a fixed proportion of the production function, which is equivalent to fixing the complement ratio of consumption to production. The overall effect is to eliminate the consumption function c_t leaving μ_t as the sole independent function to be determined.

It is a considerable task to co-jointly, intertemporally and globally optimise unknown and fully flexible functions for c_t and μ_t. As regards the shape of the amelioration and abatement function μ_t, it might be reasonable surmised that the function is likely to be a sigmoid rising at some rate through the current level 0.05, as assumed in DICE 2007, to asymptotically approach the proportion of 1.0. While in reality this sigmoid might include steps and interim delays, policy makers interested in safeguarding risks would appreciate the implications of a smooth sigmoid function as compared to synthetic piecewise functions. Furthermore, prudence suggests that policy makers would not implement interregnums in amelioration and abatement in order to maintain atmospheric temperature rise at its maximum and most damaging level for decades or longer. Therefore a smooth sigmoid is appropriate for the amelioration and abatement function μ_t.

$$\mu_t' = const\ (1-\mu_t)\ \mu_t$$
where:
$$\mu_0 = 0.05$$
Figure 4: Differential Equation for Amelioration and Abatement Function

The amelioration and abatement function μ_t is assumed to be a sigmoid characterised by an acceleration constant as defined by the differential equation in Figure 4. There are two underlying assumptions in this equation. The first is that current amelioration and abatement is already on the sigmoid trajectory at the point $\{2005, 0.05\}$ defined by DICE 2007. The second assumption is that amelioration and abatement will continue to pass smoothly and completely through the sigmoid.

It is not possible to simplify the conjoint, intertemporal, global optimisation by assuming a shape for the consumption function c_t. While a generic consumption curve might be exponential in the absence of climate change constraints, very little is known about the shape of the consumption function in the presence of atmospheric temperature rise constraints. This research assumes the shape of the consumption function c_t to be a complete Chebyshev polynomial defined through convenient time handles over the 1400 year simulation period. This function must be optimised with a global solver as there are many local maxima. The initial consumption (notionally 2005) is set to the average consumption in Figure 2 of US$ 38.1 million million, which is 40.5% of average production.

With the differential equation for amelioration and abatement in Figure 4 and the consumption function defined for the complete Chebyshev polynomial, only a small number of parameters to be optimised. These are the acceleration constant of the amelioration and abatement function and eleven consumption function handles.

2.2. Data Mining for Evidence of Savings to Production Ratio

a) Data

GTAP's Aggregated Social Accounting Matrix (ASAM) is an integrated presentation of data that facilitates testing of hypotheses at the lowest level, without aggregation by commodity or by country. Commodity variables by country were prepared using crosstab queries across the 2004 and 2007 flexagg8 databases [16].

Two files of data were prepared for analysis. The first was a “static” period dataset for a single generic year. In this case crosstab results for each country in both 2004 and 2007 were normalised by dividing all data for each country by the aggregate of all commodity production in that country in the respective year and multiplying by 10^4. The number of observations was maximised by assembling the normalised 2004 and 2007 data into a vertical set. The country identifier is retained, thereby creating two observations per country.

The second data file is a “chain” of the 2004 and 2007 years. The crosstab query results for each country in 2004 and 2007 are normalised in the same way as for the static dataset, except that both 2004 and 2007 results are divided by the aggregate of all commodities produced in that country in the 2004 year. The chain dataset is completed by horizontally augmenting the 2004 rows for each country with respective 2007 rows. Variable names relating to 2004 are identified with a “y4” suffix and those for 2007 with a “y7” suffix.

b) Static Mixed Directed Gaussian Graphical Model

Methodological Background

$$f(x)=\prod_{v\in V} f(x_v|x_{pa(v)})$$

Figure 5: DGGM factor probability distribution

A Directed Gaussian Graphical Model (DGGM) is appropriate for continuous data. This can be modelled with a product of factors probability distribution, where the factors are conditional models according to the d-separation property and Markov condition [19, p. 13]. Figure 6 shows this distribution. Each factor is associated with the distribution of a local node x_v , which depends only upon the joint distribution of the local node's parent nodes $x_{pa(v)}$.

Assumptions underlying the application of graphical models have been concisely summarised [20, pp. 8–10]. Two important assumptions are that there are no hidden (latent) variables and variables are conditionally independent, which is the sole relationship between variables. The latter implies that the global and local distributions of discrete or categorical variables follow a multinomial distribution, the global distributions of continuous variables follow a multivariate Gaussian distribution $N_d(\mu,\Sigma)$ and the local distributions of continuous variables follow a univariate or multivariate Gaussian distribution.

Methods to infer the structure of a joint graphical model generally fall into one of low order conditional independence tests of edge likelihood or log-likelihood, heuristic search through score optimisation such as the hill-climbing algorithm and Bayesian Markov Chain Monte Carlo sampling [19, p. 42].

In this investigation two directed acyclic graph (DAG) selection methods are applied, consistent with the approach of previous researchers [19, p. 62]. The first selection method for the Static model is the R pcalg() PC function, which is an example of low order conditional independence tests of edge log-likelihood [21], [22]. A skeleton of undirected edges is detected based on a threshold p-value for local edges of 0.05. Various models that are Markov equivalent have the same undirected graph skeleton and same immoralities and so cannot be distinguished in model selection [23], [24]. The PC algorithm orients edges to determine a complete partially directed acyclic graph (cpDAG) equivalence class rather than a specific Directed Gaussian Graphical Model. In addition to causal edges in the cpDAG, there are undirected edges and bidirectional edges that have one orientation in one directed acyclic graph of the equivalence class and the reverse orientation in another.

$$p_{uv}|V/\{u,v\}=\frac{-k_{uv}}{\sqrt{k_{uu}k_{vv}}}$$

Figure 6: Partial correlation between variables as a function of concentration matrix

The likelihood of an edge between two variables in the PC algorithm is a function of the empirical mutual information between the variables and thereby to the extent to which the variables are correlated. The partial correlations between random variables are calculated from the concentration matrix K, which is the inverse of the covariance matrix $K = \Sigma^{-1}$. A weighted covariance matrix Σ is calculated with the R stats package cov.wgt(), using the S-Plus “ML” method. As the covariance matrix Σ is nearly singular, the concentration matrix K is calculated from Σ using the R corpcor package pseudoinverse() function. The partial correlation matrix is then derived using the gRbase conc2pcor() function, which calculates the partial correlation between variables *u* and *v* as shown in Figure 6.

The PC algorithm is vulnerable to overfitting and measures are implemented to explicitly penalising complexity by regulation and restricting the hypothesis space [25]. Overfitting arises because the sampled *u* and *v* may have mutual information greater than zero some of the time, notwithstanding that *u* and *v* may be independent in the empirical distribution. As more edges increase the likelihood score, additional edges may be added up to the point where the likelihood score is maximised because the network is fully connected.

The second model selection method in the Static case is heuristic search through score optimisation using the R bnlearn mmhc() function [26], [27]. This function optimises a Bayesian Information Criterion (BIC) goodness-of-fit score across all possible network structures generated from the current DAG using perturbations that add, remove and reverse edges. Underfitting data is unlikely because of the asymptotic consistency of BIC scoring, and overfitting is minimised by forcing a trade-off between fit and complexity that penalises spurious edges.

The “max-min parents and children” hill-climbing mmhc() function was developed as a hybrid to the basic hill-climbing algorithm, due to the propensity of the latter to become trapped in one of the multiple local equivalence classes neighbouring the I-minimum where the BIC score doesn't change with perturbations [28]. The mmhc() algorithm restricts the hypothesis space using a forward selection of the skeleton based on maximisation of the minimum association measure observed with any subset of the nodes selected in the previous iterations [29]. Markov blankets of variables are then detected by restricting the search space using conditional independence tests using a default significance level of 0.05. The second part of the procedure is a traditional hill-climbing algorithm that finds the optimal network structure in the restricted space using BIC network scores.

c) Chain Mixed Directed Gaussian Graphical Model

Methodological Background

A chain graph is similar to a directed acyclic graph in having directed and undirected edges. The R gRapHD minForest() function optimises the network BIC score to find a BIC minimum forest using the extended Chow-Liu algorithm [30], [31], [32], [33]. This BIC minimum forest is an undirected maximum likelihood tree or forest structure closest to the true one in the probability space, under the special constraint that each parent has just one parent.

A decomposable graph is extracted from the BIC minimum forest using the R gRapHD package forward selection function stepw(). This function selects edges to be added to a triangulated graph that maximises the overall score, which by default is BIC. This leads to one or more disconnected trees, perhaps with undirected edges. Identifying decomposable graphs from undirected graphs is an NP hard problem in the same way as identifying Bayesian Networks [19, p. 166], [34].

LWF Markov properties [23], [35] specify that chain graphs factorise similarly to a directed acyclic graph, where conditional independence is represented by d-separation. In addition, chain graphs have the property that each factor (or conditional density) further factorises according to an undirected graph where conditional independence is represented by c-separation. Mixed interaction models may comprise log-linear models for discrete variables, such as country classifiers, and Gaussian models for continuous variables.

A structure that can be represented by a mixed-interaction chain model has many advantages including elegant mathematics, efficient optimisation for high dimensional problems, such as exploiting decomposability and sparse parameterisation. Being tree structures, mixed-interaction chain models have a natural resistance to overfitting, which means that models can be generalised from a small number of samples [25].

Components of a chain graph *G* are the connected components of the graph after directed edges have been removed [36, p. 68]. The components represent distributions over the variables of the component, conditioned on the parental components. The conditional distribution is itself a product over the cliques of the undirected component and moralised parental components.

The joint distribution $p(x)$ of chain graph *G* is shown in Figure 7 where C_i is the union of the cliques in component *i*, together with the moralised parental components of *i*, and Φ are the associated functions defined on each clique.

$$p(x)=\prod_i p(X_i|pa_G(X_i))$$
$$p(x)\propto\prod_i \prod_{c\in C_i} \Phi(X_{C_i})$$

Figure 7: Joint probability distribution in a chain graph

The dependence graph of the Gaussian graphical model is decomposable and model selection can exploit the closed form expressions for factor graphs. Chain graphs can be more expressive than directed acyclic graphs for marginal distributions such as undirected 4-cycles [36, p. 69].

Structure learning or model selection in chain analysis requires that the search space be restricted to the edges of a conditional model between blocks of variables. It is an appropriate technique when it is clear that variables can be classed *a priori* into meaningful blocks. This research approach has two steps. First, a model similar to the static case is developed solely for 2004 variables specifying the country classifier as a prior. Following this, a model is developed for the whole of the data (both 2004 and 2007 variables), specifying the 2004 model as a prior.

Model selection does not assume any order within blocks but respects the mutual order of the blocks. Thus the only causal edges derived in this analysis are the directed edges between the 2004 and 2007 blocks.

In the investigation of two accounting periods separated by a period of time, it is likely that the strength of any causal edges between the blocks will be impacted by a number of factors. The most obvious adverse factor is that in this research, GTAP data for the intervening periods (2005 and 2006) is unavailable, which imposes a significant intervening time period between the 2004 and 2007 class variables. Intertemporal linkages over the period 2004 to 2007 are subject to many exogenous influences in the intervening periods, particularly considering that these exogenous factors affect different countries across the globe in different ways. Therefore the relationship between the normal accounting sequence of 2004 investment and 2007 production might at best be very weak.

Technical issues also mitigate against the chain method. Statistically valid relationships will be correspondingly more difficult to identify than in the static case because chain analysis data has only half the number of observations. Furthermore, chain models cannot explicitly guarantee a minimal forest of undirected edges.

In this research, two scoring methods are used with the R gRapHD minForest() function. The first is default Chow-Liu BIC scoring. The second is Bayes Dirichlet scoring, which sometimes results in a significantly sparser graph than default BIC scoring [19, pp. 171–4]. In contrast to these two scoring methods, the traditional technique of minimising edge log-likelihoods is not used as it constrains edges weights to be non-negative, resulting in a single tree.

Bayes Dirichlet scoring Bayes Dirichlet scoring provides a maximum a posteriori (MAP) estimate using the hyper-Dirichlet distribution for conjugate priors of the decomposable graphical model and the logarithm of Bayesian independence factors for edge weights [37], [38], [39], [40], [19, pp. 171–4]. As the Dirichlet distribution is conjugate to the multinomial distribution, the posterior can be updated in closed form using sufficient statistics. This Bayesian scoring approach has sufficient statistics both from the data and from additional alpha-hyperparameters. Given a small amount of data, the sufficient statistics from the hyperparameter determine the prior beliefs and the strengths of these beliefs, which helps to smooth out random fluctuations in the data that can affect maximum likelihood estimates [25]. Real data sufficient statistics dominate at the asymptotic limit ensuring that the same result is observed for both BIC scoring and Bayes Dirichlet scoring.

3. Results

3.1. Continuous time solution over 1400 years

Discounted utility per capita over the standard 1,400 year evaluation period is shown in Figure 8, both including and excluding the Savings Ratio assumption.

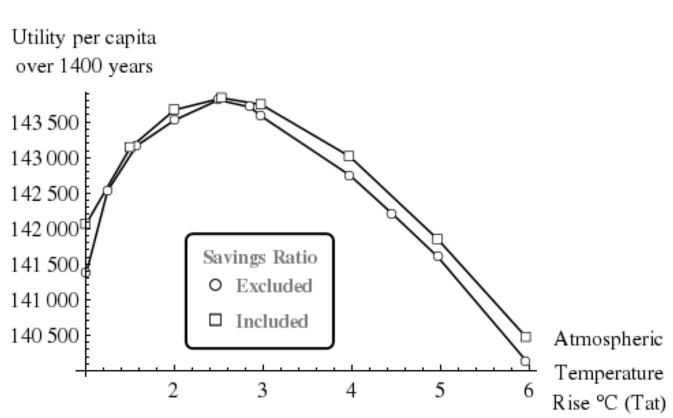

Figure 8: Discounted utility per capita as function of atmospheric temperature rise and savings ratio

While differences appear to be small given the scale, there are two issues that need to be taken into account. The first is discounting, which leads to a reduced impact from later differences while applying far less dilution to early differences. The second issue is utility offset and scale factors that reduce importance of the absolute scale.

Figure 9 illustrates the time lag between abatement measures and corresponding increase in atmospheric temperature rise with the savings ratio excluded. This figure demonstrates that just two to three decades delay in amelioration and abatement effectiveness causes temperature maxima to rise from 2°C to 6°C.

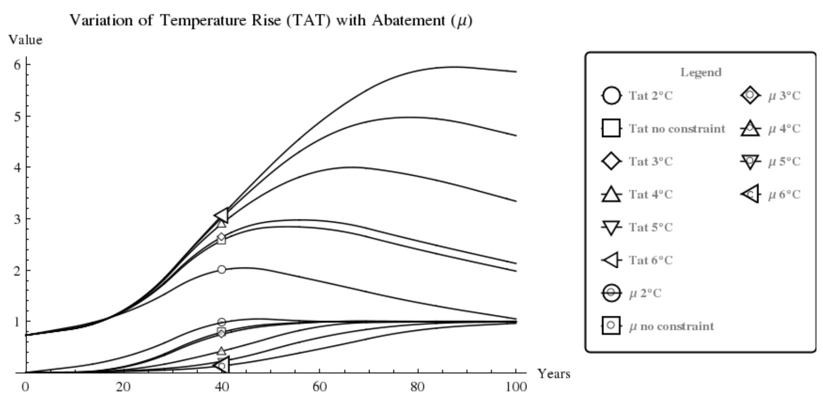

Figure 9: Relationship between atmospheric temperature rise (Tat) and the proportion of amelioration & abatement (μ)

The temperature profiles eliminate an initial decline and rise in temperature in the few years of the first decade, which is considered to be an artifact of equation fitting and initial parameters.

Figure 10 illustrates the relationship profiles between economic and geophysical factors over the first 100 years of the 1400 year projection, including and excluding the savings ratio, for the best utility outcome given no constraint is placed on atmospheric temperature rise. The worst case atmospheric temperature rise is approximately 2.85ºC. It may be noted that the best utility outcome profile requires approximately 50% amelioration and abatement within about 30 years, approaching 100% within 40 years, which are respectively 2035 and 2045 given the commencement year of 2005.

While the profiles for consumption and abatement both including and excluding the savings ratio are similar in each of these cases, two there are two significant differences are apparent. Firstly, while capital profiles converge toward the end of the 100 year period, the profiles are quite different in earlier years.

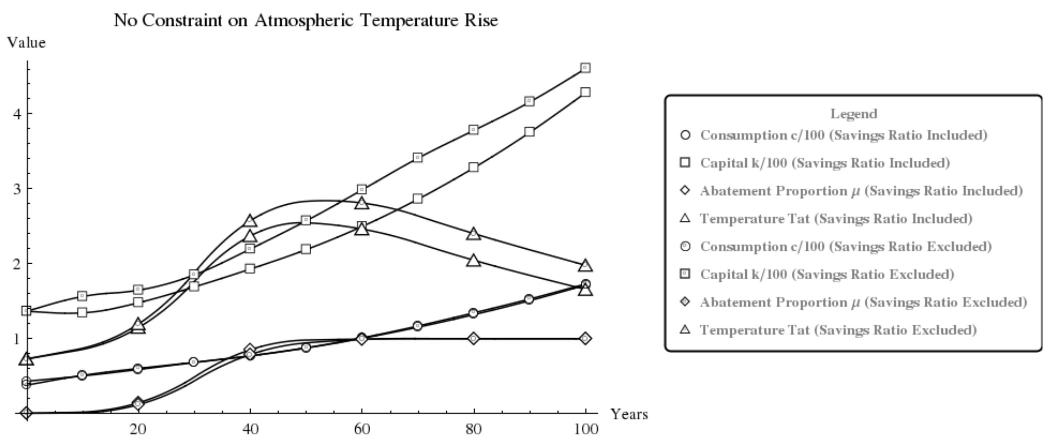

Figure 10: Relationships between economic and geophysical factors over the first 100 years of a 1400 year projection with Savings Ratio included and excluded (for the best utility outcome given no constraint on temperature rise)

More importantly, allowing the best outcome to be sought in the absence of a temperature constraint, the maximum atmospheric temperature with savings ratio included of 2.55°C is 12% less than 2.85°C with savings ratio excluded. In each case an amelioration and abatement proportion of approximately 15% needs to be achieved by year 10 and 85% by year 20.

3.2. *Mining GTAP data for evidence of Savings Ratio*

a) Static Mixed Directed Gaussian Graphical Model

The number of causal edges involving investment in Figure 11 for the two algorithms pcalg() and mmhc(). It may be noted that the mmhc algorithm with BIC scoring is significantly more parsimonious than the PC algorithm.

In general, selected models with fewer edges are preferred, which indicates that BIC scoring is favoured. This is consistent with the well known disadvantage of the PC algorithm in overfitting data.

Non−intertemporal Single Commodity Edges Involving Investment (Causal Edge → Or Bidiretional Edge ↔)

Commodity	PC DAG Oriented Equivalence Class	Max−Min Hill−Climbing (BIC)
Animal products nec	Industry Use of Comodity ↔ Investment	
Beverages & tobacco products	Capital Returns → Investment	
Bovine cattle, sheep & goats, horses	Land Returns → Investment	
Business services nec	Investment → Industry Use of Comodity	
Coal	Investment → End Consumption	
Communication	Capital Returns → Investment	
Construction	Industry Production → Investment Capital Returns → Investment	Industry Production → Investment
Crops nec	Land Returns → Investment	
Electronic equipment	Investment → Industry Production End Consumption → Investment Capital Returns → Investment	
Manufactures nec	Labour → Investment	Labour → Investment
Mineral products nec	Investment → Capital Returns	
Motor vehicles & parts	Investment → End Consumption	Investment → End Consumption
Raw milk	Labour → Investment	
Sugar cane, sugar beet	Investment → End Consumption	
Transport equipment nec	End Consumption ↔ Investment	End Consumption → Investment
Water	Capital Returns → Investment	Capital Returns → Investment
Wood products	Investment → Labour	
Wool, silk−worm cocoons	Land Returns → Investment End Consumption ↔ Investment	

Figure 11: Causal Edges for Static Model by PC algorithm pcalg() with p<0.05 & bnlearn algorithm mmhc() with edge strength > 0.85 & direction ≥ 0.5

Although there are 57 sectors in the analysis, it may be noted that the pcalg() algorithm detected associations between investment and production in the Construction and Electronic Equipment sectors. In contrast, the BIC scored mmhc() algorithm detected only the Construction sector.

These results suggest that a strong relationship between investment and production in a single time period is unable to be established.

b) Chain Mixed Directed Gaussian Graphical Model

Single Commodity Causal Edges Involving Investment

Commodity	BIC Scoring	Dirichlet Scoring
Animal products nec	2004 Investment → 2007 Labour 2004 Investment → 2007 Land Returns	2004 Investment → 2007 Land Returns
Beverages & tobacco products	2004 Investment → 2007 Labour	2004 Investment → 2007 Labour
Chemical, rubber, plastic products	2004 Investment → 2007 Capital Returns	2004 Investment → 2007 Capital Returns
Coal		2004 Investment → 2007 Capital Returns
Construction	2004 Investment → 2007 Capital Returns	2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Labour
Crops nec	2004 Investment → 2007 Industry Use of Comodity 2004 Investment → 2007 Land Returns	2004 Investment → 2007 Industry Production 2004 Investment → 2007 Industry Use of Comodity 2004 Investment → 2007 Labour 2004 Investment → 2007 Land Returns
Electricity	2004 Investment → 2007 Industry Production	2004 Investment → 2007 Industry Production 2004 Investment → 2007 Industry Use of Comodity 2004 Investment → 2007 Labour
Fishing	2004 Investment → 2007 Industry Production 2004 Investment → 2007 Land Returns	
Forestry	2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Industry Production 2004 Investment → 2007 Industry Use of Comodity 2004 Investment → 2007 Labour 2004 Investment → 2007 Land Returns	2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Industry Production 2004 Investment → 2007 Industry Use of Comodity 2004 Investment → 2007 Labour 2004 Investment → 2007 Land Returns
Leather products	2004 Industry Use of Comodity → 2007 Investment 2004 Investment → 2007 Labour	2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Labour
Metal products	2004 Investment → 2007 Labour	2004 Investment → 2007 Labour
Minerals nec	2004 Investment → 2007 Industry Use of Comodity	2004 Investment → 2007 Industry Use of Comodity
Motor vehicles & parts	2004 Investment → 2007 Capital Returns	2004 Investment → 2007 Capital Returns
Oil	2004 Industry Use of Comodity → 2007 Investment 2004 Investment → 2007 Labour	2004 Investment → 2007 Labour
Oil seeds		2004 Investment → 2007 Land Returns
Paddy rice		2004 Industry Use of Comodity → 2007 Investment
Plant−based fibers	2004 Investment → 2007 Industry Use of Comodity	2004 Investment → 2007 Industry Use of Comodity
Public admin. & defence, education, health	2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Industry Use of Comodity	
Raw milk	2004 Investment → 2007 Industry Production	2004 Investment → 2007 Industry Production
Sugar		2004 Capital Returns → 2007 Investment 2004 Labour → 2007 Investment
Sugar cane, sugar beet	2004 Investment → 2007 Labour	
Transport equipment nec	2004 Investment → 2007 Industry Production 2004 Investment → 2007 Industry Use of Comodity 2004 Investment → 2007 Labour	2004 Investment → 2007 Industry Production 2004 Investment → 2007 Industry Use of Comodity 2004 Investment → 2007 Labour
Transport nec		2004 Industry Use of Comodity → 2007 Investment
Vegetables, fruit, nuts	2004 Investment → 2007 Industry Use of Comodity	
Water	2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Industry Production 2004 Investment → 2007 Labour	2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Labour
Wheat	2004 Capital Returns → 2007 Investment 2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Industry Production 2004 Investment → 2007 Industry Use of Comodity 2004 Investment → 2007 Labour 2004 Land Returns → 2007 Investment	2004 Capital Returns → 2007 Investment 2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Industry Production 2004 Investment → 2007 Industry Use of Comodity 2004 Investment → 2007 Labour 2004 Land Returns → 2007 Investment
Wool, silk−worm cocoons	2004 Investment → 2007 Capital Returns 2004 Investment → 2007 Labour 2004 Labour → 2007 Investment 2004 Land Returns → 2007 Investment	2004 Labour → 2007 Investment 2004 Land Returns → 2007 Investment

Figure 12:Causal Edges 2004→2007 for a Chain Model using gRapHD ChowLiu Algorithm using BIC scoring and Dirichlet scoring

Chain graph causal edges associated with investment across the time period 2004 to 2007 are shown in Figure 12.

It may be noted that BIC scoring detected seven associations between 2004 Investment and 2007 Production (Electricity, Fishing, Forestry, Raw Milk, Transport equipment, Water and Wheat).

These seven cases represent 12% of the 57 sectors in the analysis, which is approaching a significant proportion of sectors.

Dirichlet-Bayes scoring is consistent in detecting six associations, with 5 sectors in common with BIC scoring (Electricity, Forestry, Raw Milk, Transport equipment and Wheat).

As 2007 data is normalised by 2004 production, the normalised 2007 production values represent growth. Therefore these findings results are consistent with the expectation of a relationship, albeit somewhat dilute, between investment in 2004 and production growth from 2004 to 2007.

4. Discussion

The results of the continuous time solution of the DICE 2007 model show that removal of the savings ratio constraint increases potential rise by 12% to 2.85°C when no constraint is imposed on atmospheric temperature rise allowing the best utility outcome to be sought. This implies that models employing a savings ratio understate potential atmospheric temperature rise.

In the absence of governments imposing a savings ratio constraint, the issue arises as to whether any evidence exists for the presence of a savings ratio in economic data. Analysis of GTAP data shows that the ratio between savings (or investment) and production is approximately 0.11 or 0.12, compared to the DICE 2007 constraint of value of 0.22.

Furthermore, data mining of GTAP data showed that a relationship between savings and production within a single year existed in only one or two of 57 commodity sectors.

The main sector exhibiting a causal edge is Construction, which is an industry where a single relationship between investment and production might be expected.

Similarly, the number of sectors having causal relationships across the three years between 2004 and 2007 increased to six or seven, which is a marginally significant proportion of the 57 sectors analysed. Furthermore the detection of residual causal edges connecting investment and production in significant sectors such as Electricity, Fishing, Forestry, Raw Milk, Transport equipment, Water and Wheat does confirm the potential for an intertemporal relationship. The importance of model selection techniques determining relationships in these sectors is underlined by the difficulty in detecting such relationships given the dilution in the accounting chain by intervening events.

5. Conclusion

The use of a savings ratio constraint in the DICE 2007 model has been investigated in the continuous domain using a novel Runge-Kutta sampling technique for solving differential transcendental equations. It is found that the introduction of a savings ratio constraint leads to a 12% understatement in the ensuing maximum atmospheric temperature rise, in the important case where a best utility outcome is sought in the absence of a constraint on temperature rise.

Evidence for the presence of a relationship between savings and output was sought in unaggregated GTAP 8.1 economic data, using investment as a proxy for savings and model selection techniques for mixed Gaussian probabilistic graphical models.

A relationship between investment and production within a single generic annual period comprising the normalised 2004 and 2007 years was established in only one of 57 sectors, the Construction sector. However, using a 2004 graphical model as the Bayesian prior for a 2007 graphical model an intertemporal relationship between investment in 2004 and the increased production in 2007 was established within seven sectors (Electricity, Fishing, Forestry, Raw Milk, Transport equipment, Water and Wheat) suggesting the relationship is marginally significant.

These data mining outcomes are consistent with prima facie expectations of an absence of any direct relationship between savings (or investment) and production within a single time period but a dilute relationship between investment and a consequent increase in production across time periods.

The results of this research show that the DICE 2007 model is solvable in the continuous domain without the use of a savings ratio constraint and suggest deprecation of such a constraint in discrete implementations.

6. References

[1] W. D. Nordhaus, *New base DICE-2006 as of November 16, 2006 (D_111606_alt_base.gms)*. 2006 [Online]. Available: http://www.econ.yale.edu/~nordhaus/homepage/D_111606_alt_base.GMS. [Accessed: 04-Mar-2013]

[2] W. D. Nordhaus, “Notes on how to run the DICE model,” Oct. 2007 [Online]. Available: http://www.econ.yale.edu/~nordhaus/homepage/DICE2007.htm. [Accessed: 26-Jun-2008]

[3] W. D. Nordhaus, *A Question of balance: weighing the options on global warming policies*. Yale University Press, 2008.

[4] Y. Cai, K. L. Judd, and T. S. Lontzek, “Open science is necessary,” *Nat. Clim. Change*, vol. 2, no. 5, pp. 299–299, 2012.

[5] M. Webster, N. Santen, and P. Parpas, “An approximate dynamic programming framework for modeling global climate policy under decision-dependent uncertainty,” *Comput. Manag. Sci.*, pp. 1–24, 2012.

[6] A. Haurie, “Integrated assessment modeling for global climate change: an infinite horizon optimization viewpoint,” *Environ. Model. Assess.*, vol. 8, no. 3, pp. 117–132, 2003.

[7] A. Smirnov, “Attainability Analysis of the DICE Model,” International Institute for Applied Sytems Analysis, Laxenburg, Austria, Interim Report IR-05-049, Sep. 2005 [Online]. Available: http://www.iiasa.ac.at/publication/more_IR-05-049.php. [Accessed: 05-Jul-2013]

[8] Wolfram Inc., “NDSolve—Wolfram Mathematica 9 Documentation,” 2013. [Online]. Available: http://reference.wolfram.com/mathematica/ref/NDSolve.html. [Accessed: 07-Jun-2013]

[9] Y. Cai, K. L. Judd, and T. S. Lontzek, *CJL -- one year*. National Bureau of Economic Research, 2012 [Online]. Available: https://sites.google.com/site/opensciencеletter/results/cjl----one-year/gams-code. [Accessed: 07-Mar-2013]

[10] W. Nordhaus, “The Challenge of Global Warming:Economic Models and Environmental Policy,” Yale University, New Haven, CT, 24-Jul-2007 [Online]. Available: http://www.econ.yale.edu/~nordhaus/homepage/dice_mss_072407_all.pdf

[11] S. H. Penman, *Financial statement analysis and security valuation*. McGraw-Hill/Irwin New York, NY, 2007 [Online]. Available: http://www.ifm.wi.tum.de/files/courses/ss04/financial_statement_analysis/financial_statement_analysis_ch13.pdf. [Accessed: 22-Oct-2012]

[12] T. C. Davis, “How the Du Pont organization appraises its performance,” *Am. Manag. Assoc.*, vol. 94, no. 7, 1950.

[13] A. D. Chandler, *The visible hand: The managerial revolution in American business*. Belknap Pr, 1977 [Online]. Available: http://books.google.com.au/books?hl=en&lr=&id=hUkqx76sF6oC&oi=fnd&pg=PA1&dq=%22The+Visible+Hand%22+Chandler&ots=CJvmzltSiY&sig=_SyEVEgLc1bxOlSFCnto_FKtubw. [Accessed: 23-Oct-2012]

[14] A. Sloan, *My years with general motors*. Crown Business, 1990.

[15] S. J. Nettleton, *Personal communication with senior executives of E. I. du Pont de Nemours And Company*. 2012.

[16] S. McDonald and K. Thierfelder, “Deriving a Global Social Accounting Matrix from GTAP Versions 5 and 6 Data,” Global Trade Analysis Project (GTAP), GTAP Technical Paper No 22, Dec. 2004.

[17] Purdue University Department of Agricultural Resources, “GTAP 8.1 Database,” 2013.

[Online]. Available: https://www.gtap.agecon.purdue.edu/databases/v8/default.asp

[18] Y. Cai, K. L. Judd, and T. S. Lontzek, “Continuous-Time Methods for Integrated Assessment Models,” National Bureau of Economic Research, 2012 [Online]. Available: http://www.nber.org/papers/w18365. [Accessed: 07-Mar-2013]

[19] S. Højsgaard, D. Edwards, and S. Lauritzen, *Graphical Models with R*. Springer Verlag, 2012 [Online]. Available: http://books.google.com.au/books?hl=en&lr=&id=em5QdpWmljAC&oi=fnd&pg=PR3&dq=%22Graphical+Models+with+R%22&ots=-YWA25vcVy&sig=naG_GBOQ0eWIvRz4vZRD-JF_es8. [Accessed: 16-Jul-2012]

[20] M. Scutari and K. Strimmer, “Introduction to Graphical Modelling,” *Arxiv Prepr. Arxiv10051036*, 2010 [Online]. Available: http://arxiv.org/abs/1005.1036. [Accessed: 20-Oct-2012]

[21] P. Spirtes and C. Glymour, “An algorithm for fast recovery of sparse causal graphs,” *Soc. Sci. Comput. Rev.*, vol. 9, no. 1, pp. 62–72, 1991.

[22] P. Spirtes, C. Glymour, and R. Scheines, *Causation, Prediction, and Search*. Springer-Verlag, 1993.

[23] M. Frydenberg, “The chain graph Markov property,” *Scand. J. Stat.*, pp. 333–353, 1990.

[24] J. Pearl and T. S. Verma, “A theory of inferred causation,” 1991 [Online]. Available: http://citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1.51.7221. [Accessed: 12-Oct-2012]

[25] D. Koller, “Probabilistic Graphcial Models (Stanford University),” 19-Mar-2012 [Online]. Available: https://class.coursera.org/pgm/. [Accessed: 12-Oct-2012]

[26] M. Scutari, “Learning Bayesian Networks with the bnlearn R Package,” *J. Stat. Softw.*, vol. 35, no. 3, Jul. 2010 [Online]. Available: http://www.jstatsoft.org/v35/i03/. [Accessed: 12-Oct-2012]

[27] M. Scutari, *CRAN - Package bnlearn*. 2013 [Online]. Available: http://cran.r-project.org/web/packages/bnlearn/. [Accessed: 19-Jun-2013]

[28] I. Tsamardinos, C. F. Aliferis, A. Statnikov, and E. Statnikov, “Algorithms for large scale Markov blanket discovery,” in *Proceedings of the sixteenth international Florida artificial intelligence research society conference*, 2003, pp. 376–381 [Online]. Available: http://www.aaai.org/Papers/FLAIRS/2003/Flairs03-073.pdf. [Accessed: 12-Oct-2012]

[29] I. Tsamardinos, L. E. Brown, and C. F. Aliferis, “The max-min hill-climbing Bayesian network structure learning algorithm,” *Mach. Learn.*, vol. 65, no. 1, pp. 31–78, 2006.

[30] G. C. G. de Abreu, D. Edwards, and R. Labouriau, “High-Dimensional Graphical Model Search with the gRapHD R Package,” *J. Stat. Softw.*, vol. 37, no. 1, pp. 1–18, Nov. 2010.

[31] G. C. G. de Abreu, D. Edwards, and R. Labouriau, *gRapHD: Efficient selection of undirected graphical models for high-dimensional datasets*. 2013 [Online]. Available: http://http://cran.r-project.org/web/packages/gRapHD/index.html. [Accessed: 19-Jun-2013]

[32] C. Chow and C. Liu, “Approximating discrete probability distributions with dependence trees,” *Inf. Theory Ieee Trans.*, vol. 14, no. 3, pp. 462–467, 1968.

[33] S. Kirshner, P. Smyth, and A. Robertson, “Conditional Chow-Liu tree structures for modeling discrete-valued vector time series,” in *Proceedings of the Twentieth Conference on Uncertainty in Artificial Intelligence*, Arlington, 2004, pp. 317–324 [Online]. Available: http://arxiv.org/abs/1207.4142. [Accessed: 13-Oct-2012]

[34] D. M. Chickering, “Learning Bayesian networks is NP-complete,” *Lect. Notes Stat. New York Springer Verl.*, pp. 121–130, 1996.

[35] S. L. Lauritzen and N. Wermuth, “Graphical models for associations between variables, some of which are qualitative and some quantitative,” *Ann. Stat.*, pp. 31–57, 1989.

[36] D. Barber, *Bayesian reasoning and machine learning*. Cambridge University Press, 2011.

[37] W. Buntine, “Theory refinement on Bayesian networks,” in *Proceedings of the Seventh Conference on Uncertainty in Artificial Intelligence*, 1991, pp. 52–60 [Online]. Available: http://dl.acm.org/citation.cfm?id=2100669. [Accessed: 18-Oct-2012]

[38] G. F. Cooper and E. Herskovits, “A Bayesian method for the induction of probabilistic networks from data,” *Mach. Learn.*, vol. 9, no. 4, pp. 309–347, 1992.
[39] A. P. Dawid and S. L. Lauritzen, “Hyper Markov laws in the statistical analysis of decomposable graphical models,” *Ann. Stat.*, vol. 21, no. 3, pp. 1272–1317, 1993.
[40] D. Heckerman, D. Geiger, and D. M. Chickering, “Learning Bayesian networks: The combination of knowledge and statistical data,” *Mach. Learn.*, vol. 20, no. 3, pp. 197–243, 1995.