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The inner structure of basis spinors.

Abstract

At first the 4-dimensional real basis spinors are set in accordance to usual 2dimensional complex basis spinors. This is the first step. Then the 4-dimensional space of real basis spinors represents as the tensor product of two 2-demensional spaces. Further the sum of these two 2-dimensional spaces represents as the tensor product of two new 2-dimensional spaces. And this operation repeats infinitely. The metric tensor defines for each of derived in this process spaces (as for 2dimensional, so for 4-dimensional). The shortage of data at this process compensates by the simplicity principle.

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1). The inner complex structure of 2-dimensional basis spinors.

Let us consider ordinary 4-dimensional Minkowski space. It's metric tensor is : $n_{\mu\nu} = (n_{\mu}, n_{\nu})$

	n_1	n_2	n ₃	n_4	
n_1	1	0	0	0	
n_2	0	-1	0	0	
n ₃	0	0	-1	0	
n_4	0	0	0	-1	
(1.1)					

From [1] we'll take the formule (3.1.20) for the connection of basis 4-vectors with the basis complex 2-spinors :

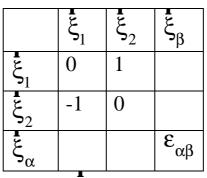
$$\mathbf{r}_{n_{1}} = \frac{1}{\sqrt{2}} \cdot (\mathbf{\xi}_{1} \otimes \mathbf{\xi}_{1}^{*} + \mathbf{\xi}_{2} \otimes \mathbf{\xi}_{2}^{*})$$

$$\mathbf{r}_{n_{2}} = \frac{1}{\sqrt{2}} \cdot (\mathbf{\xi}_{1} \otimes \mathbf{\xi}_{2}^{*} + \mathbf{\xi}_{2} \otimes \mathbf{\xi}_{1}^{*}) \qquad \mathbf{r}_{\mu} = n_{\mu}^{\alpha\beta} \cdot \mathbf{\xi}_{\alpha} \otimes \mathbf{\xi}_{\beta}^{*} \quad (1.2)$$

$$\mathbf{r}_{n_{3}} = \frac{i}{\sqrt{2}} \cdot (\mathbf{\xi}_{1} \otimes \mathbf{\xi}_{2}^{*} - \mathbf{\xi}_{2} \otimes \mathbf{\xi}_{1}^{*})$$

$$\mathbf{r}_{n_{4}} = \frac{1}{\sqrt{2}} \cdot (\mathbf{\xi}_{1} \otimes \mathbf{\xi}_{1}^{*} - \mathbf{\xi}_{2} \otimes \mathbf{\xi}_{2}^{*})$$

From that formule we get the metric tensors for $\hat{\xi}_{\alpha}$ and $\hat{\xi}_{\alpha}^{*}$: $(\hat{\xi}_{\alpha}, \hat{\xi}_{\beta}) = \varepsilon_{\alpha\beta}$ $(\hat{\xi}_{\alpha}^{*}, \hat{\xi}_{\beta}^{*}) = \varepsilon_{\alpha\beta}$ (1.3)



It is written in [1] that [(2.5.26)] ξ_{α}^{*} did not decompose along ξ_{β} . Then we do so. Let us form vectors \hat{a}_{α} and \hat{b}_{α} so :

$$\mathbf{r}_{a_{\alpha}} = \frac{1}{\sqrt{2}} \cdot (\mathbf{\xi}_{\alpha} + \mathbf{\xi}_{\alpha}^{*}) \qquad \mathbf{r}_{a_{\alpha}}^{*} = \mathbf{r}_{\alpha} \qquad \alpha = 1, 2$$

$$\mathbf{r}_{b_{\alpha}} = \frac{i}{\sqrt{2}} \cdot (\mathbf{\xi}_{\alpha}^{*} - \mathbf{\xi}_{\alpha}) \qquad \mathbf{r}_{\alpha}^{*} = \mathbf{r}_{\alpha}$$
Then
$$\mathbf{r}_{\delta_{\alpha}} = \frac{1}{\sqrt{2}} \cdot (\mathbf{r}_{\alpha}^{*} + \mathbf{r}_{\alpha}^{*})$$

$$\mathbf{r}_{\delta_{\alpha}} = \frac{1}{\sqrt{2}} \cdot (\mathbf{r}_{\alpha}^{*} + \mathbf{r}_{\alpha}^{*})$$

$$\mathbf{r}_{\delta_{\alpha}}^{*} = \frac{1}{\sqrt{2}} \cdot (\mathbf{r}_{\alpha}^{*} - \mathbf{r}_{\alpha}^{*})$$
(1.5)

And the equations (1.3) will take the form :

$$(\stackrel{\mathbf{r}}{a_{\alpha}}, \stackrel{\mathbf{r}}{a_{\beta}}) - (\stackrel{\mathbf{r}}{b_{\alpha}}, \stackrel{\mathbf{r}}{b_{\beta}}) + i \cdot ((\stackrel{\mathbf{r}}{a_{\alpha}}, \stackrel{\mathbf{r}}{b_{\beta}}) + (\stackrel{\mathbf{r}}{b_{\alpha}}, \stackrel{\mathbf{r}}{a_{\beta}})) = 2 \cdot \varepsilon_{\alpha\beta}$$

$$(\stackrel{\mathbf{r}}{a_{\alpha}}, \stackrel{\mathbf{r}}{a_{\beta}}) - (\stackrel{\mathbf{r}}{b_{\alpha}}, \stackrel{\mathbf{r}}{b_{\beta}}) - i \cdot ((\stackrel{\mathbf{r}}{a_{\alpha}}, \stackrel{\mathbf{r}}{b_{\beta}}) + (\stackrel{\mathbf{r}}{b_{\alpha}}, \stackrel{\mathbf{r}}{a_{\beta}})) = 2 \cdot \varepsilon_{\alpha\beta}$$

$$(1.6)$$

From the idea of simplicity we choose the next formule :

$$(\overset{\mathbf{r}}{a}_{\alpha}, \overset{\mathbf{r}}{b}_{\beta}) = (\overset{\mathbf{r}}{b}_{\alpha}, \overset{\mathbf{r}}{a}_{\beta}) = 0$$
(1.7)

From the idea of simplicity we adopt :

$$(\stackrel{\mathbf{r}}{a}_{\alpha},\stackrel{\mathbf{r}}{a}_{\beta}) = \varepsilon_{\alpha\beta} \qquad (\stackrel{\mathbf{r}}{b}_{\alpha},\stackrel{\mathbf{r}}{b}_{\beta}) = -\varepsilon_{\alpha\beta} \qquad (1.8)$$

Let us designate :

$$\mathbf{r}_{1} = \mathbf{a}_{1}$$
 $\mathbf{r}_{2} = \mathbf{a}_{2}$ $\mathbf{r}_{3} = \mathbf{b}_{1}$ $\mathbf{r}_{4} = \mathbf{b}_{2}$ (1.9)

Then

$$(c_{\mu}, c_{\nu}) = c_{\mu\nu}$$
 (1.10)

	a_1	a_2	b_1	b_2	c _μ
a_1	0	1	0	0	\vec{c}_1
a_2	-1	0	0	0	\ddot{c}_2
b_1	0	0	0	-1	<i>c</i> ₃
b_2	0	0	1	0	<i>c</i> ₄
C _v	C_1	\dot{c}_2	<i>c</i> ₃	c_4	C _{μν}

2) The second step inside the structure of basis spinors.

Let us represent the resulting 4-dimensional space of real spinors with the basis \dot{c}_{μ} as the tensor product of two 2-dimensional spaces with bases \dot{d}_{α} and \dot{e}_{β} , and their connection with \dot{c}_{μ} we'll take in this form :

$$\begin{split} \mathbf{r}_{c_{1}} &= \frac{1}{\sqrt{2}} \cdot (\mathbf{r}_{1} \otimes \mathbf{e}_{1}^{\mathbf{r}} + \mathbf{r}_{2}^{\mathbf{r}} \otimes \mathbf{e}_{2}^{\mathbf{r}}) \\ \mathbf{r}_{c_{2}} &= \frac{1}{\sqrt{2}} \cdot (\mathbf{r}_{1} \otimes \mathbf{e}_{2}^{\mathbf{r}} - \mathbf{r}_{2}^{\mathbf{r}} \otimes \mathbf{e}_{1}^{\mathbf{r}}) \\ \mathbf{r}_{c_{3}} &= \frac{1}{\sqrt{2}} \cdot (\mathbf{r}_{1} \otimes \mathbf{e}_{2}^{\mathbf{r}} + \mathbf{r}_{2}^{\mathbf{r}} \otimes \mathbf{e}_{1}^{\mathbf{r}}) \\ \mathbf{r}_{c_{3}} &= \frac{1}{\sqrt{2}} \cdot (\mathbf{r}_{1}^{\mathbf{r}} \otimes \mathbf{e}_{2}^{\mathbf{r}} + \mathbf{r}_{2}^{\mathbf{r}} \otimes \mathbf{e}_{1}^{\mathbf{r}}) \\ \mathbf{r}_{c_{4}} &= \frac{1}{\sqrt{2}} \cdot (\mathbf{r}_{1}^{\mathbf{r}} \otimes \mathbf{e}_{1}^{\mathbf{r}} - \mathbf{r}_{2}^{\mathbf{r}} \otimes \mathbf{e}_{2}^{\mathbf{r}}) \end{split}$$

$$\begin{aligned} \mathbf{r}_{c_{4}} &= \frac{1}{\sqrt{2}} \cdot (\mathbf{r}_{1}^{\mathbf{r}} \otimes \mathbf{e}_{1}^{\mathbf{r}} - \mathbf{r}_{2}^{\mathbf{r}} \otimes \mathbf{e}_{2}^{\mathbf{r}}) \end{aligned}$$

Then we have for metric tensors : $(d_{\alpha}, d_{\beta}) = d_{\alpha\beta}$

	d_1	d_2	d_{β}	
d_1	1	0		
d_2	0	1		
d_{α}			$d_{\alpha\beta}$	
(2.2)				

$$(\dot{e}_{\alpha}, \dot{e}_{\beta}) = e_{\alpha\beta}$$

	e_1	e_2	e_{β}	
e_1	0	1		
e_2	-1	0		
e_{α}			$e_{\alpha\beta}$	
(2.3)				

We got the situation the same as with complex 2-dimensional basis spinors, before the input into consideration the real 4-dimensional basis spinors. Let us try to input the 4-dimensional objects and now :

spinors. Let us try to input the 4-dimensional objects and now : $f_1 = d_1$ $f_2 = d_2$ $f_3 = e_1$ $f_4 = e_2$ (2.4) And the missing components of metric tensor $f_{\mu\nu} = (f_{\mu}, f_{\nu})$ (from idea of simplicity) we will consider a zeros :

	d_1	d_2	e_1	e_2	f_{μ}
d_1	1	0	0	0	f_1
d_2	0	1	0	0	f_2
e_1	0	0	0	1	f_3
e_2	0	0	-1	0	f_4
f_{v}	f_1	f_2	f_3	f_4	$f_{\mu\nu}$
(2.5)					

3) The third step inside basis spinors.

Let us represent the resulting 4-dimensional space with the basis f_{μ} as the tensor product of two 2-dimensional spaces with bases g_{α} and h_{β} , and their connection with f_{μ} we'll take in the form : $\mathbf{r}_{f_1} = \frac{1}{2} \cdot (\mathbf{r}_1 \otimes \mathbf{h}_1 + \mathbf{r}_2 \otimes \mathbf{h}_2)$ $\mathbf{r}_{f_2} = \frac{1}{2} \cdot (\mathbf{r}_1 \otimes \mathbf{h}_2 - \mathbf{r}_2 \otimes \mathbf{h}_1)$ $\mathbf{r}_{f_3} = \frac{1}{2} \cdot (\mathbf{r}_1 \otimes \mathbf{h}_2 + \mathbf{r}_2 \otimes \mathbf{h}_1)$ $\mathbf{r}_{f_4} = \frac{1}{2} \cdot (\mathbf{r}_1 \otimes \mathbf{h}_1 - \mathbf{r}_2 \otimes \mathbf{h}_2)$ (3.1)

Then we have for metric tensors : $(\dot{g}_{\alpha}, \dot{g}_{\beta}) = g_{\alpha\beta}$

	g_1	g_2	g _β		
g_1	1	-1			
\dot{g}_2	1	1			
\dot{g}_{α}			$g_{\alpha\beta}$		
(3.2)					

$$(\dot{h}_{\alpha}, \dot{h}_{\beta}) = h_{\alpha\beta}$$

	h_1	h_2	h _β	
\dot{h}_1	1	-1		
h_2	1	1		
h_2 h_{α}			$h_{\alpha\beta}$	
(3.3)				

Let us again input the 4-dimensional objects, now \vec{i}_{il} :

$$i_1 = g_1$$
 $i_2 = g_2$
 $i_3 = h_1$
 $i_4 = h_2$
(3.4)

And the missing components for metric tensor $i_{\mu\nu} = (i_{\mu}, i_{\nu})$ wish themselves to be such, that :

	g_1	<i>8</i> ₂	h_1	h_2	i _µ
g_1	1	-1	-1	1	\dot{i}_1
\dot{g}_2	1	1	-1	-1	\dot{i}_2
h_1	1	-1	1	-1	\dot{i}_3
h_2	1	1	1	1	\dot{i}_4
i_{ν}	\vec{i}_1	\vec{i}_2	i_3	i ₄	i _{μν}
(3.5)					

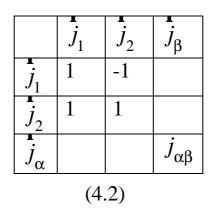
4) The fourth step inside basis spinors.

Let us represent the resulting 4-dimensional space with the basis i_{μ} as the tensor product of two 2-dimensional spaces with bases j_{α} and k_{β} . And their connection with i_{μ} we'll take in the form :

$$\begin{split} \mathbf{i}_{1} &= \mathbf{j}_{1} \otimes \mathbf{k}_{1} \\ \mathbf{i}_{2} &= \mathbf{j}_{1} \otimes \mathbf{k}_{2} \\ \mathbf{i}_{3} &= \mathbf{j}_{2} \otimes \mathbf{k}_{1} \\ \mathbf{i}_{4} &= \mathbf{j}_{2} \otimes \mathbf{k}_{2} \end{split} \qquad \mathbf{i}_{\mu} = \mathbf{i}_{\mu}^{\alpha\beta} \cdot \mathbf{j}_{\alpha} \otimes \mathbf{k}_{\beta} \qquad (4.1)$$

Then we have for metric tensors :

$$(\dot{j}_{\alpha}, \dot{j}_{\beta}) = j_{\alpha\beta}$$



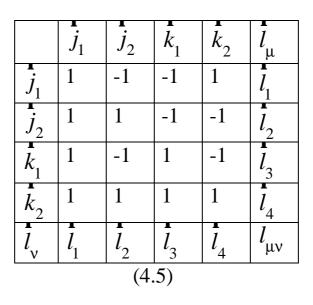
$$(k_{\alpha}, k_{\beta}) = k_{\alpha\beta}$$

	k_1	k_2	k _β	
k_1	1	-1		
k_2	1	1		
k _α			$k_{\alpha\beta}$	
(4.3)				

Let us again enter the 4-dimensional space with basis \bar{l}_{μ} :

 $\vec{l}_1 = \vec{j}_1$ $\vec{l}_2 = \vec{j}_2$ $\vec{l}_3 = \vec{k}_1$ $\vec{l}_4 = \vec{k}_2$ (4.4)

And the missing components for metric tensor $l_{\mu\nu} = (l_{\mu}, l_{\nu})$ wish themselves to be such, that:



5) The further steps inside basis spinors.

Now it's clear how to do the fifth step, the sixth, and so on Let us write the metric tensor for 4-dimensional space $_pV$, which arise after p-th step inside spinors ($p \ge 3$):

$$({}_{p}m_{\mu}, {}_{p}m_{\nu}) = {}_{p}m_{\mu\nu}$$

Literature:

1). R. Penrose, W. Rindler "Spinors and space-time." Volume 1, 1984

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