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The inner structure of basis spinors.

Abstract

At first the 4-dimensional real basis spinors are set in accordance to usual 2-dimensional complex basis spinors. This is the first step. Then the 4-dimensional space of real basis spinors represents as the tensor product of two 2-demensional spaces. Further the sum of these two 2-dimensional spaces represents as the tensor product of two new 2-dimensional spaces. And this operation repeats infinitely. The metric tensor defines for each of derived in this process spaces (as for 2-dimensional, so for 4-dimensional). The shortage of data at this process compensates by the simplicity principle.

Content

1).The inner complex structure of 2-dimensional basis spinors.

Let us consider ordinary 4-dimensional Minkowski space. It's metric tensor is : $n_{\mu\nu} = (\vec{n}_\mu, \vec{n}_\nu)$

	$\vec{n}_1$	$\vec{n}_2$	$\vec{n}_3$	$\vec{n}_4$
$\vec{n}_1$	1	0	0	0
$\vec{n}_2$	0	-1	0	0
$\vec{n}_3$	0	0	-1	0
$\vec{n}_4$	0	0	0	-1

(1.1)

From [1] we'll take the formule (3.1.20) for the connection of basis 4-vectors with the basis complex 2-spinors :

$$\vec{n}_1 = \frac{1}{\sqrt{2}} \cdot (\vec{\xi}_1 \otimes \vec{\xi}_1^{\,*} + \vec{\xi}_2 \otimes \vec{\xi}_2^{\,*})$$

$$\vec{n}_2 = \frac{1}{\sqrt{2}} \cdot (\vec{\xi}_1 \otimes \vec{\xi}_2^{\,*} + \vec{\xi}_2 \otimes \vec{\xi}_1^{\,*}) \qquad \vec{n}_\mu = n_\mu^{\ \alpha\beta} \cdot \vec{\xi}_\alpha \otimes \vec{\xi}_\beta^{\,*} \qquad (1.2)$$

$$\vec{n}_3 = \frac{i}{\sqrt{2}} \cdot (\vec{\xi}_1 \otimes \vec{\xi}_2^{\,*} - \vec{\xi}_2 \otimes \vec{\xi}_1^{\,*})$$

$$\vec{n}_4 = \frac{1}{\sqrt{2}} \cdot (\vec{\xi}_1 \otimes \vec{\xi}_1^{\,*} - \vec{\xi}_2 \otimes \vec{\xi}_2^{\,*})$$

From that formule we get the metric tensors for $\vec{\xi}_\alpha$ and $\vec{\xi}_\alpha^{\,*}$:

$$(\vec{\xi}_\alpha, \vec{\xi}_\beta) = \varepsilon_{\alpha\beta} \qquad (\vec{\xi}_\alpha^{\,*}, \vec{\xi}_\beta^{\,*}) = \varepsilon_{\alpha\beta} \qquad (1.3)$$

	$\vec{\xi}_1$	$\vec{\xi}_2$	$\vec{\xi}_\beta$
$\vec{\xi}_1$	0	1	
$\vec{\xi}_2$	-1	0	
$\vec{\xi}_\alpha$			$\varepsilon_{\alpha\beta}$

It is written in [1] that [(2.5.26)] $\vec{\xi}_\alpha^{\,*}$ did not decompose along $\vec{\xi}_\beta$. Then we do so. Let us form vectors $\vec{a}_\alpha$ and $\vec{b}_\alpha$ so :

$$\vec{a}_\alpha = \frac{1}{\sqrt{2}} \cdot (\vec{\xi}_\alpha + \vec{\xi}_\alpha^{\,*}) \qquad \vec{a}_\alpha^{\,*} = \vec{a}_\alpha \qquad \alpha = 1, 2$$

$$\vec{b}_\alpha = \frac{i}{\sqrt{2}} \cdot (\vec{\xi}_\alpha^{\,*} - \vec{\xi}_\alpha) \qquad \vec{b}_\alpha^{\,*} = \vec{b}_\alpha$$

Then

$$\vec{\xi}_\alpha = \frac{1}{\sqrt{2}} \cdot (\vec{a}_\alpha + i \cdot \vec{b}_\alpha)$$

$$\vec{\xi}_\alpha^{\,*} = \frac{1}{\sqrt{2}} \cdot (\vec{a}_\alpha - i \cdot \vec{b}_\alpha) \qquad (1.5)$$

And the equations (1.3) will take the form :

$$(\vec{a}_\alpha,\vec{a}_\beta)-(\vec{b}_\alpha,\vec{b}_\beta)+i\cdot((\vec{a}_\alpha,\vec{b}_\beta)+(\vec{b}_\alpha,\vec{a}_\beta))=2\cdot\varepsilon_{\alpha\beta}$$
$$(\vec{a}_\alpha,\vec{a}_\beta)-(\vec{b}_\alpha,\vec{b}_\beta)-i\cdot((\vec{a}_\alpha,\vec{b}_\beta)+(\vec{b}_\alpha,\vec{a}_\beta))=2\cdot\varepsilon_{\alpha\beta} \qquad (1.6)$$

From the idea of simplicity we choose the next formule :

$$(\vec{a}_\alpha,\vec{b}_\beta)=(\vec{b}_\alpha,\vec{a}_\beta)=0 \qquad (1.7)$$

From the idea of simplicity we adopt :

$$(\vec{a}_\alpha,\vec{a}_\beta)=\varepsilon_{\alpha\beta} \qquad (\vec{b}_\alpha,\vec{b}_\beta)=-\varepsilon_{\alpha\beta} \qquad (1.8)$$

Let us designate :

$$\vec{c}_1=\vec{a}_1 \qquad \vec{c}_2=\vec{a}_2 \qquad \vec{c}_3=\vec{b}_1 \qquad \vec{c}_4=\vec{b}_2 \qquad (1.9)$$

Then
$$(\vec{c}_\mu,\vec{c}_\nu)=c_{\mu\nu} \qquad (1.10)$$

	$\vec{a}_1$	$\vec{a}_2$	$\vec{b}_1$	$\vec{b}_2$	$\vec{c}_\mu$
$\vec{a}_1$	0	1	0	0	$\vec{c}_1$
$\vec{a}_2$	-1	0	0	0	$\vec{c}_2$
$\vec{b}_1$	0	0	0	-1	$\vec{c}_3$
$\vec{b}_2$	0	0	1	0	$\vec{c}_4$
$\vec{c}_\nu$	$\vec{c}_1$	$\vec{c}_2$	$\vec{c}_3$	$\vec{c}_4$	$c_{\mu\nu}$

2) The second step inside the structure of basis spinors.

Let us represent the resulting 4-dimensional space of real spinors with the basis $\vec{c}_\mu$ as the tensor product of two 2-dimensional spaces with bases $\vec{d}_\alpha$ and $\vec{e}_\beta$, and their connection with $\vec{c}_\mu$ we'll take in this form :

$$\vec{c}_1 = \frac{1}{\sqrt{2}} \cdot (\vec{d}_1 \otimes \vec{e}_1 + \vec{d}_2 \otimes \vec{e}_2)$$
$$\vec{c}_2 = \frac{1}{\sqrt{2}} \cdot (\vec{d}_1 \otimes \vec{e}_2 - \vec{d}_2 \otimes \vec{e}_1)$$
$$\vec{c}_3 = \frac{1}{\sqrt{2}} \cdot (\vec{d}_1 \otimes \vec{e}_2 + \vec{d}_2 \otimes \vec{e}_1) \qquad \vec{c}_\mu = c_\mu{}^{\alpha\beta} \cdot \vec{d}_\alpha \otimes \vec{e}_\beta \quad (2.1)$$
$$\vec{c}_4 = \frac{1}{\sqrt{2}} \cdot (\vec{d}_1 \otimes \vec{e}_1 - \vec{d}_2 \otimes \vec{e}_2)$$

Then we have for metric tensors : $(\vec{d}_\alpha, \vec{d}_\beta) = d_{\alpha\beta}$

	$\vec{d}_1$	$\vec{d}_2$	$\vec{d}_\beta$
$\vec{d}_1$	1	0	
$\vec{d}_2$	0	1	
$\vec{d}_\alpha$			$d_{\alpha\beta}$

(2.2)

$$(\vec{e}_\alpha, \vec{e}_\beta) = e_{\alpha\beta}$$

	$\vec{e}_1$	$\vec{e}_2$	$\vec{e}_\beta$
$\vec{e}_1$	0	1	
$\vec{e}_2$	-1	0	
$\vec{e}_\alpha$			$e_{\alpha\beta}$

(2.3)

We got the situation the same as with complex 2-dimensional basis spinors, before the input into consideration the real 4-dimensional basis spinors. Let us try to input the 4-dimensional objects and now :

$$\vec{f}_1 = \vec{d}_1 \qquad \vec{f}_2 = \vec{d}_2 \qquad \vec{f}_3 = \vec{e}_1 \qquad \vec{f}_4 = \vec{e}_2 \qquad (2.4)$$

And the missing components of metric tensor $f_{\mu\nu} = (\vec{f}_\mu, \vec{f}_\nu)$ (from idea of simplicity) we will consider a zeros :

	$\vec{d}_1$	$\vec{d}_2$	$\vec{e}_1$	$\vec{e}_2$	$\vec{f}_\mu$
$\vec{d}_1$	1	0	0	0	$\vec{f}_1$
$\vec{d}_2$	0	1	0	0	$\vec{f}_2$
$\vec{e}_1$	0	0	0	1	$\vec{f}_3$
$\vec{e}_2$	0	0	-1	0	$\vec{f}_4$
$\vec{f}_\nu$	$\vec{f}_1$	$\vec{f}_2$	$\vec{f}_3$	$\vec{f}_4$	$f_{\mu\nu}$

(2.5)

3) The third step inside basis spinors.

Let us represent the resulting 4-dimensional space with the basis $\vec{f}_\mu$ as the tensor product of two 2-dimensional spaces with bases $\vec{g}_\alpha$ and $\vec{h}_\beta$, and their connection with $\vec{f}_\mu$ we'll take in the form :

$$\vec{f}_1 = \frac{1}{2}\cdot(\vec{g}_1 \otimes \vec{h}_1 + \vec{g}_2 \otimes \vec{h}_2)$$

$$\vec{f}_2 = \frac{1}{2}\cdot(\vec{g}_1 \otimes \vec{h}_2 - \vec{g}_2 \otimes \vec{h}_1)$$

$$\vec{f}_3 = \frac{1}{2}\cdot(\vec{g}_1 \otimes \vec{h}_2 + \vec{g}_2 \otimes \vec{h}_1) \qquad \vec{f}_\mu = f_\mu{}^{\alpha\beta}\cdot\vec{g}_\alpha \otimes \vec{h}_\beta \qquad (3.1)$$

$$\vec{f}_4 = \frac{1}{2}\cdot(\vec{g}_1 \otimes \vec{h}_1 - \vec{g}_2 \otimes \vec{h}_2)$$

Then we have for metric tensors : $(\vec{g}_\alpha, \vec{g}_\beta) = g_{\alpha\beta}$

	$\vec{g}_1$	$\vec{g}_2$	$\vec{g}_\beta$
$\vec{g}_1$	1	-1	
$\vec{g}_2$	1	1	
$\vec{g}_\alpha$			$g_{\alpha\beta}$

(3.2)

$$(\vec{h}_\alpha, \vec{h}_\beta) = h_{\alpha\beta}$$

	$\vec{h}_1$	$\vec{h}_2$	$\vec{h}_\beta$
$\vec{h}_1$	1	-1	
$\vec{h}_2$	1	1	
$\vec{h}_\alpha$			$h_{\alpha\beta}$

(3.3)

Let us again input the 4-dimensional objects, now $\vec{i}_\mu$:

$$\vec{i}_1 = \vec{g}_1 \qquad \vec{i}_2 = \vec{g}_2 \qquad \vec{i}_3 = \vec{h}_1 \qquad \vec{i}_4 = \vec{h}_2 \qquad (3.4)$$

And the missing components for metric tensor $i_{\mu\nu} = (\vec{i}_\mu, \vec{i}_\nu)$ wish themselves to be such, that :

	$\vec{g}_1$	$\vec{g}_2$	$\vec{h}_1$	$\vec{h}_2$	$\vec{i}_\mu$
$\vec{g}_1$	1	-1	-1	1	$\vec{i}_1$
$\vec{g}_2$	1	1	-1	-1	$\vec{i}_2$
$\vec{h}_1$	1	-1	1	-1	$\vec{i}_3$
$\vec{h}_2$	1	1	1	1	$\vec{i}_4$
$\vec{i}_\nu$	$\vec{i}_1$	$\vec{i}_2$	$\vec{i}_3$	$\vec{i}_4$	$i_{\mu\nu}$

(3.5)

4) The fourth step inside basis spinors.

Let us represent the resulting 4-dimensional space with the basis $\vec{i}_\mu$ as the tensor product of two 2-dimensional spaces with bases $\vec{j}_\alpha$ and $\vec{k}_\beta$. And their connection with $\vec{i}_\mu$ we'll take in the form :

$$\begin{aligned} \vec{i}_1 &= \vec{j}_1 \otimes \vec{k}_1 \\ \vec{i}_2 &= \vec{j}_1 \otimes \vec{k}_2 \\ \vec{i}_3 &= \vec{j}_2 \otimes \vec{k}_1 \\ \vec{i}_4 &= \vec{j}_2 \otimes \vec{k}_2 \end{aligned} \qquad \vec{i}_\mu = i_\mu{}^{\alpha\beta} \cdot \vec{j}_\alpha \otimes \vec{k}_\beta \qquad (4.1)$$

Then we have for metric tensors : $(\vec{j}_\alpha, \vec{j}_\beta) = j_{\alpha\beta}$

	$\vec{j}_1$	$\vec{j}_2$	$\vec{j}_\beta$
$\vec{j}_1$	1	-1	
$\vec{j}_2$	1	1	
$\vec{j}_\alpha$			$j_{\alpha\beta}$

(4.2)

$(\vec{k}_\alpha, \vec{k}_\beta) = k_{\alpha\beta}$

	$\vec{k}_1$	$\vec{k}_2$	$\vec{k}_\beta$
$\vec{k}_1$	1	-1	
$\vec{k}_2$	1	1	
$\vec{k}_\alpha$			$k_{\alpha\beta}$

(4.3)

Let us again enter the 4-dimensional space with basis $\vec{l}_\mu$:

$$\vec{l}_1 = \vec{j}_1 \qquad \vec{l}_2 = \vec{j}_2 \qquad \vec{l}_3 = \vec{k}_1 \qquad \vec{l}_4 = \vec{k}_2 \qquad (4.4)$$

And the missing components for metric tensor $l_{\mu\nu} = (\vec{l}_\mu, \vec{l}_\nu)$ wish themselves to be such, that:

	$\vec{j}_1$	$\vec{j}_2$	$\vec{k}_1$	$\vec{k}_2$	$\vec{l}_\mu$
$\vec{j}_1$	1	-1	-1	1	$\vec{l}_1$
$\vec{j}_2$	1	1	-1	-1	$\vec{l}_2$
$\vec{k}_1$	1	-1	1	-1	$\vec{l}_3$
$\vec{k}_2$	1	1	1	1	$\vec{l}_4$
$\vec{l}_\nu$	$\vec{l}_1$	$\vec{l}_2$	$\vec{l}_3$	$\vec{l}_4$	$l_{\mu\nu}$

(4.5)

5) The further steps inside basis spinors.

Now it's clear how to do the fifth step, the sixth, and so on … . Let us write the metric tensor for 4-dimensional space ${}_pV$, which arise after p-th step inside spinors ($p \geq 3$) :

$$({}_p\dot{m}_\mu, {}_p\dot{m}_\nu) = {}_pm_{\mu\nu}$$

${}_pm_{\mu\nu}$					${}_p\dot{m}_\mu$
	1	-1	-1	1	${}_p\dot{m}_1$
	1	1	-1	-1	${}_p\dot{m}_2$
	1	-1	1	-1	${}_p\dot{m}_3$
	1	1	1	1	${}_p\dot{m}_4$
${}_p\dot{m}_\nu$	${}_p\dot{m}_1$	${}_p\dot{m}_2$	${}_p\dot{m}_3$	${}_p\dot{m}_4$	

(5.1)

Literature:

1). R. Penrose, W. Rindler "Spinors and space-time." Volume 1, 1984

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