EXTERIOR AND INTERIOR DYNAMIC ISO-SPHERE HOLOGRAPHIC RINGS WITH AN INVERSE ISO-DUALITY

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Abstract

In this preliminary work, we use a dynamic iso-unit function to iso-topically lift the "static" Inopin holographic ring (IHR) of the unit sphere to an interconnected pair of "dynamic iso-sphere IHRs" (iso-DIHR), where the IHR is simultaneously iso-dual to both a magnified "exterior iso-DIHR" and de-magnified "interior iso-DIHR". For both the continuously-varying and discretely-varying cases, we define the dynamic iso-amplitude-radius of one iso-DIHR as being equivalent to the dynamic iso-amplitude-curvature of its counterpart, and conversely. These initial results support the hypothesis that a new IHR-based mode of iso-geometry and iso-topology may be in order, which is significant because the interior and exterior zones delineated by the IHR are fundamentally "iso-dual inverses" and may be inferred from one another.

Keywords: Santilli iso-mathematics; Inopin holographic ring; Iso-geometry; Iso-topology; Dynamic iso-sphere; Iso-duality.

To the memory of my good friend Gavin Koester-Backstrom.

1 Introduction

In a forward attempt to establish order in chaos, A.E. Inopin introduced the dual space-time IHR topology in a proof of quark confinement [1], which received a preliminary topological upgrade in the triplex generalization of [2]. In Euclidean complex space, Inopin's dual 3D space-time IHR topology comprises a 1-sphere IHR "time zone" that delineates two spatial 2-branes, whereas in Euclidean triplex space, Inopin's dual 4D space-time IHR topology generalizes the 1-sphere IHR to a 2-sphere IHR that delineates two spatial 3-branes [1, 2]. In other words, the brane states can be inferred from the IHR states and vice-versa because the IHR acquires Berry phase transitions for (spontaneous gauge symmetry breaking) topological deformation order parameters and is simultaneously dual to both branes [1, 2].

Recently, R.M. Santilli's iso-mathematics [3, 4, 5, 6, 7] was applied to the dual 4D space-time IHR topology (with the 2-sphere IHR) [1, 2, 8] to initiate the *iso-dual* 4D space-time IHR topology (with the iso-2-sphere IHR) [9]. Subsequently, the new class of *dynamic iso-spaces* was constructed [10]; a dynamic iso-space is an iso-space that is characterized by constant *change* [10]. More specifically, a dynamic iso-space is built with a dynamic isotopic lifting that arises due to a dynamic iso-unit function that varies over time [10]. Santilli's discovery of iso-mathematics gave way to these dynamic constructs because he proved that his iso-unit can be, among many things, a function [3, 4, 5, 6, 7]. Therefore, in this paper, we engage this cutting-edge notion of dynamic iso-spaces [10] with the emerging iso-dual 4D space-time IHR topology [9] to define an interconnected pair of iso-DIHRs, where the exterior iso-DIHR is iso-dual to an interior iso-DIHR.

We launch our investigation with Section 2, where we augment the isosphere IHR (iso-IHR) [9] by initiating definitions for the *exterior iso-IHR* and the *interior iso-IHR*—for this, we propose a fundamental and critical *inverse iso-duality* between the exterior and interior iso-IHRs that are both locally iso-morphic to the original IHR. Next, in Section 3, we deploy the dynamic iso-topic lifting of [10] to upgrade the initial results of Section 2, where we mobilize general, continuous, and discrete definitions for the exterior iso-DIHR and interior iso-DIHR that preserve the original iso-morphism. Finally, we conclude our exploration with the recapitulation of results and future outlook of Section 4.

2 Inverse iso-duality between the exterior and interior iso-sphere IHRs

Here, we discuss and extend the iso-IHR [9] by proposing an inverse iso-duality between the exterior and interior branes.

Following [1, 2], let T_r^1 be the *unit 1-sphere IHR* of amplitude-radius $r = 1$ and amplitude-curvature $\kappa = \frac{1}{r}$ that is iso-metrically embedded in the complex space S^2, such that eq. (13) of [2] identifies

$$T_r^1 = \{\vec{s} \in S^2 : |\vec{s}| = r\}, \tag{1}$$

where $T_r^1 \subset S^2$ is the multiplicative group of all non-zero complex coordinate-vectors of normalized amplitude-radius r. Next, let T_r^2 be a unit 2-sphere IHR that is iso-metrically embedded in the triplex space S^3, such that eq. (40) of [2] identifies

$$T_r^2 = \{\vec{s} \in S^3 : |\vec{s}| = r\}, \tag{2}$$

where $T_r^2 \subset S^3$ is the multiplicative group of all non-zero triplex coordinate-vectors of normalized amplitude-radius r; T_r^1 is the great circle of T_r^2 so both *non-linear* structures share the same amplitude-radius r with the constant characterizing curvature of κ, where $S^2 \subset S^3$ and $T_r^1 = T_r^2 \cap S^2$ [2]. In this IHR-based topology [1, 2], eqs. (14) and (40) in [2] demonstrate that the micro 2-brane sub-space $S_-^2 \subset S^2$ and the micro 3-brane sub-space $S_-^3 \subset S^3$ correspond to *interior* dynamical systems, while the macro 2-brane sub-space $S_+^2 \subset S^2$ and the micro 3-brane sub-space $S_+^3 \subset S^3$ correspond to *exterior* dynamical systems, where T_r^1 delineates S_-^2 and S_+^2, and T_r^2 delineates S_-^3 and S_+^3.

Now, following Santilli's iso-number methodology [3, 4, 5, 6, 7] and the iso-IHR definition [9], we select some positive-definite iso-unit $\hat{r}_+ > r$ with the corresponding positive-definite inverse $\hat{r}_- = \frac{1}{\hat{r}_+} < r$ to establish the array of *exterior* iso-topic liftings

$$\begin{array}{rccc} f(\hat{r}_+): & T_r^n & \rightarrow & T_{\hat{r}_+}^n \\ f^{-1}(\hat{r}_+): & T_{\hat{r}_+}^n & \rightarrow & T_r^n \end{array}, \quad n \in \{1, 2\}, \tag{3}$$

for the magnified

1. *exterior iso-1-sphere IHR (iso-1-IHR)* $T^1_{\hat{r}_+}$ and
2. *exterior iso-2-sphere IHR (iso-2-IHR)* $T^2_{\hat{r}_+}$.

In this case of eq. (3), a given $T^n_{\hat{r}_+}$ is "outside" T^n_r because $\hat{r}_+ > r$. Thus, $\hat{r}_+$ is termed the *exterior iso-unit*, which serves as the *exterior iso-amplitude-radius* for both $T^1_{\hat{r}_+}$ and $T^2_{\hat{r}_+}$, while $\hat{r}_-$ serves as the *exterior iso-amplitude-curvature* for both $T^1_{\hat{r}_+}$ and $T^2_{\hat{r}_+}$, such that $T^1_{\hat{r}_+} \subset T^2_{\hat{r}_+}$ is the great circle of $T^2_{\hat{r}_+}$.

So a question comes to mind: how might the exterior iso-amplitude-curvature $\hat{r}_-$ fit into the structure and function of the said iso-IHR topology? Our *hypothesis* is that the selected iso-unit relation $\hat{r}_+ = \frac{1}{\hat{r}_-} > r$ identifies a critical and fundamental iso-duality for IHR-based topological implementations in terms of spherical radii and curvature. In the iso-IHR topology introduction of [9], we recall that the iso-amplitude-curvature property was only mentioned in a brief context due to the limited scope of that analysis. Therefore, in this section, we wish to further probe the applicability of the iso-amplitude-curvature by deploying it to define an additional topological iso-structure. Hence, in addition to being the exterior iso-amplitude-curvature of $T^1_{\hat{r}_+}$ and $T^2_{\hat{r}_+}$, we furthermore define $\hat{r}_-$ as the *interior iso-amplitude-radius* and *interior iso-unit* of two *new* iso-IHRs, namely the de-magnified

1. *interior iso-1-IHR* $T^1_{\hat{r}_-}$ and
2. *interior iso-2-IHR* $T^2_{\hat{r}_-}$,

with the corresponding array of *interior* iso-topic liftings

$$\begin{array}{rccc} f(\hat{r}_-): & T^n_r & \rightarrow & T^n_{\hat{r}_-} \\ f^{-1}(\hat{r}_-): & T^n_{\hat{r}_-} & \rightarrow & T^n_r \end{array}, \quad n \in \{1, 2\}. \tag{4}$$

In this case of eq. (4), a given $T^n_{\hat{r}_-}$ is "inside" T^n_r because $\hat{r}_- < r$. Hence, upon recalling the relation $\hat{r}_- = \frac{1}{\hat{r}_+}$, we realize that $\hat{r}_+$ is also the *interior iso-amplitude-curvature* of both $T^1_{\hat{r}_-}$ and $T^2_{\hat{r}_-}$! Thus, in terms of iso-amplitude-radius and iso-amplitude-curvature, we've identified a *fundamental iso-duality* between $T^n_{\hat{r}_+}$ and $T^n_{\hat{r}_-}$ written as

$$T^n_{\hat{r}_-} \leftarrow T^n_r \rightarrow T^n_{\hat{r}_+} \tag{5}$$

because the iso-amplitude-radii and iso-amplitude-curvatures are interdependent with respect to T^n_r. Therefore, in addition to the lemmas of [9], the results of eqs. (3–5) indicate the trichotomy:

1. ($\hat{r}_- < 1$): interior iso-amplitude-radius of $T^n_{\hat{r}_-}$, exterior iso-amplitude-curvature of $T^n_{\hat{r}_+}$, interior iso-multiplicative iso-unit of $T^n_{\hat{r}_-}$, exterior iso-multiplicative iso-unit inverse of $T^n_{\hat{r}_+}$;

2. ($r = 1$): amplitude-radius, amplitude-curvature, multiplicative unit; and

3. ($\hat{r}_+ > 1$): exterior iso-amplitude-radius of $T^n_{\hat{r}_+}$, interior iso-amplitude-curvature of $T^n_{\hat{r}_-}$, exterior iso-multiplicative iso-unit of $T^n_{\hat{r}_+}$, interior iso-multiplicative iso-unit inverse of $T^n_{\hat{r}_-}$.

Therefore, we have establish the following:

Lemma 1. *An n-sphere IHR T^n_r of amplitude-radius (and unit) $r = 1$ that is iso-topically lifted via $T^n_r \rightarrow T^n_{\hat{r}_+}$ to the exterior iso-n-IHR $T^n_{\hat{r}_+}$ of exterior iso-amplitude-radius (and exterior iso-multiplicative iso-unit) $\hat{r}_+ > r$ can be simultaneously lifted via $T^n_r \rightarrow T^n_{\hat{r}_-}$ to the interior iso-n-IHR $T^n_{\hat{r}_-}$ of interior iso-amplitude-radius (and interior iso-multiplicative iso-unit) $\hat{r}_- < r$ if $\hat{r}_- = \frac{1}{\hat{r}_+}$, where $\hat{r}_+$ is the interior iso-amplitude-curvature of $T^n_{\hat{r}_-}$ and $\hat{r}_-$ is the exterior iso-amplitude-curvature of $T^n_{\hat{r}_+}$, such that $T^n_{\hat{r}_+}$ and $T^n_{\hat{r}_-}$ are iso-dual inverses and locally iso-morphic to T^n_r.*

At this point, we've discussed and extended the iso-IHR of [9] to include the exterior and interior iso-IHRs of eqs. (3–4) and Lemma 1. See Figure 1 for a depiction of this scenario.

3 Dynamic iso-sphere IHR

Here, we apply the dynamic iso-topic lifting of [10] to the iso-IHR results of Section 2, where we'll introduce the general definitions for the exterior and interior iso-DIHRs in Section 3.1. Subsequently, in Section 3.2, we'll push beyond the general form to construct the continuous and discrete cases.

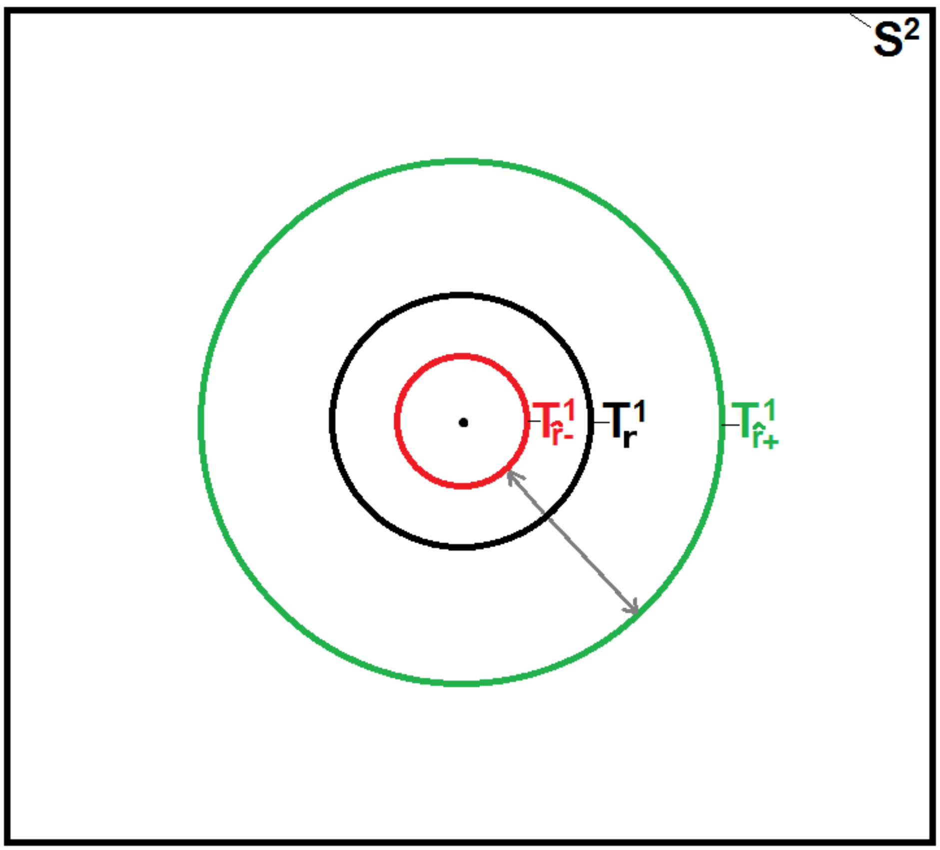

Fig. 1: The iso-1-IHR T^1_r is iso-topically lifted to both the exterior iso-1-IHR $T^1_{\hat{r}_+}$ and the interior iso-1-IHR $T^1_{\hat{r}_-}$ simultaneously, where $T^1_{\hat{r}_+}$ and $T^1_{\hat{r}_-}$ are iso-dual.

3.1 General

Thus, following the dynamic methodology of [10], we define the positive-definite dynamic iso-unit function as

$$\hat{r}_+ \equiv \hat{\delta}_+(t) > r \tag{6}$$

with its corresponding positive-definite inverse

$$\hat{r}_- \equiv \frac{1}{\hat{\delta}_+(t)} \equiv \hat{\delta}_-(t) < r, \tag{7}$$

where $\hat{\delta}_+(t)$ increases and $\hat{\delta}_-(t)$ decreases simultaneously as the parameter t varies as $t \to \infty$, for the *general* form. Hence, eq. (3) can be rewritten to establish the *exterior dynamic iso-topic lifting* form

$$\begin{array}{rlcl} f(\hat{\delta}_+(t)): & T_r^n & \to & T^n_{\hat{\delta}_+(t)} \\ f^{-1}(\hat{\delta}_+(t)): & T^n_{\hat{\delta}_+(t)} & \to & T_r^n \end{array}, \quad n \in \{1,2\}, \tag{8}$$

to define the

1. *exterior iso-1-DIHR* $T^1_{\hat{\delta}_+(t)}$ and
2. *exterior iso-2-DIHR* $T^2_{\hat{\delta}_+(t)}$.

Similarly, eq. (4) can be rewritten to express the *interior dynamic iso-topic lifting* form

$$\begin{array}{rlcl} f(\hat{\delta}_-(t)): & T_r^n & \to & T^n_{\hat{\delta}_-(t)} \\ f^{-1}(\hat{\delta}_-(t)): & T^n_{\hat{\delta}_-(t)} & \to & T_r^n \end{array}, \quad n \in \{1,2\}, \tag{9}$$

to define the

1. *interior iso-1-DIHR* $T^1_{\hat{\delta}_-(t)}$ and
2. *interior iso-2-DIHR* $T^2_{\hat{\delta}_-(t)}$.

Therefore, the implications and results of eqs. (6–9) authorize us to establish the following:

Lemma 2. *An n-sphere IHR* T^n_r *of amplitude-radius (and unit)* $r = 1$ *that is dynamically iso-topically lifted via* $T^n_r \to T^n_{\hat{\delta}_+(t)}$ *to the exterior iso-n-DIHR* $T^n_{\hat{\delta}_+(t)}$ *of exterior dynamic iso-amplitude-radius (and exterior dynamic iso-unit)* $\hat{\delta}_+(t) > r$ *can be simultaneously lifted via* $T^n \to T^n_{\hat{\delta}_-(t)}$ *to the interior iso-n-DIHR* $T^n_{\hat{\delta}_-(t)}$ *of interior dynamic iso-amplitude-radius (and interior dynamic iso-unit)* $\hat{\delta}_-(t) < r$ *if* $\hat{\delta}_-(t) = \frac{1}{\hat{\delta}_+(t)}$ *as the parameter* t *varies, where* $\hat{\delta}_+(t)$ *is the interior dynamic iso-amplitude-curvature of* $T^n_{\hat{\delta}_-(t)}$ *and* $\hat{\delta}_-(t)$ *is the exterior dynamic iso-amplitude-curvature of* $T^n_{\hat{\delta}_+(t)}$*, such that* $T^n_{\hat{\delta}_+(t)}$ *and* $T^n_{\hat{\delta}_-(t)}$ *are dynamically, inversely iso-dual and locally iso-morphic to* T^n_r*.*

At this point, we've successfully applied the *general* dynamic iso-topic lifting definitions of [10] to the iso-IHR results of Section 2 by introducing the definitions for the exterior and interior iso-DIHRs in Section 3.1, where the resulting constructions of eqs. (6–9) are characterized by Lemma 2. See Figure 2 for a depiction of this scenario.

3.2 Continuous and discrete

Next, we combine the continuous and discrete dynamic iso-space definitions of [10] with the general iso-DIHR definitions of Section 3.1 to assemble the continuous and discrete iso-DIHR implementations.

First, we will show that $T^n_{\hat{\delta}_-(t)}$ and $T^n_{\hat{\delta}_+(t)}$ can be defined as *continuous iso-n-DIHRs* if the dynamic iso-unit functions $\hat{\delta}_{+_c}(t)$ and $\hat{\delta}_{-_c}(t)$ are both continuous as their parameter t varies, where insert the additional subscript label c to denote the "continuous" case. Hence, for example, let t be the continuously varying parameter for the *continuous exterior and interior dynamic iso-unit* functions

$$\left.\begin{array}{lllll} \hat{r}_+ & \equiv & \hat{\delta}_{+_c}(t) & \in & \mathbb{R}_c \\ \hat{r}_- & \equiv & \hat{\delta}_{-_c}(t) & \equiv & \frac{1}{\hat{\delta}_{+_c}(t)} \in \mathbb{R}_c \end{array}\right. ,\ 0 < \hat{\delta}_{-_c}(t) < r < \hat{\delta}_{+_c}(t) < \infty,\ t \to \infty, \tag{10}$$

such that $\mathbb{R}_c$ is a positive-definite continuous set (i.e. the positive real numbers), to consequently define the

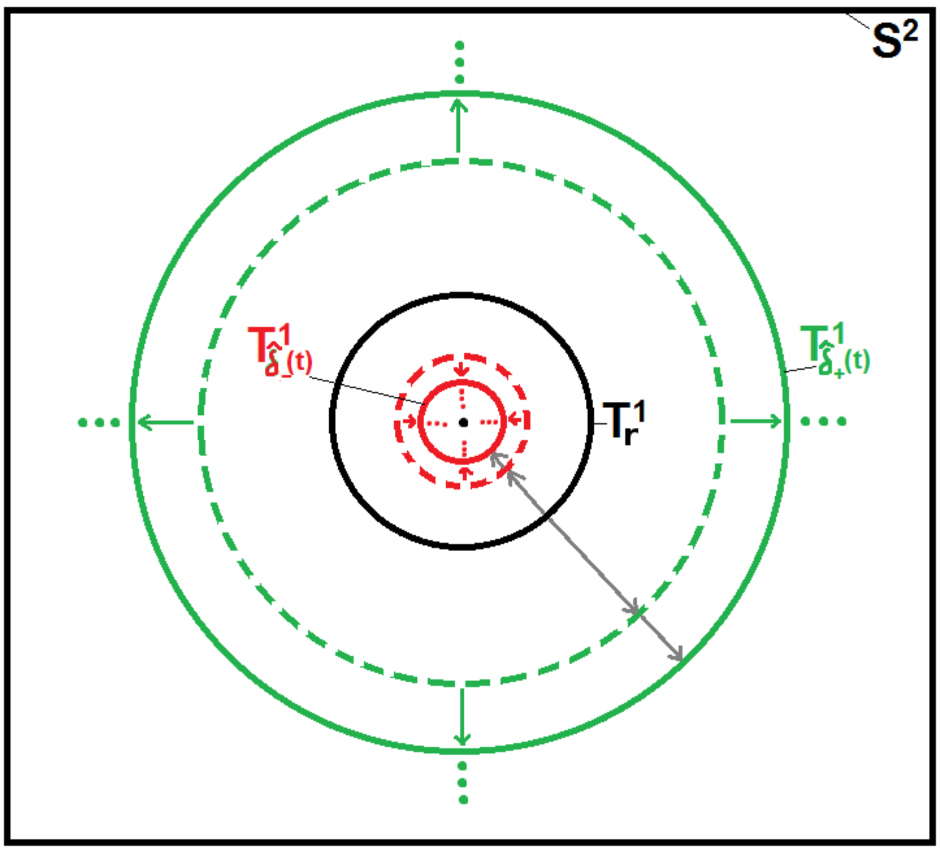

Fig. 2: The iso-1-IHR T_r^1 is dynamically iso-topically lifted to both the exterior iso-1-DIHR $T^1_{\hat{\delta}_+(t)}$ and the interior iso-1-DIHR $T^1_{\hat{\delta}_-(t)}$ simultaneously as the parameter t varies as $t \to \infty$, where $T^1_{\hat{\delta}_+(t)}$ and $T^1_{\hat{\delta}_-(t)}$ are iso-dual inverses.

1. *continuous exterior iso-n-DIHR* $T^n_{\hat{\delta}_{+c}(t)}$ and
2. *continuous interior iso-n-DIHR* $T^n_{\hat{\delta}_{-c}(t)}$,

where we rewrite eqs. (8–9) in the *continuous exterior and interior dynamic iso-topic lifting* form

$$\left.\begin{array}{rccc} f(\hat{\delta}_{+_c}(t)): & T^n_r & \to & T^n_{\hat{\delta}_{+c}(t)} \\ f^{-1}(\hat{\delta}_{+_c}(t)): & T^n_{\hat{\delta}_{+c}(t)} & \to & T^n_r \end{array}\right., \quad n \in \{1,2\}, \tag{11}$$

and

$$\left.\begin{array}{rccc} f(\hat{\delta}_{-_c}(t)): & T^n_r & \to & T^n_{\hat{\delta}_{-c}(t)} \\ f^{-1}(\hat{\delta}_{-_c}(t)): & T^n_{\hat{\delta}_{-c}(t)} & \to & T^n_r, \end{array}\right., \quad n \in \{1,2\}, \tag{12}$$

respectively. In eqs. (11–12), T^n_r remains locally iso-morphic to both $T^n_{\hat{\delta}_{-c}(t)}$ and $T^n_{\hat{\delta}_{+c}(t)}$ as t continuously varies. Thus, the results of eqs. (10–12) permit us to identify the following:

Lemma 3. *An exterior iso-n-DIHR* $T^n_{\hat{\delta}_{+c}(t)}$ *is a continuous exterior iso-n-DIHR if the exterior dynamic iso-unit function* $\hat{\delta}_{+_c}(t)$ *is continuous as its parameter* t *varies.*

Lemma 4. *An interior iso-n-DIHR* $T^n_{\hat{\delta}_{-c}(t)}$ *is a continuous interior iso-n-DIHR if the interior dynamic iso-unit function* $\hat{\delta}_{-_c}(t)$ *is continuous as its parameter* t *varies.*

Second, we will show that $T^n_{\hat{\delta}_{-}(t)}$ and $T^n_{\hat{\delta}_{+}(t)}$ can also be defined as *discrete iso-n-DIHRs* if the dynamic iso-unit functions $\hat{\delta}_{+_d}(t)$ and $\hat{\delta}_{-_d}(t)$ are both discrete as their parameter t varies, where let d denote the "discrete" case. Hence, for example, let t be the discretely varying parameter for the *discrete exterior and interior dynamic iso-unit* functions

$$\left.\begin{array}{rcccl} \hat{r}_+ & \equiv & \hat{\delta}_{+_d}(t) & \in & \mathbb{R}_d \\ \hat{r}_- & \equiv & \hat{\delta}_{-_d}(t) & \equiv & \frac{1}{\hat{\delta}_{+_d}(t)} \in \mathbb{R}_d \end{array}\right., \ 0 < \hat{\delta}_{-_d}(t) < r < \hat{\delta}_{+_d}(t) < \infty, \ t \to \infty, \tag{13}$$

such that $\mathbb{R}_d$ is a positive-definite discrete set (i.e. positive Fibonacci numbers), to consequently define the

1. *discrete exterior iso-n-DIHR* $T^n_{\hat{\delta}_{+_d}(t)}$ and
2. *discrete interior iso-n-DIHR* $T^n_{\hat{\delta}_{-_d}(t)}$,

where we rewrite eqs. (8–9) in the *discrete exterior and interior dynamic iso-topic lifting* form

$$\begin{array}{rlll} f(\hat{\delta}_{+_d}(t)): & T^n_r & \to & T^n_{\hat{\delta}_{+_d}(t)} \\ f^{-1}(\hat{\delta}_{+_d}(t)): & T^n_{\hat{\delta}_{+_d}(t)} & \to & T^n_r \end{array}, \quad n \in \{1,2\}, \tag{14}$$

and

$$\begin{array}{rlll} f(\hat{\delta}_{-_d}(t)): & T^n_r & \to & T^n_{\hat{\delta}_{-_d}(t)} \\ f^{-1}(\hat{\delta}_{-_d}(t)): & T^n_{\hat{\delta}_{-_d}(t)} & \to & T^n_r, \end{array}, \quad n \in \{1,2\}, \tag{15}$$

respectively. In eqs. (14–15), $T_)r^n$ remains locally iso-morphic to both $T^n_{\hat{\delta}_{-_d}(t)}$ and $T^n_{\hat{\delta}_{+_d}(t)}$ as t discretely varies. Thus, the results of eqs. (13–15) enable us to identify the following:

Lemma 5. *An exterior iso-n-DIHR $T^n_{\hat{\delta}_{+_d}(t)}$ is a discrete exterior iso-n-DIHR if the exterior dynamic iso-unit function $\hat{\delta}_{+_d}(t)$ is discrete as its parameter t varies.*

Lemma 6. *An interior iso-n-DIHR $T^n_{\hat{\delta}_{-_d}(t)}$ is a discrete interior iso-n-DIHR if the interior dynamic iso-unit function $\hat{\delta}_{-_d}(t)$ is discrete as its parameter t varies.*

At this point, we've successfully combined the continuous and discrete dynamic iso-space definitions of [10] with the general iso-DIHR definitions of Section 3.1 to assemble the continuous and discrete iso-DIHR implementations, where the resulting constructions of eqs. (10–15) are characterized by Lemmas 3–6.

4 Conclusion

The results of this work include original definitions and lemmas for continuous and discrete iso-DIHRs. Through this process, we proposed an "inverse iso-duality" that fundamentally relates the exterior iso-DIHRs to the interior iso-DIHRs, which are simultaneously, locally iso-morphic to the original IHR. This emerging array of iso-DIHRs is significant because it extends the Santilli's pioneering work [3, 4, 5, 6, 7] to new realms of exploration with potential (near future) application to the disciplines of science, technology, and engineering.

Thus, there is still much work to do, as we must continue to relentlessly scrutinize, challenge, and upgrade this emerging framework via the Scientific Method. In particular, we suggest that in order to test the validity of our results and advance the general capability and applicability of these dynamic systems to subsequent levels, a thorough and rigorous iso-mathematical investigation should be conducted along this research trajectory. For this, we must prove the said lemmas and expand the framework by instantiating additional pertinent IHR families of dynamic iso-spheres, and furthermore the dynamic geno-spheres, dynamic hyper-spheres, and dynamic iso-dual-spheres.

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