DYNAMIC ISO-TOPIC LIFTING WITH APPLICATION TO FIBONACCI'S SEQUENCE AND MANDELBROT'S SET

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Abstract

In this exploration, we introduce and define "dynamic iso-spaces", which are cutting-edge iso-mathematical constructions that are built with "dynamic iso-topic liftings" for "dynamic iso-unit functions". For this, we consider both the continuous and discrete cases. Subsequently, we engineer two simple examples that engage Fibonacci's sequence and Mandelbrot's set to define a "Fibonacci dynamic iso-space" and a "Mandelbrot dynamic iso-space", respectively. In total, this array of resulting iso-structures indicates that a new branch of iso-mathematics may be in order.

Keywords: Geometry and topology; Santilli iso-number; Santilli iso-space; Dynamic iso-space; Fractal geometry; Fibonacci sequence; Mandelbrot set.

1 Introduction

A *gigantic* problem of pure mathematics, in the context of number theory, is to establish a *universal* number classification system for describing the physical laws of nature, such as the identification of all sets that occur with numeric field axioms. During his relentless pursuit of this problem, scientific pioneer R.M. Santilli successfully circumvented the original five number field axioms [1] to create the new field of iso-mathematics [2, 3, 4, 5], which became the foundation of hadronic mechanics [6].

Since its initiation, Santilli's iso-mathematics [2, 3, 4, 5] has continued to gain momentum in application and popularity, but this emergent discipline is—to say the least—*vast*, relatively adolescent, and is currently not considered to be "mainstream" in the community of science, technology, engineering, and mathematics. Therefore, the iso-mathematics realm remains largely unexplored, with unknown bounds on its utilization potential.

In this paper, we venture into iso-mathematics [2, 3, 4, 5] by considering iso-spaces that systematically *change*. In Section 2, we introduce the notion of *dynamic iso-topic liftings* by assembling definitions for *continuous dynamic iso-spaces* and *discrete dynamic iso-spaces*. Afterwards, we give two examples that demonstrate how dynamic iso-spaces can be lifted for fractal structures in Section 3. Finally, we conclude our exploration with Section 4, where we recapitulate the results with brief discussion and outlook on future modes of research.

2 Dynamic iso-spaces

Here, we introduce and define the dynamic iso-space with its dynamic iso-topic lifting. For this, we follow Santilli's methodology [2, 3, 4, 5] and consider the following procedure:

1. Let S be a space.

2. Let $S_{\hat{t}}$ be an iso-space that results from the iso-topic lifting $S \to S_{\hat{t}}$ for the iso-unit $\hat{t} > 0$ with the corresponding inverse $\hat{\tau} = \frac{1}{\hat{t}} > 0$, such that
$$\begin{array}{rccc} f(\hat{t}): & S & \to & S_{\hat{t}} \\ f^{-1}(\hat{t}): & S_{\hat{t}} & \to & S, \end{array} \qquad (1)$$
so clearly S and $S_{\hat{t}}$ are locally iso-morphic.

3. **Dynamic iso-space.** Now, we wish to show that $S_{\hat{t}}$ is an iso-space that is characterized by constant change, meaning that $S_{\hat{t}}$ is a dynamic iso-space. Hence, we will show that $S_{\hat{t}}$ varies in accordance to some function; here, this positive-definite function will serve as the *dynamic iso-unit* that varies because its parameter varies.

4. Thus, let $r > 0$ be the parameter that varies as $r \to \infty$ for the *generic form* of the *dynamic iso-unit function*

$$\hat{t} \equiv \hat{\delta}(r) > 0, \ r \to \infty, \tag{2}$$

to consequently define the dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}(r)}. \tag{3}$$

5. **Continuous dynamic iso-space.** First, we will show that the $S_{\hat{\delta}(r)}$ of eq. (3) can be rewritten and defined as a *continuous* dynamic iso-space if its corresponding dynamic iso-unit function is continuous as its parameter r varies; for this, the constraint is that r must take on the values of an infinite continuous sequence of positive-definite numbers. Hence, for example, if $\mathbb{R}^+$ is the set of all positive-definite real numbers, then let $r \in \mathbb{R}^+$ be the positive-definite, real-valued, varying parameter for the *continuous dynamic iso-unit function*

$$\hat{t} \equiv \hat{\delta}_{\mathbb{R}^+}(r) \equiv 2\pi r > 0, \ r \in \mathbb{R}^+, \ r \to \infty_{\mathbb{R}^+}, \tag{4}$$

to consequently define the continuous dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}_{\mathbb{R}^+}(r)}, \tag{5}$$

where we rewrite eq. (1) to encompass the continuous dynamic structure of eqs. (4–5) as

$$\begin{array}{rlcl} f(\hat{\delta}_{\mathbb{R}^+}(r)): & S & \to & S_{\hat{\delta}_{\mathbb{R}^+}(r)} \\ f^{-1}(\hat{\delta}_{\mathbb{R}^+}(r)): & S_{\hat{\delta}_{\mathbb{R}^+}(r)} & \to & S \end{array} \tag{6}$$

so S and $S_{\hat{\delta}_{\mathbb{R}^+}(r)}$ remain locally iso-morphic as $r \to \infty_{\mathbb{R}^+}$. Note that the 2π circumference scaling of eq. (4) is arbitrary for this example, as we can make eq. (4) anything we want as long as the resulting iso-unit $\hat{t}$ is positive-definite and varies continuously as $r \to \infty_{\mathbb{R}^+}$.

6. **Discrete dynamic iso-space.** Second, we will show that the $S_{\hat{\delta}(r)}$ of eq. (3) can be rewritten and defined as a *discrete* dynamic iso-space if its corresponding dynamic iso-unit function is discrete as its parameter r varies; for this, the constraint is that r must take on the values of an infinite discrete sequence of positive-definite numbers. Hence, for example, if $\mathbb{N}$ is the set of all natural numbers (positive integers), then let $r \in \mathbb{N}$ be the positive-definite, natural-valued, varying parameter for the *discrete dynamic iso-unit function*

$$\hat{t} \equiv \hat{\delta}_{\mathbb{N}}(r) \equiv 2\pi r > 0, \; r \in \mathbb{N}, \; r \to \infty_{\mathbb{N}}, \tag{7}$$

to consequently define the discrete dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}_{\mathbb{N}}(r)}, \tag{8}$$

where we rewrite eq. (1) to encompass the discrete dynamic structure of eqs. (7–8) as

$$\begin{array}{rlcl} f(\hat{\delta}_{\mathbb{N}}(r)): & S & \to & S_{\hat{\delta}_{\mathbb{N}}(r)} \\ f^{-1}(\hat{\delta}_{\mathbb{N}}(r)): & S_{\hat{\delta}_{\mathbb{N}}(r)} & \to & S \end{array} \tag{9}$$

so S and $S_{\hat{\delta}_{\mathbb{N}}(r)}$ remain locally iso-morphic as $r \to \infty_{\mathbb{N}}$. Again, note that the 2π circumference scaling of eq. (7) is arbitrary for this example, as we can make eq. (7) anything we want as long as the resulting iso-unit $\hat{t}$ is positive-definite and varies discretely as $r \to \infty_{\mathbb{N}}$.

At this point, the results of eqs. (1–9) indicate the existence of a new family of iso-spaces called dynamic iso-spaces, where a member of this family (continuously or discretely) deforms in accordance to Santilli's topological-preservations as r varies. Hence, we establish the following:

Lemma 1. *A space S is locally iso-morphic to the dynamic iso-space $S_{\hat{\delta}(r)}$ under the topologically-preserving dynamic iso-topic lifting $S \to S_{\hat{\delta}(r)}$ for the dynamic iso-unit function $\hat{\delta}(r) > 0$ with inverse $\hat{\delta}^{-1}(r) = \frac{1}{\hat{\delta}(r)} > 0$, where the positive-definite parameter r varies indefinitely.*

Lemma 2. *A dynamic iso-space $S_{\hat{\delta}(r)}$ is a continuous dynamic iso-space if the dynamic iso-unit function $\hat{\delta}(r)$ is continuous as its parameter r varies.*

Lemma 3. *A dynamic iso-space $S_{\hat{\delta}(r)}$ is a discrete dynamic iso-space if the dynamic iso-unit function $\hat{\delta}(r)$ is discrete as its parameter r varies.*

3 Application Examples

Here, we provide two short examples that engage the dynamic iso-space concept. For this, we opt that both applications include fractals.

3.1 Fibonacci dynamic iso-space

First, we introduce and define a Fibonacci dynamic iso-space, which is a specific type of discrete dynamic iso-space. The well-known Fibonacci sequence is [7]

$$F = \{0, 1, 1, 2, 3, 5, 8, ...\}. \tag{10}$$

Hence, we identify the nth Fibonacci number in the sequence F with the "Fibonacci function" $F(n)$, which returns the initial values

$$\begin{array}{l} F(0) = 0,\ F(1) = 1,\ F(2) = 1,\ F(3) = 2,\ F(4) = 3, \\ F(5) = 5,\ F(6) = 8,\ ... \ . \end{array} \tag{11}$$

Now suppose that we begin with the positive-definite $r = F(1)$ and let $r \in F - \{0\}$ discretely varies or "jumps" according to Fibonacci's eqs. (10–11)—so in this Fibonacci case, as $r \to \infty_{Fib}$, r takes on only Fibonacci numbers (excluding zero), which satisfies the discrete iso-unit constraints that $F - \{0\}$ must be an infinite discrete sequence positive-definite numbers. Thus, the discrete dynamic iso-unit function of eq. (7) can be rewritten to define the *Fibonacci dynamic iso-unit function* as

$$\hat{t} \equiv \hat{\delta}_{Fib}(r) \equiv 2\pi r > 0,\ r \in F - \{0\},\ r \to \infty_{Fib}, \tag{12}$$

to consequently define the Fibonacci dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}_{Fib}(r)} \tag{13}$$

with the Fibonacci dynamic iso-topic liftings

$$\begin{array}{rlcl} f(\hat{\delta}_{Fib}(r)) : & S & \to & S_{\hat{\delta}_{Fib}(r)} \\ f^{-1}(\hat{\delta}_{Fib}(r)) : & S_{\hat{\delta}_{Fib}(r)} & \to & S. \end{array} \tag{14}$$

Here, we observe that $\hat{t}$ strictly acquires "Fibonacci circumference" values for the dynamic lifting because r is always treated as a "Fibonacci radius", which indicates that $S_{\hat{\delta}_{Fib}(r)}$ iteratively deforms as $r \to \infty_{Fib}$, where S and $S_{\hat{\delta}_{Fib}(r)}$ remain locally iso-morphic.

At this point, we've completed our first fractal example because we've successfully defined a Fibonacci dynamic iso-space.

3.2 Mandelbrot dynamic iso-space

Second, we introduce and define a Mandelbrot dynamic iso-space, which is a specific type of discrete dynamic iso-space because we obtain each value, one-by-one, as we systematically generate Mandelbrot's set by iterating his complex quadratic polynomial [8, 9]

$$z_{n+1} = z_n^2 + c, \tag{15}$$

where $z_n, z_{n+1}, c \in \mathbb{C}$ are complex numbers. Thus, imagine that we select some c and z_0 and continuously iterate eq. (15) to obtain the discrete Mandelbrot sequence

$$M = \{z_0, z_1, z_2, z_3, z_4, z_5, ...\}. \tag{16}$$

Hence, similarly to the Fibonacci case of eq. (11), we identify the nth Mandelbrot number in the sequence M with the "Mandelbrot function" $M(n)$ that depends on c, which returns the initial values

$$\begin{aligned} &M(0) = z_0,\ M(1) = z_1,\ M(2) = z_2,\ M(3) = z_3,\ M(4) = z_4, \\ &M(5) = z_5,\ \ldots\ . \end{aligned} \tag{17}$$

Now we need positive-definite values for our dynamic iso-units, so how do we translate the *complex-valued* (and possibly negative-valued) results of Mandelbrot's eq. (17) as we did to Fibonacci's eq. (11)? Well, there are multiple ways to approach this rich problem, so honestly at this point it is a matter of preference. Thus, for the sake of example simplicity, we opt to use the real-valued modulus

$$|z_n| = \sqrt{z_{n_{\mathbb{R}}}^2 + z_{n_{\mathbb{I}}}^2} \tag{18}$$

for the nth iterate of eq. (17) because it is positive-definite. Consequently, we list the moduli of eqs. (16) and (17) as

$$M_{Mod} = \{|z_0|, |z_1|, |z_2|, |z_3|, |z_4|, |z_5|, ...\} \tag{19}$$

and

$$\begin{array}{rcl} M_{Mod}(0) & = & |z_0|,\ M_{Mod}(1) = |z_1|,\ M_{Mod}(2) = |z_2|,\ M_{Mod}(3) = |z_3|, \\ M_{Mod}(4) & = & |z_4|,\ M_{Mod}(5) = |z_5|,\ ...\ , \end{array} \tag{20}$$

respectively.

Next, suppose that we begin with the positive-definite $r = M_{Mod}(0)$ and let $r \in M_{Mod}$ discretely vary according to Mandelbrot's moduli of eqs. (19–20)—so in this selected Mandelbrot case, as $r \to \infty_{Man}$, r takes on only the moduli of Mandelbrot numbers, which satisfies the discrete iso-unit constraints that M_{Mod} must be an infinite discrete sequence of positive-definite numbers. Thus, the discrete dynamic iso-unit function of eq. (7) can be rewritten to define the *Mandelbrot dynamic iso-unit function* as

$$\hat{t} \equiv \hat{\delta}_{Man}(r) \equiv 2\pi r > 0,\ r \in M_{Mod},\ r \to \infty_{Man}, \tag{21}$$

to consequently define the Mandelbrot dynamic iso-space

$$S_{\hat{t}} \equiv S_{\hat{\delta}_{Man}(r)} \tag{22}$$

with the Mandelbrot dynamic iso-topic liftings

$$\begin{array}{rlcl} f(\hat{\delta}_{Man}(r)): & S & \to & S_{\hat{\delta}_{Man}(r)} \\ f^{-1}(\hat{\delta}_{Man}(r)): & S_{\hat{\delta}_{Man}(r)} & \to & S. \end{array} \tag{23}$$

Here, we observe that $\hat{t}$ strictly acquires "Mandelbrot circumference" values for the dynamic lifting because r is always treated as a "Mandelbrot radius", which indicates that $S_{\hat{\delta}_{Man}(r)}$ iteratively deforms as $r \to \infty_{Man}$, where S and $S_{\hat{\delta}_{Man}(r)}$ remain locally iso-morphic.

At this point, we've completed our second fractal example because we've successfully defined one version of a Mandelbrot dynamic iso-space.

4 Conclusion

The results of this work include original definitions and lemmas for continuous and discrete dynamic iso-spaces, and example constructions of Fibonacci and Mandelbrot dynamic iso-spaces. These dynamic iso-spaces may serve as important tools for future analysis and exploration. But there is still much work to do, as we must continue to relentlessly scrutinize, challenge, and upgrade this emerging framework via the scientific method. In particular, we suggest that in order to test the validity of our results and advance the general capability and applicability of these dynamic iso-systems to subsequent levels, a thorough and rigorous iso-mathematical investigation should be conducted along this research trajectory. For this, we must prove the said lemmas and expand the framework by instantiating additional pertinent families of dynamic iso-structures, and furthermore the dynamic geno-structures, dynamic hyper-structures, and dynamic iso-dual-structures.

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